Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 241 – Calculus III

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Oct. 17, 1-3 PM Andy, Aditya, Francesco

Session 2: Oct. 18, 1-3 PM Gabe, Tommy, Vedansh

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/845
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Consider the vector function $\vec{r}(t) = \langle 2\cos t, 2\sin t, t\rangle$. Find the length from $(2, 0, 0)$ to $(2, 0, 4\pi)$ of the vector function.

2. A tiny spaceship is orbiting a path given by $x^2 + y^2 = 4$. The solar radiation at a point (x, y) in the plane of the orbit is $f(x, y) = xy + 2y$.

   Use the method of *Lagrange multipliers* to find the maximum value and minimum value of solar radiation experienced by the tiny spaceship in its orbit.

3. Let $f(x, y)$ be a differentiable function on the disk $\{D : x^2 + y^2 \leq 400\}$, where:

(I) $f(x, y) = 19$ for every point on the boundary of the disk $x^2 + y^2 = 400$

(II) $f(0, 0) = 7$

(III) $f(x, y)$ has only one critical point which is at $(-1, 2)$

Decide which statement is true:

A) $f(-1, 2) > 7$

B) $f(-1, 2) < 7$

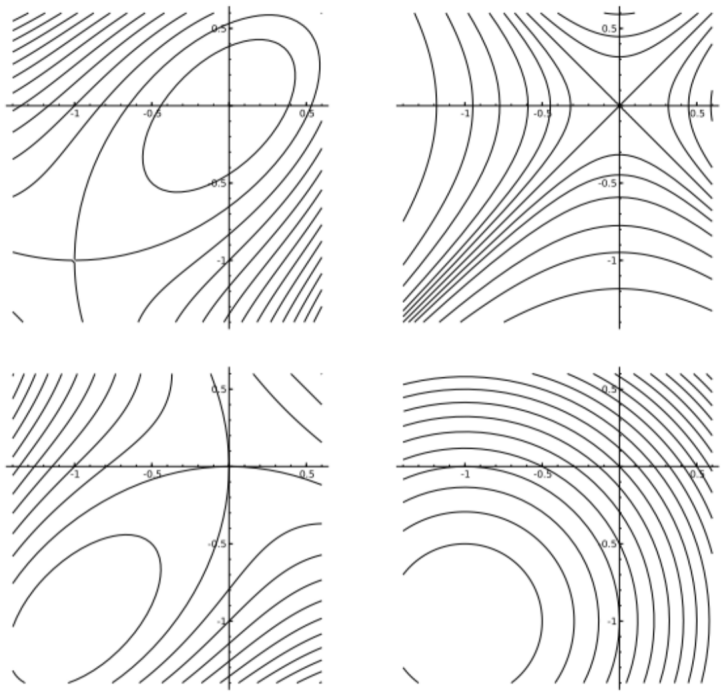
C) $f(-1, 2) = 7$

D) Not enough information is given

4. Consider the function $f(x, y) = x^3 + y^3 + 3xy$

(a) The critical points of f are $(0, 0)$ and $(-1, -1)$. Classify them into local minima, local maxima and/or saddle points

(b) Based on your answer in (a), identify the correct contour diagram of f



5. Consider a function $f(t) = f(x(t), y(t), z(t)) = xyz - z^2$, where $x(t)$, $y(t)$, $z(t)$ are defined as followed:

$$x(t) = 2t^2 + 1 \qquad y(t) = 3 - \frac{1}{t} \qquad z(t) = 3$$

Find the following values:

(a) $f_z(3, 1, 2)$ (b) $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t=0)}$ (c) $\left.\frac{df}{dt}\right|_{(t=1)}$

6. If $f(x, y, z) = xye^z$, find the gradient of f and the directional derivative at (2,5,0) in the direction of $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$.

7. Find min/max of f(x,y,z) = $3x^2 + 8y^2 + z^2 - 2z$ defined on the domain $x^2 + 4y^2 + 2z \leq 8$ and $z \geq 0$

(a) The domain is (select all that apply)

I) open
II) closed
III) bounded
IV) unbounded

(b) Where are the critical points inside the domain? Evaluate the function value on these points.

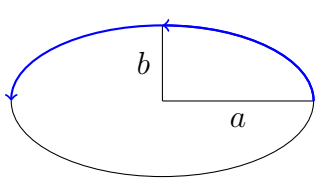
(c) What is the minimum and maximum on $x^2 + 4y^2 + 2z = 8$?

(d) What is the minimum and maximum on $z = 0$?

(e) What is the global minimum and maximum of the whole domain?

8. The vector field $\vec{F} = \langle 2xy + 2x + y^2, 2xy + 2y + x^2 \rangle$ is conservative. Find a potential function f for $\vec{F}$ (a function with $\nabla f = \vec{F}$)

9. A particle moves along the upper part of an ellipse in the xy-plane that has its center at the origin with semi-major and semi-minor axes $a = 4$ and $b = 3$, respectively. Starting at $(a, 0)$ and ending at $(-a, 0)$ and subject to the following force field, what is the total work done?



$$\vec{F} = (3x - 4y + 2z)\hat{\imath} + (4x + 2y - 3z^2)\hat{\jmath} + (2xz - 4y^2 + z^3)\hat{k}$$

10. Evaluate $\int_C \nabla f \cdot d\vec{r}$ where $f(x,y) = ye^{x^2-1} + 4x\sqrt{y}$ and C is given by $\vec{r}(t) = \langle 1-t, 2t^2-2t \rangle$ with $0 \leq t \leq 2$.