Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 285 – Intro Differential Equations

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: October 20 from 6:30-7:50pm in 4039 CIF - Sarah and Rodrigo

Session 2: October 21 from 6:30-7:50pm in 4039 CIF - Amy and Alex

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/846
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. The differential equation $mu'' + 4u' + 8u = 0$ describes a mass-spring system. For what values of m is the system underdamped?

   A) $m < \frac{1}{2}$
   B) $m < 2$
   C) $m > \frac{1}{2}$
   D) $m > 2$

2. For what forcing frequency ω is it possible for the solution to $18y''+2y = 81\cos(\omega t)$ to experience resonance?

   A) $\omega = 3$
   B) $\omega = 9$
   C) $\omega = \frac{1}{3}$
   D) $\omega = \frac{1}{9}$

3. What is the correct expression for the characteristic solution of the following ODE?

$$y'' - 8y' + 17y = 0$$

   A) $y(t) = e^{-4t}[c_1 cos(t) + c_2 sin(t)]$
   B) $y(t) = e^{4t}[c_1 cos(t) + c_2 sin(t)]$
   C) $y(t) = e^{t}[c_1 cos(4t) + c_2 sin(4t)]$
   D) $y(t) = e^{-t}[c_1 cos(-4t) + c_2 sin(-4t)]$

4. Consider the following statements

(i) If Y_1 is a particular solution to the DE

$$y'' + p(t)y' + q(t)y = g(t)$$

then $Y_2 = cY_1$ is also a solution to the differential equation, where c is a constant

(ii) If y_1 and y_2 are solutions to

$$ay'' + by' + cy = g(t)$$

then y_1 - y_2 is a solution to the homogeneous differential equation $ay'' + by' + cy = 0$ where a, b, and c are constants.

(iii) The solution to

$$ay'' + by' + cy = 0$$

with initial conditions $y(0) = A$ and $y'(0) = B$, is unique on $t \in (-\infty, \infty)$ where a, b, and c are constants.

Which of the above statements are always true?

A) (i) and (ii)

B) (i) and (iii)

C) (ii) and (iii)

D) None

5. Consider the following initial value problem $(t-5)y'' + \csc(t)y' + y = e^t$, $y'(4) = 1$ and $y(4) = \pi$. What is the largest interval on which the initial value problem is guaranteed to exist?

A) (0, 5)

B) (0, 2π)

C) (π, 5)

D) (π, 2π)

6. Identify the form of the particular solution for $y'' - 16y = (t-3)e^{-4t} + (4t+3)\sin(2t)$

A) $(At+B)te^{-4t} + (Ct+D)t\sin(2t) + (Et+F)t\cos(2t)$

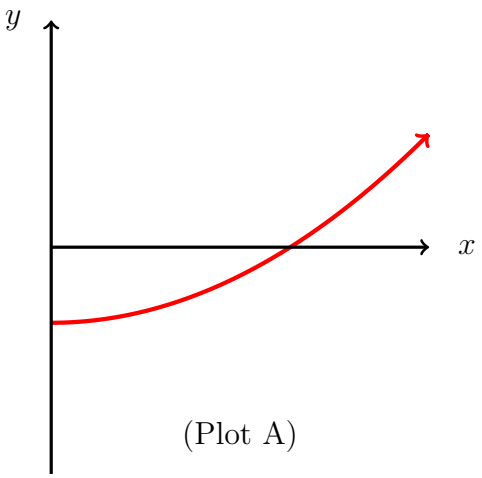
B) $(At+B)e^{-4t} + (Ct+D)\sin(2t) + (Et+F)\cos(2t)$

C) $(At+B)e^{-4t} + (Ct+D)t\sin(2t) + (Et+F)t\cos(2t)$

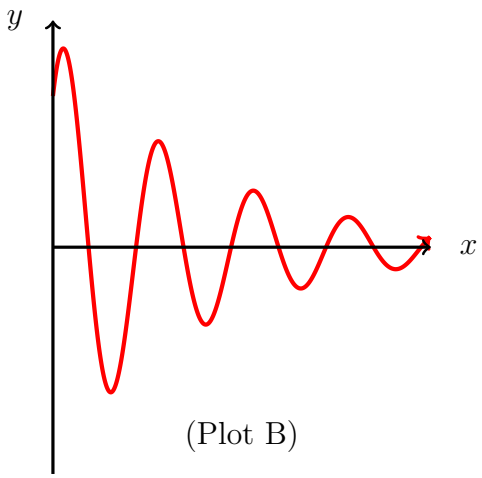
D) $(At+B)te^{-4t} + (Ct+D)\sin(2t) + (Et+F)\cos(2t)$

7. Find the general solution for the differential equation $y'' + 4y' + 4y = 0$

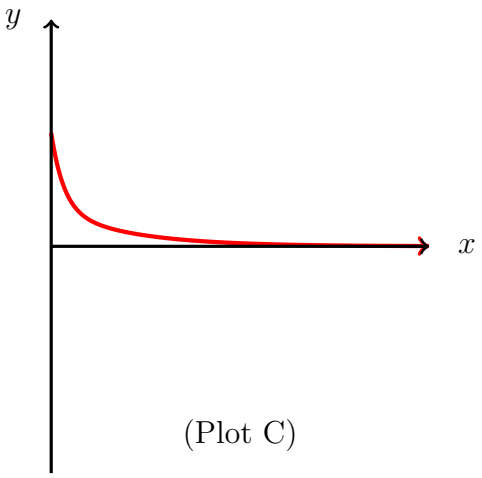
8. Which of the following plots represents a solution to the ODE $y'' + 7y' + 6y = 0$?



(Plot A)



(Plot B)



(Plot C)

A) Plot A
B) Plot B
C) Plot C
D) None of the Above

9. Use the method of undetermined coefficients to find the general solution to the following ODE:

$$y'' - 4y = e^{-2t}$$

10. Find the solution $y(t)$ for the following ODE with a Laplace transformation. Assume all initial conditions are zero.

$$3y' + 4y = t$$

11. Find the general solution to the following ODE using variation of parameters.

$$y'' + 2y' + y = e^{-t}$$

12. Calculate the Wronskian of $y_1(t) = e^{-3t^2-3t+6}$ and $y_2(t) = e^{-3t^2-3t-3}$ and explain if your result shows linear independence, linear dependance, or neither: