



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

MATH 257 – Linear Algebra with Computational Applications

Extras Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: May 6, 5:30 - 7:00 PM Aman, Maheen, Serge

Session 2: May 7. 3:30 - 5:00 PM Rohan, Aidan, Siddh

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/955>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. For any $\mathbf{A}_{n \times n}$ matrix, there exists an invertible matrix P and diagonal matrix D , such that $A = PDP^{-1}$ for all square matrices.

2. True/False : Another way of representing matrix diagonalization is $L_{BB} = I_{BE}L_{EE}I_{EB}$, where the basis B represents the eigenbasis of a matrix A in the standard basis

Let A be a collection of vectors $[\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_k] \in \mathbb{R}^n$. A is linearly independent iff $\text{Nul}(A) \neq 0$.

True/False : Matrix A that has $\lambda_1 = \lambda_2 = 2$ and linearly independent eigenvectors V_1 and V_2 is

diagonalizable.

3. If A is a 3×3 matrix such that: $A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$, $A \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{2}} \\ 0 \\ \frac{2}{\sqrt{2}} \end{bmatrix}$, $A \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ What is A^2 ?

4. What is the limit of $A^k v$ for all $v \in \mathbb{R}^n$ as $k \rightarrow \infty$ if $|\lambda_i| < 1$, where λ_i is an eigenvalue of matrix A ?

5. SVD given Diagonalization. **it is not necessary to calculate matrix inverses**

a) Matrix A is diagonalized with the following matrices: $P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 17 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

Find the SVD matrices U , Σ , and V so that $A = U\Sigma V^T$

b) Matrix A is diagonalized with the following matrices: $P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -16 \end{bmatrix}$

Find the Compact SVD matrices U_C , Σ_C , and V_C so that $A = U\Sigma V^T$

6. A is a 3×4 matrix with singular values σ_1 and σ_2 . The column space of A is given $\text{col}(A) = \text{span} \left\{ \begin{bmatrix} 9 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \right\}$, and the null space is given $\text{null}(A) = \text{span} \left\{ \begin{bmatrix} -4 \\ 9 \\ -2 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -1 \\ 4 \end{bmatrix} \right\}$

Produce the compact SVD matrices of A