Exam 4 Review Session
Math 231E



Please join the queue for attendance!

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Outline

1. Please join the queue
2. Mini review of some topics covered
3. Practice! → CARE Worksheet, Practice Exams
   a. Please raise hands for questions rather than put them



Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	6pm-9pm	12pm-2pm	3pm-8pm			1pm-3pm	

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Applications

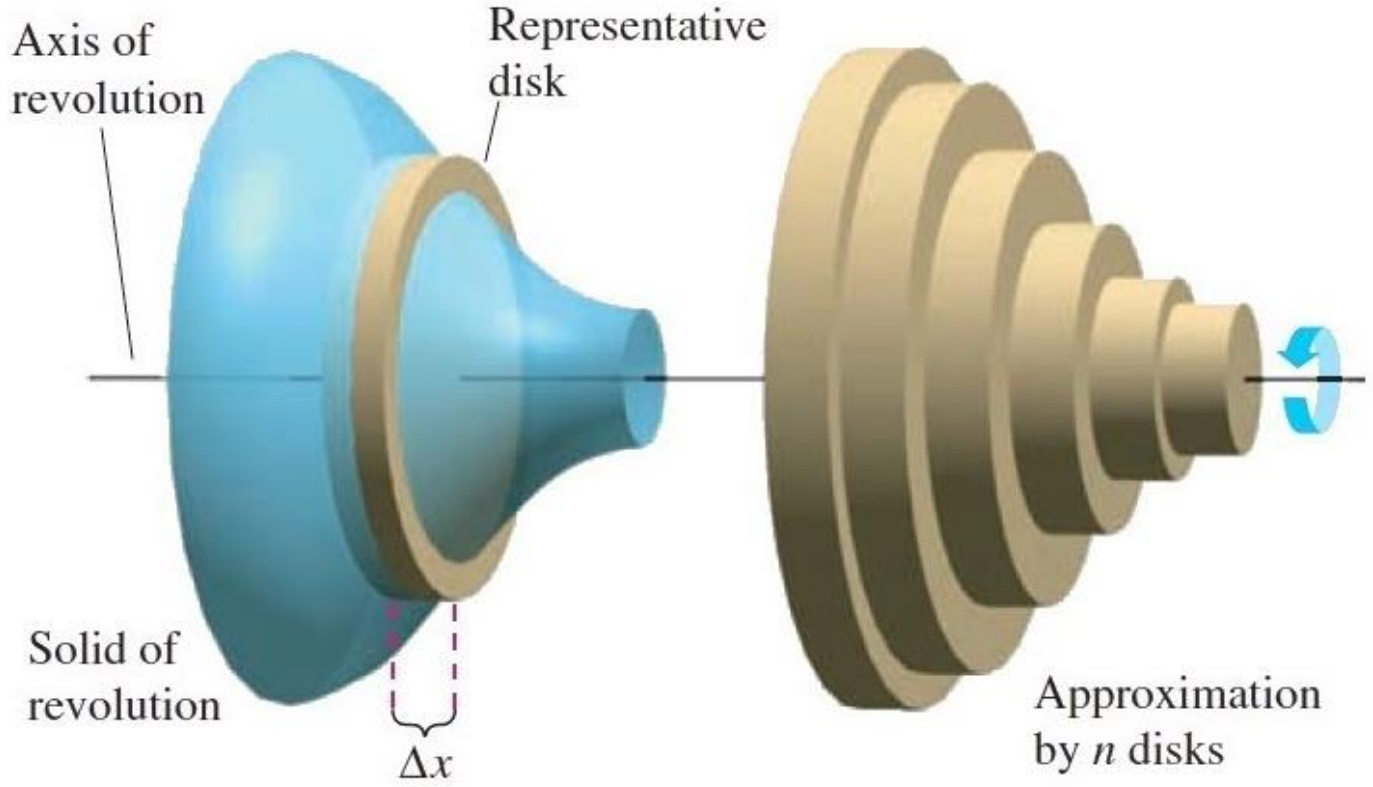
- Volume

Disk Method

- Using the formula for area of a circle
  - Putting it in the integral adds each circle in the bounds

$$V = \int_a^b \pi r^2 dx$$

Where the $r = f(x)$



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Applications

- Volume

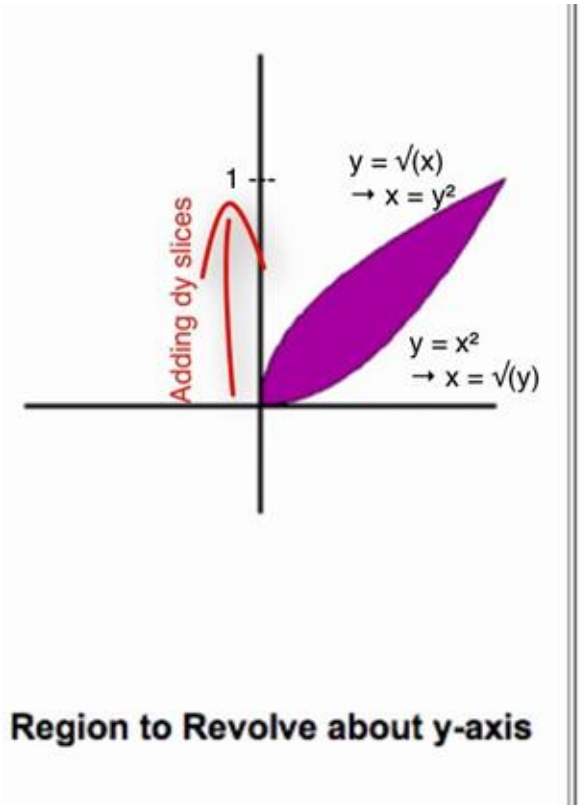
Washer Method

- Must subtract big function minus small function to find the in between region

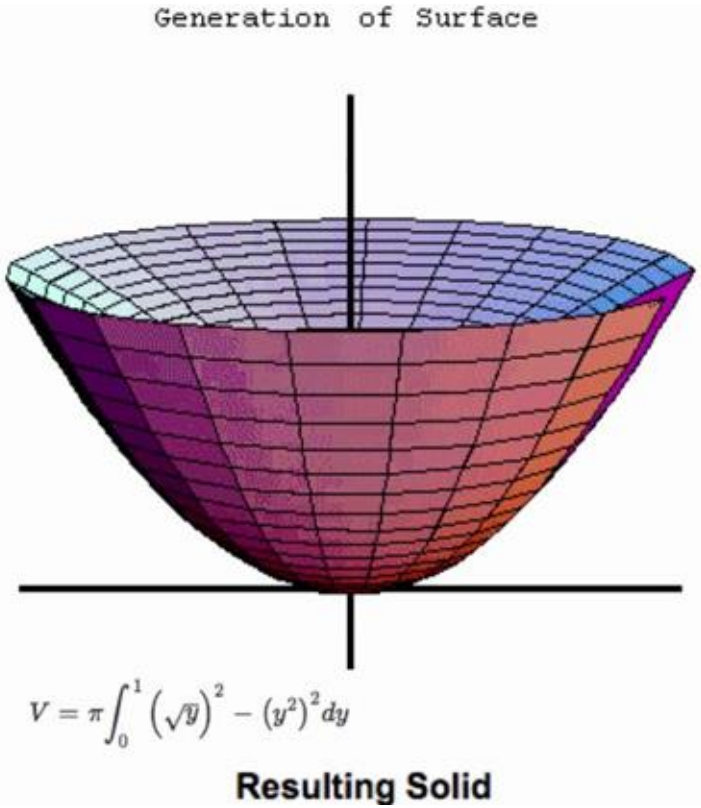
$$(Top - Bottom)$$
$$(Right - Left)$$

$$V = \int_a^b \pi(R^2 - r^2)dx$$

Where R $= f(x)$
$r = g\,(x)$
$f(x) > g(x)$



Region to Revolve about y-axis



Resulting Solid

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Applications

- Arclength ("height"):

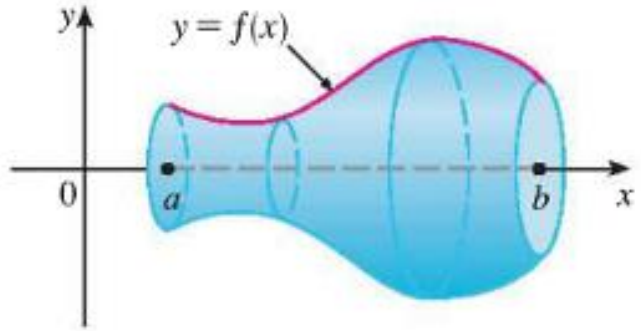
$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \text{ if } y = f(x)$$

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \text{ if x} = h(y)$$

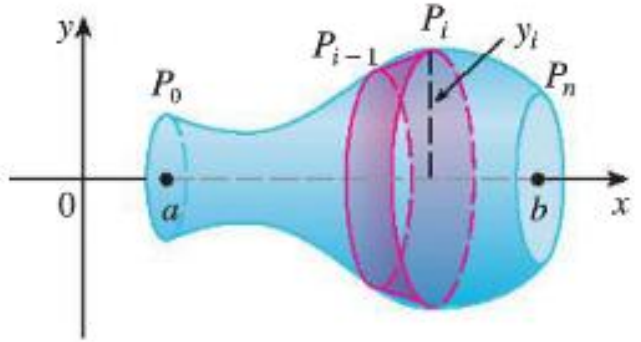
- Surface of Revolution:

$$SA = \int 2\pi y ds \text{ rotation about x-axis}$$

$$SA = \int 2\pi x ds \text{ rotation about y-axis}$$



(a) Surface of revolution



(b) Approximating band

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Applications Example

How to set up the surface area equation when rotating $y = \sqrt{9 - x^2}$ about the y-axis?

- Because about y-axis, using $SA = \int 2\pi x ds$
  - We need x and ds

To get x, rearrange the given equation

$$y = \sqrt{9 - x^2}$$
$$y^2 = 9 - x^2$$
$$x^2 = 9 - y^2$$
$$x = \sqrt{9 - y^2}$$

Substituting...

$$SA = \int 2\pi\sqrt{9 - y^2} ds$$

To get ds, use the equation for ds that gives us a dy (since x is in terms of y)

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \text{ if x} = h(y)$$

Derivative of x...

$$\frac{dx}{dy} = \frac{-y}{\sqrt{9-y^2}}$$

Plug back in...

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Work

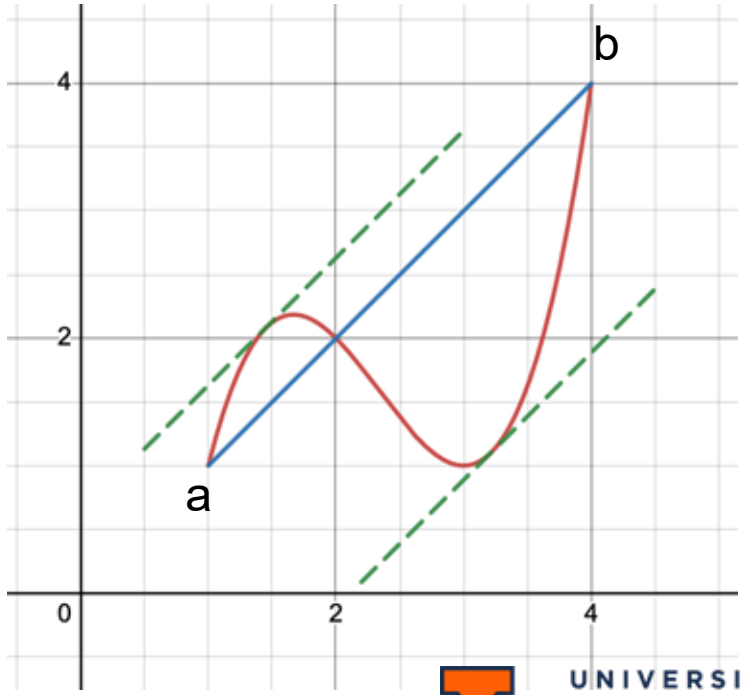
- Work: Force over a distance

$$W = \int F(x)dx$$

  - If the force is not constant.

- Average Value of a function over an interval

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$



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Series / Sequences

- **Sequence:** Just the list of the numbers
  - Limits of sequences
    - Treat it like a function
  - Convergence
    - Treat it like a function
    - Derivative can tell you if it is always increasing or decreasing
- **Series**: The sum of a sequence
  - If a series converges, then the sequence must converge as well.
  - **However:** If sequence converges, then the series may or may not converge.
  - Σa_n converges if the limit of the series converges.

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Integral Test

- Let an = f(n):
  - $\int f(x)dx$ (from k to infinity) converges if the series converges (Σa_k).

    Must be:
    - Continuous
    - Positive
    - Decreasing

1. If $\int_k^\infty f(x)\,dx$ is convergent so is $\sum_{n=k}^{\infty} a_n$.

2. If $\int_k^\infty f(x)\,dx$ is divergent so is $\sum_{n=k}^{\infty} a_n$.

- P-test:
  - The series $\Sigma \frac{1}{n^p}$ converges if p > 1 and diverges if p ≤ 1.

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Good luck on your exam!

You can use the rest of the time to practice



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