

Exam 3 Review Session

Math 231E



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queue for
attendance!



UNIVERSITY OF
ILLINOIS
URBANA-CHAMPAIGN

Outline

- 1. Please join the queue
- 2. Mini review of some topics covered
- 3. Practice! → CARE Worksheet, Practice Exams
 - a. Please raise hands for questions rather than put them



Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	6pm-9pm	12pm-2pm	3pm-8pm			1pm-3pm	

Partial Fractions

- If you have a rational function with a polynomial in the denominator ...

Rational Function	Partial Fraction
$\frac{px + q}{(x - a)(x - b)}, a \neq b$	$\frac{A}{(x - a)} + \frac{B}{(x - b)}$
$\frac{px + q}{(x - a)^2}$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2}$
$\frac{px^2 + qx + r}{(x - a)(x - b)(x - c)}$	$\frac{A}{(x - a)} + \frac{B}{(x - b)} + \frac{C}{(x - c)}$
$\frac{px^2 + qx + r}{(x - a)^2(x - b)}$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2} + \frac{C}{(x - b)}$
$\frac{px^2 + qx + r}{(x - a)(x^2 - bx - c)}$	$\frac{A}{(x - a)} + \frac{Bx + C}{(x^2 - bx - c)}$

... from the original numerator you can solve for the A, B, C, etc.



Trigonometric Integrals

- If you have an integrand with trig functions, you can use known trig identities to simplify the integral and use another method if needed

$$\cos^2(x) + \sin^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\tan^2(x) = \sec^2(x) - 1$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\sin^2 x = \frac{1}{2} (1 - \cos(2x))$$

$$\cos^2 x = \frac{1}{2} (1 + \cos(2x))$$

$$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 1 - 2\sin^2(x) \\ &= 2\cos^2(x) - 1\end{aligned}$$

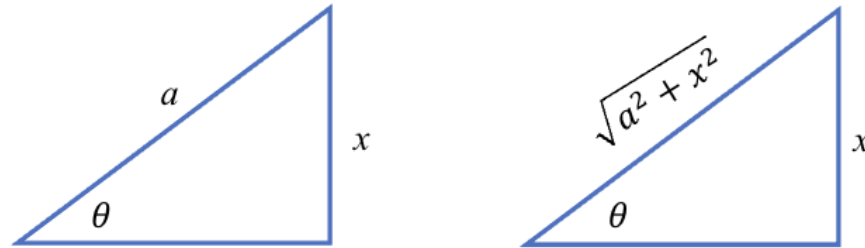


Trigonometric Substitution

- If you notice a complicated function inside of a square root ...

Format	Substitution	Derivative Substitution	Trig Identity
$\sqrt{a^2 - x^2}$	$x = a * \sin(\theta)$	$dx = a * \cos(\theta) d\theta$	$\cos^2(\theta) + \sin^2(\theta) = 1$
$\sqrt{a^2 + x^2}$	$x = a * \tan(\theta)$	$dx = a * \sec^2(\theta) d\theta$	$\tan^2(\theta) + 1 = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a * \sec(\theta)$	$dx = a * \sec(\theta) \tan(\theta) d\theta$	$\tan^2(\theta) = \sec^2(\theta) - 1$

... you can substitute a known trig identity and solve the integral...



... and can convert back using a triangle



Partial Fractions

- If you have a rational function with a polynomial in the denominator ...

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$\frac{px + q}{(x - a)(x - b)}, a \neq b$	$\frac{A}{(x - a)} + \frac{B}{(x - b)}$
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... from the original numerator you can solve for the A,B,C, etc.



Improper Integrals

- Improper Integrals: FTC does not hold since functions are **not continuous along the interval of integration**.
- Type I: Infinite Interval

$$\int_a^{\infty} f(x)dx = \lim_{t \rightarrow \infty} \int_a^t f(x)dx$$
$$\int_{-\infty}^b f(x)dx = \lim_{t \rightarrow -\infty} \int_t^b f(x)dx$$

- Type II: Discontinuous Interval

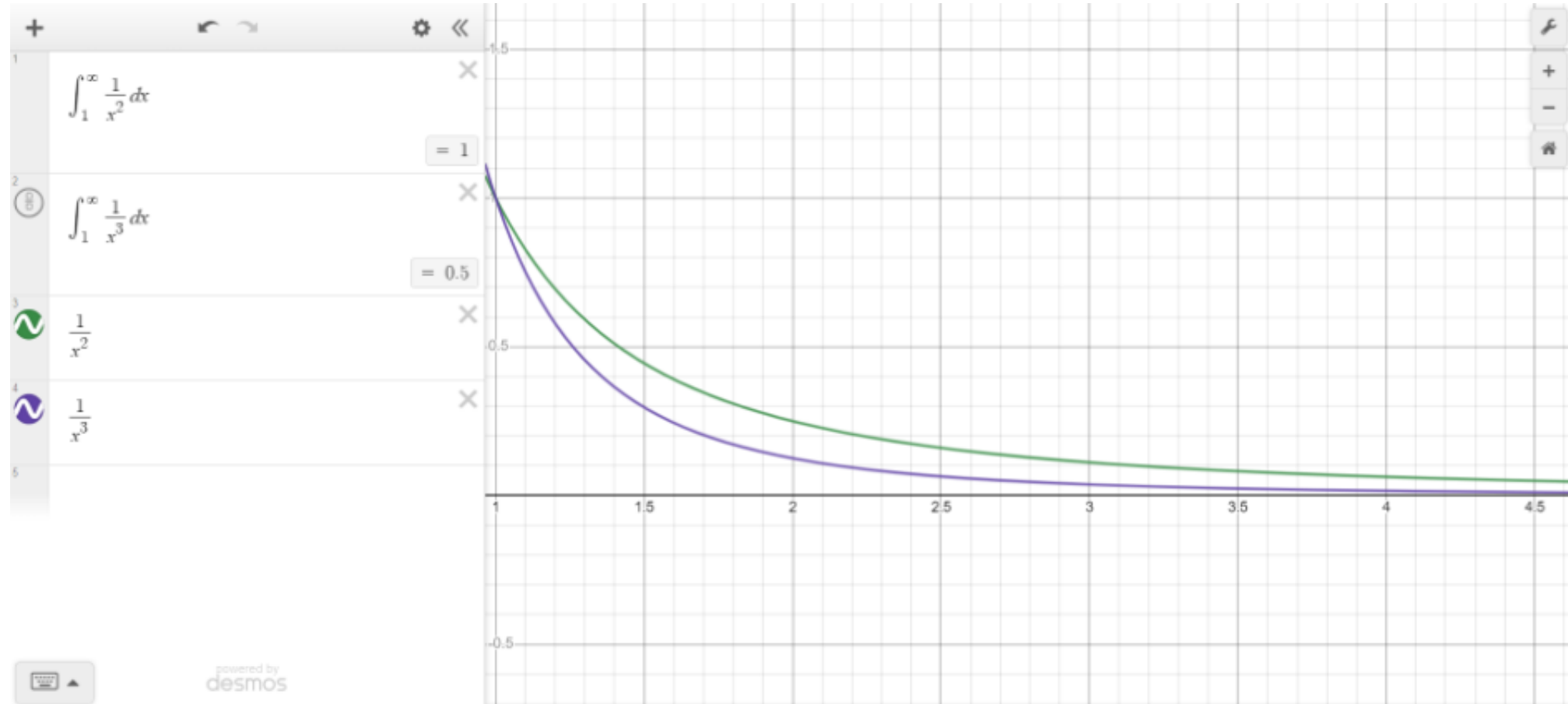
$$\int_{a_b}^b f(x)dx = \lim_{t \rightarrow b^-} \int_{a_b}^t f(x)dx$$
$$\int_a^{a_b} f(x)dx = \lim_{t \rightarrow a^+} \int_t^{a_b} f(x)dx$$
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$



Improper Integrals

- Comparison Principal: If given two functions $g(x)$ and $h(x)$ and we want to take the integral to infinity, **and we know that $g(x)$ is always smaller than $h(x)$** , then:
 - If $g(x)$ diverges, then $h(x)$ must as well.
 - If $h(x)$ converges, then $g(x)$ must as well.

Improper Integrals



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