

Exam 2 Review Session

Math 231E



Please join the queue
for attendance!



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Outline

1. Please join the queue
2. Mini review of some topics covered
3. Practice! → CARE Worksheet, Practice Exams
 - a. Please raise hands for questions rather than put them in the queue



Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	6pm-9pm	12pm-2pm	3pm-8pm			1pm-3pm	



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Content Review



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Derivatives

- Interpretation: the derivative of a function $f(x)$ at a represents the instantaneous rate of change at a

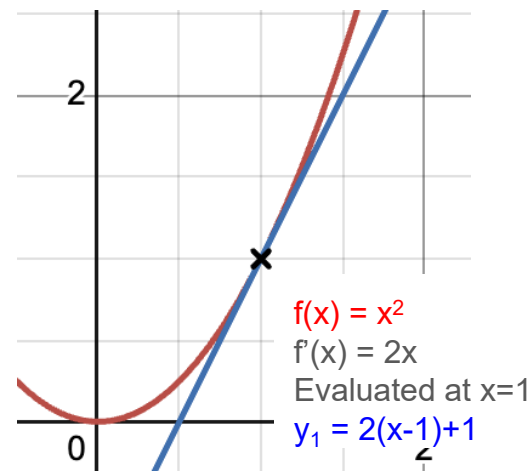
- Slope of tangent line at a : $y_a = f'(a)(x - a) + f(a)$

- Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$

- Product Rule: $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

- Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

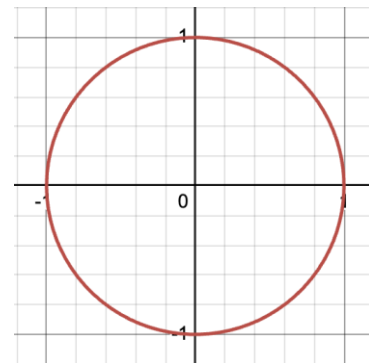
- **Chain Rule:** $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$ (unwrap the layers of the function)



Implicit Differentiation and Logarithmic Differentiation

Implicit Differentiation

- Used when functions are defined in terms of both x and y
- Take the derivative of everything with respect to x
 - What is $\frac{dx}{dx}$? What is $\frac{dy}{dx}$?



Logarithmic Differentiation

- Used to deal with functions with functions of x to the power of x

$$\begin{aligned}y &= x^x \\ \ln y &= x \ln x \\ \frac{y'}{y} &= \ln x + 1 \\ y' &= x^x (\ln x + 1)\end{aligned}$$



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Related rates and differentials

Related Rates

- Relate quantities $\rightarrow$ differentiate in $t \rightarrow$ substitute.
- Keep units and signs: increasing (+), decreasing (-).

For a spherical
balloon:

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

At $r = 10$ cm and $dV/dt = 50$ cm³/s:

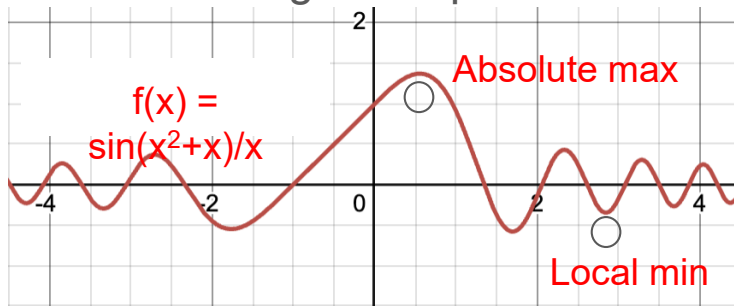
$$\frac{dr}{dt} = \frac{1}{8\pi} \text{ cm/s}$$

Differentials

- For a small change in input Δx , the function will change proportionally to the rate of change:

Minima and Maxima

- Absolute minima/maxima: smallest/largest outputs a function can produce on an interval
- Local minima/maxima: smallest/largest outputs of a function around a certain point

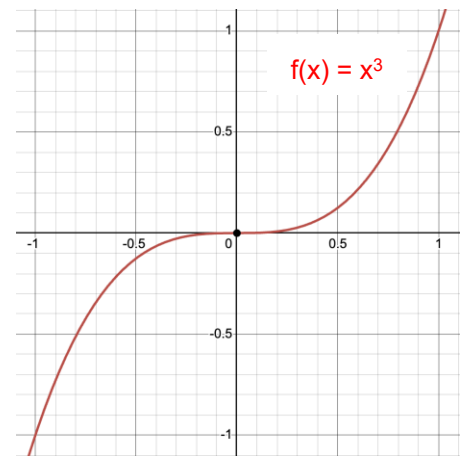


- Extreme Value Theorem: If $f(x)$ is continuous on $[a,b]$, then f achieves both an **absolute maximum and absolute minimum** on $[a,b]$
- Fermat's Theorem: If $f(x)$ has a **local minimum/maximum** at c and f' exists at c , then $f'(c) = 0$
 - We call c a critical point



Finding Extreme Values

1. Take the first derivative and set it equal to zero
 - a. Solve for all the critical points



1. Plug each critical point back into $f(x)$ and see which is the largest/smallest

-or-

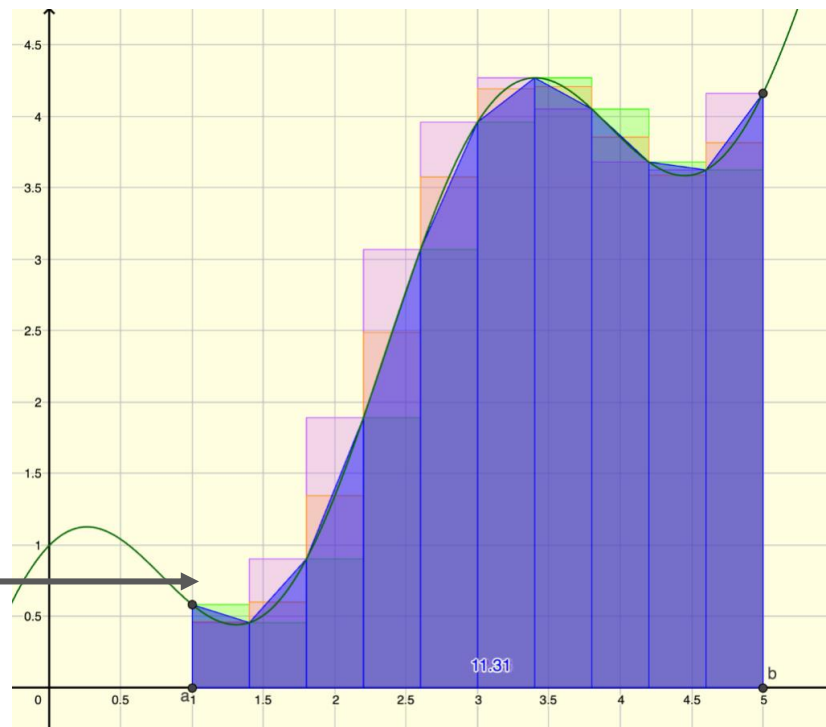
2. Take the second derivative
 - a. If $f''(c)$ is $> 0 \rightarrow$ function is concave up $\rightarrow$ local minimum
 - b. If $f''(c)$ is $< 0 \rightarrow$ function is concave down $\rightarrow$ local maximum
 - c. If $f''(c) = 0 \rightarrow$ inconclusive



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Riemann Sums

- Approximation for the area under a curve
- General form: $\sum_{x=a}^b f(x_k^*) \Delta x$
 - $\Delta x = \frac{(b-a)}{n} \rightarrow$ difference of the two values divided by the number partitions
 - $x_k^* = a + n\Delta x \rightarrow$ the leftmost value plus the difference times the number partition you are on
- Types
 - Left endpoint
 - Right endpoint
 - Midpoint
 - Trapezoidal



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Integrals and the Fundamental Theorem of Calculus

- What exactly is an integral? → Riemann sum where each “rectangle” has an infinitely small width

$$\lim_{n \rightarrow \infty} \sum_{x=a}^b f(x_k^*) \Delta x = \int_a^b f(x) dx$$

- Representations: area under a curve, accumulation of change

- Fundamental Theorem of Calculus: $\int_a^b f'(x) dx = f(b) - f(a)$

- An integral “undoes” differentiation → the resulting function from integrating is an antiderivative



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Known Antiderivatives

- For some common functions the antiderivative is straightforward:

Constant: $\int A dx = Ax + C$

Exponential: $\int e^x dx = e^x + C$

Power rule: $\int x^n dx = \frac{x^{n+1}}{(n+1)} + C$

Sin(x): $\int \sin(x) dx = -\cos(x) + C$

1/x: $\int \frac{1}{x} dx = \ln(x) + C$

Cos(x): $\int \cos(x) dx = \sin(x) + C$

Is the function I'm integrating a known derivative of another function?



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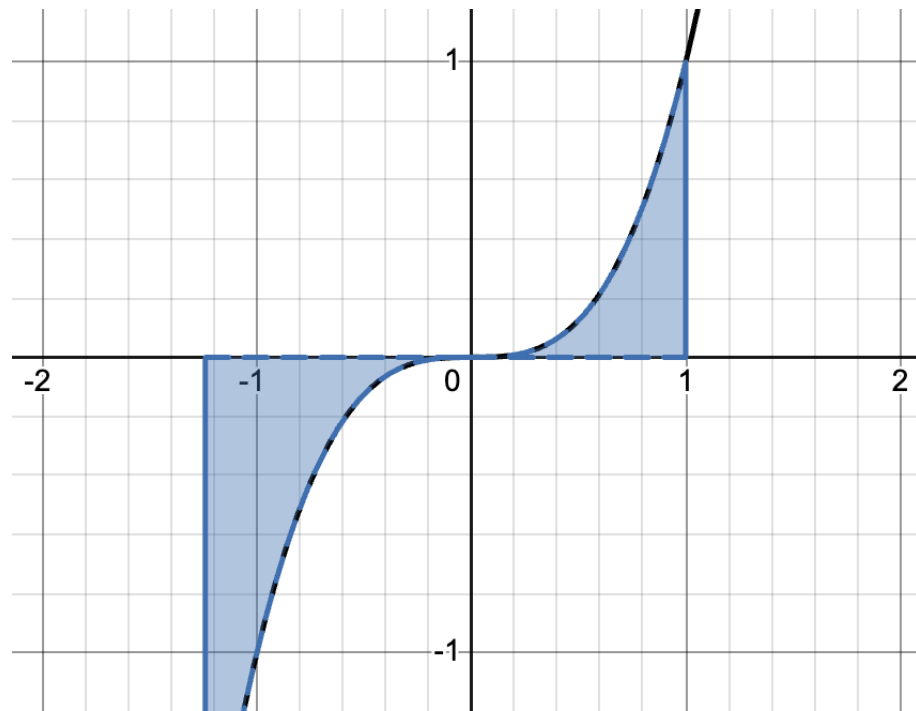
Integral Properties

$$1. \int C f(x) dx = C \int f(x)$$

$$2. \int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

$$3. \int_a^a f(x) dx = 0$$

$$4. \int_a^b f(x) dx = - \int_b^a f(x) dx$$



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Methods for Evaluating More Complicated Integrals



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U-Substitution

- If you notice one part of the integrand is the derivative of another part ...

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du$$

$$u = g(x)$$

$$du = g'(x) dx$$

...you can substitute a new variable “u” and take its derivative to plug back into the integral

Example: $\int \frac{x}{\sqrt{1-x^2}} dx$ $\begin{matrix} u = 1 - x^2 \\ du = -2x \end{matrix}$ $\longrightarrow$ $\int \frac{-1}{2\sqrt{u}} du$

Remember to put your answer back in terms of x



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Integration by Parts

- If you notice that two functions that are not derivatives of each other are multiplied together ...

$$\int u dv = uv - \int v du$$

...you can decompose the integrand such where ...

$$u = f(x)$$

$$v = \int g(x) dx$$

$$du = f'(x)$$

$$dv = g(x)$$

You may need to do multiple integration by parts to fully work through the integral!



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Integration by Parts → LIATE Method



L - logarithmic

I - inverse trigonometric

A - algebraic

T - trigonometric

E - exponential

u



dv

Examples

L - $\ln(x)$

I - $\sin^{-1}(x)$

A - $x^2 + 3x$

T - $\cos(x)$

E - e^x

You may need to do multiple integration by parts to fully work through the integral!



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Good luck on your exam!

You can use the rest of the time to practice



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