Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 231E – Engineering Calculus

Midterm 5 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Tutors: Nov. 8, 2-3:50pm Yash and Aditya

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/979
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Test the following improper integral for convergence. If it converges, find the value

$$\int_0^\infty e^{-\frac{4}{3}y} dy$$

(a) Diverges
(b) Converges, value is $-\frac{3}{4}$
(c) Converges, value is $\frac{4}{3}$
(d) Converges, value is $\frac{3}{4}$
(e) None of the above

2. For which value of C does the following series converge?

$$\sum_{n=3}^{\infty} \left(\frac{C}{n+2} - \frac{n}{3n^2+1} \right)$$

(a) It does not converge for any value of C.
(b) $\frac{1}{2}$
(c) 1
(d) $\frac{1}{3}$
(e) 0

3. If $a_n = 3 + \frac{\ln(n)^6}{\sqrt{n}}$, then the sequence a_n:

(a) Converges to 3.

(b) Diverges to infinity.

(c) Diverges, but not to infinity.

(d) Converges to 6.

(e) Converges to 4.

4. Determine if the following integral converges or diverges

$$\int_1^\infty \frac{x^3 + 12x - 2}{x^6 + 5x^5 + 3x^2 - 1} dx$$

5. Determine which of the following is true about the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

(a) Not enough information to make a determination

(b) The series converges to $\frac{2}{5}$

(c) The series diverges

(d) The series converges absolutely

(e) The series converges only conditionally

6. Find the sum of the geometric series:

$$12 + 12\left(\frac{7}{19}\right) + 12\left(\frac{7}{19}\right)^2 + 12\left(\frac{7}{19}\right)^3 ...$$

(a) 20

(b) 8

(c) 19

(d) 14

(e) $\frac{7}{19}$

7. Find the sum of the series:

$$\sum_{n=0}^{\infty} \frac{\cos^n(x)}{3^n}$$

(a) $\frac{3}{3-\cos(x)}$

(b) $\frac{1}{3-\cos(x)}$

(c) $\frac{2}{3-\cos(x)}$

(d) $\frac{3}{3-\sin(x)}$

(e) $\frac{1}{3-\sin(x)}$

8. Determine the value of the series, if it converges:

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

(a) 64

(b) The series diverges

(c) 2

(d) 125

(e) 25

9. Does the series converge or diverge?

$$\sum_{n=1}^{\infty} e^{\frac{3}{n}}$$

10. Determine whether the sequence

$$a_n = \left(\frac{n+1}{n+3}\right)^{2n}$$

converges or diverges. If it converges, find its limit.

(a) 0

(b) e^{-2}

(c) e^{-4}

(d) 1

(e) The sequence diverges.

11. Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+1}$$

converges absolutely, converges conditionally, or diverges.

(a) It converges absolutely by the ratio test.

(b) It converges conditionally.

(c) It diverges because its terms do not approach zero.

(d) It converges absolutely by comparison with $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

(e) It diverges because the magnitudes of its terms increase.

12. Use the ratio test to analyze the convergence of

$$\sum_{n=1}^{\infty}(-1)^{n}\frac{3^n n!}{n^n}.$$

Which option gives the correct value of

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

and the corresponding conclusion?

(a) $L=0$; the series converges absolutely.

(b) $L=1$; the test is inconclusive.

(c) $L=e/3$; the series converges absolutely.

(d) $L=3/e$; the series diverges.

(e) $L=1/3$; the series converges absolutely.

13. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n\,3^n}.$$

(a) $R = 3, \quad I = [-1, 5)$
(b) $R = 3, \quad I = (-1, 5]$
(c) $R = 3, \quad I = (-1, 5)$
(d) $R = 3, \quad I = [-1, 5]$
(e) $R = 1, \quad I = (1, 3)$

14. Find a power series representation for

$$f(x) = \frac{1}{3-x}$$

centered at $x = 0$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3^n}$
(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^{n+1}}$
(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$
(d) $\displaystyle\sum_{n=0}^{\infty} 3^n x^n$
(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{3^{n+1}}$

15. By differentiating a geometric series, find a power series representation for

$$f(x) = \frac{1}{(2-x)^2}$$

centered at $x = 0$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)x^n}{2^{n+1}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)x^n}{2^{n+2}}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^{n+2}}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)x^n}{2^{n+2}}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{nx^n}{2^{n+2}}$

16. By integrating a geometric series, find a power series representation for

$$f(x) = \ln(1+2x)$$

centered at $x = 0$.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}(2x)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2x)^n}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2x)^n}{n+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2x)^n}{n}$