Q2: Consider the reflection of an electromagnetic wave normally incident on a transparent dielectric material.

a) Write down Maxwell's equations in your chosen system of units, and use them to derive the wave equations for the $\mathbf{E}$ and $\mathbf{B}$ fields in a region with vacuum magnetic permeability μ_0 and dielectric constant $\varepsilon = \varepsilon_r \varepsilon_0$, where ε_0 is the vacuum permittivity and ε_r the relative permittivity. What is the refractive index n for this material?

b) The region $x < 0$ is occupied by vacuum, while the region $x > 0$ is occupied by a dielectric with the properties given in part (a). Write down the boundary conditions at $x = 0$ that relate the x, y, z components of the electric field in the two regions, and similarly for the three components of the magnetic field $\mathbf{B}$. (You may use the notation $\mathbf{E}_\parallel$ and $\mathbf{E}_\perp$ to shorten your answer.)

c) An electromagnetic wave of wavenumber k and angular frequency ω is propagating parallel to the x axis, and is polarized in the z direction:

$$\mathbf{E}_{\text{incoming}} = \hat{\mathbf{z}} E_0 \cos(kx - \omega t).$$

The wave is incident on the surface of the dielectric at $x = 0$ and is partially reflected. Compute the reflection and transmission coefficients $r = E_{\text{reflected}}/E_0$ and $t = E_{\text{transmitted}}/E_0$. Express your answer in terms of the refractive index of the dielectric.