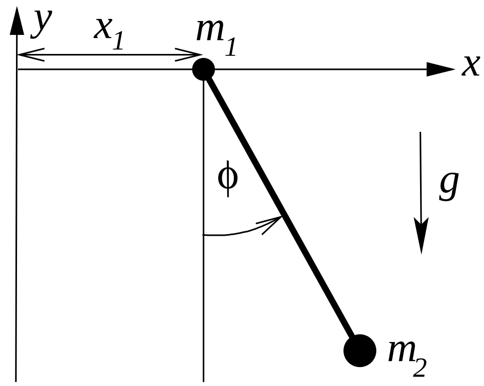
Q1: The suspension point of a planar pendulum has mass m_1 and can slide without friction on the x axis. The pendulum rod is massless and has length L. The bob at its lower end has mass m_2. The suspension point is moved back and forth by the oscillatory motion of m_2. Gravity g acts in the $-y$ direction.



a) Starting from Newton's equations, without any use of the Lagrangian formalism, derive the equations of motion that allow you to solve for the coordinate, x_1, of the suspension point and the angle $\phi(t)$ of the pendulum from the vertical.

b) Solve the equations of motion to find $x_1(t)$ and $\phi(t)$ for the $t = 0$ initial conditions $x_1(0) = 0$, $\phi(0) = \phi_0$ where ϕ_0 is a non-zero constant, and

$$\left.\frac{dx_1}{dt}\right|_{t=0} = 0, \quad \left.\frac{d\phi}{dt}\right|_{t=0} = 0.$$

You may assume, *in this part only*, that the angles $\phi(t)$ are small enough that we can approximate $\sin(\phi) \approx \phi$, *etc.*

c) Write down the Lagrangian for this problem using x_1 and ϕ as the generalized coordinates.

d) Write down the corresponding Euler-Lagrange equations. Do they agree with the equations of motion you derived directly in (a)?

e) From your equations of motion show that the component p_x of the total momentum (*i.e.* the momentum of mass m_1 plus the x-component of the momentum of mass m_2) is conserved.

f) Show that for $p_x = 0$ the mass m_2 moves along part of an ellipse and find the lengths of the semi-major and semi-minor axes of the ellipse.