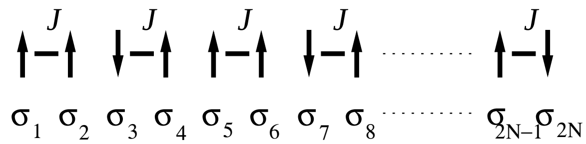
Q4: Consider the classical statistical mechanics of a one-dimensional chain of N pairs of spins σ.



The spins take only values of $\sigma_i = \pm 1$. The system energy is determined by the Hamiltonian

$$H = -J \sum_{i=1,3,\ldots,2N-1} \sigma_i \sigma_{i+1} - h \sum_{i=1}^{2N} \sigma_i,$$

where both the exchange-coupling J and the externally imposed magnetic field h are non-negative. Note that the sum over i in the first term in H is only over <u>odd numbers</u> while in the second term it is over <u>all</u> numbers from 1 to $2N$.

a) Determine the partition function $Z(T, h)$ of this system as a function of the temperature T and h. Do this by thinking about pairs of spins.

b) Determine the free energy $f(T, h)$ per particle and use it to compute the magnetization m (*i.e.* the thermal average of σ_i) per particle and the susceptibility

$$\chi(T, h) = \left(\frac{\partial m}{\partial h}\right)_T .$$

Sketch a graph of $\chi(T, h = 0)$ as a function of T.

c) Still with $h = 0$, determine the number, $\Omega(E, N)$, of states of the system associated with a given energy E for the given number of pairs N.

d) Using the $\Omega(E, N)$ derived in item (c) recompute the partition function for $h = 0$. Does your new Z reduce to the $h = 0$ version of your answer in item (a)?