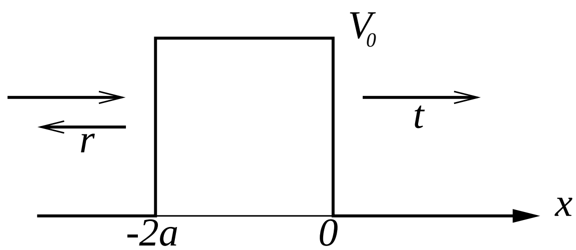
Q3: Consider the quantum mechanics of an electron of mass m experiencing the potential barrier shown in the figure.



For $x < -2a$, the potential is zero. For $-2a < x < 0$ the potential is V_0 and for $x > 0$ the potential is again zero. The electron is incident on the barrier from the left. You are going to compute the reflection and transmission coefficients for the case that the energy E of the incoming electron is *less* than the height of the barrier V_0. The wavefunction takes the form

$$\psi(x) = \begin{cases} e^{ikx} + re^{-ikx}, & \text{for } x < -2a \\ te^{ikx}, & \text{for } x > 0. \end{cases}$$

a) Assuming that the energy of the incoming electron is *less* than V_0, write down the general form of a wavefunction that satisfies the time-independent Schrödinger equation in the region $-2a < x < 0$.

b) State the boundary conditions that your wavefunction from part (a) must satisfy at $x = 0$, and use it to express the wavefunction in the region $-2a < x < 0$ as a linear combination of $\cosh \kappa x$ and $\sinh \kappa x$ with complex coefficients. Give an expression for κ.

c) Now write down the conditions that $\psi(x)$ must satisfy at $x = -2a$. Solve the resulting equations and so determine r and t, in terms of V_0, m, and a.

d) Define $R = |r|^2$ and $T = |t|^2$ and evaluate $R + T$. Explain why this sum takes the value that it does.