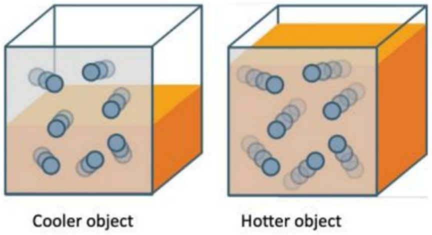
C.A.R.E. PHYS 213 Final Exam Review Session

C.A.R.E. PHYS 213 Quiz 1 Review Session

Units for the Exam

- Internal Energy
- Temperature
- Heat Capacity
- Entropy

Internal Energy



- Total energy is <u>**ALWAYS**</u> conserved

- **Positive work on a system increases the system's internal energy**

- **Higher temperature → More Internal Energy**

- First law of thermodynamics:

$$\Delta U = W_{on} + Q$$

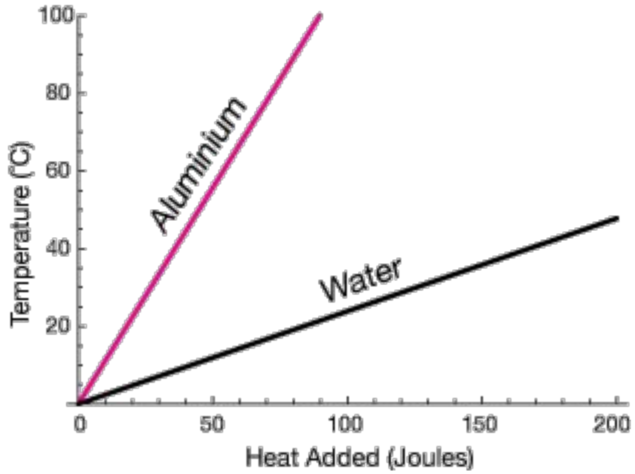
Change in internal energy | Work done on the system | Heat added to the system

Temperature & Heat Capacity

- **Heat Capacity** (*C*) - how much energy it takes to **increase** the temperature of a substance by 1 K/°C
  - Units of J/K

- **Larger *C*** $\rightarrow$ **More energy** is required to **increase the temperature** of the object

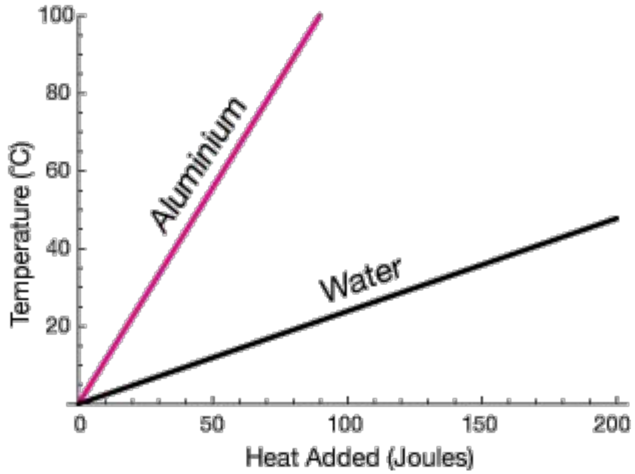
$$C = dQ/dT$$



Temperature & Heat Capacity

- **Heat Capacity** (*C*) - how much energy it takes to **increase** the temperature of a substance by 1 K/°C
  - Units of J/K
- **Larger *C*** $\rightarrow$ **More energy** is required to **increase the temperature** of the object
- Water has a larger heat capacity than Aluminum

$$C = dQ/dT$$



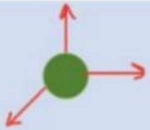
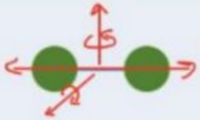
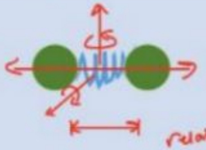
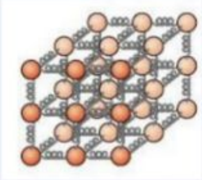
Types of Heat Capacity

- **Molar Heat Capacity [J/mol K]:** The amount of heat required to raise the temperature of **1 mole** of a substance by 1 K/°C
  - $c_M = C/n$, where n is the number of moles

- **Specific Heat Capacity [J/kg K]:** The amount of heat required to raise the temperature of **1 kg** of a substance by 1 K/°C
  - $c = C/m$, where m is the mass [kg]

- **Heat Capacity at a Constant Volume and Constant Pressure:**
  - $C_v = dU/dT$
  - $C_p = dU/dT + p\, dV/dT$

Equipartition

	Molecule	Degrees of freedom	U	C_V
Monatomic: DOF = 3		$N_{DOF} = 3$ x, y, z momentum	$U = \frac{3}{2} NkT$	$C_V = \frac{3}{2} Nk$
Diatomic: DOF = 5		$N_{DOF} = 5$ x, y, z momentum 2 rotation axes	$U = \frac{(3+2)}{2} NkT$	$C_V = \frac{5}{2} Nk$
Vibrational: DOF = 7	 relative	x, y, z momentum 2 rotation axes vibration mode (momentum+potential) $N_{DOF} = 7$	$U = \frac{(3+2+2)}{2} NkT$	$C_V = \frac{7}{2} Nk$
Solid: DOF = 6		x, y, z momentum x, y, z spring modes	$U = \frac{(3+3)}{2} NkT$	$C_V = 3Nk$

Equipartition & Heat Capacity

- Only need to memorize DOFs for monatomic gas, diatomic gas, and solids (3, 5, and 6, respectively)

- For substances under the **equipartition assumption:**

$$U = \frac{N_{\text{DOF}}}{2} NkT \implies C = \frac{\mathrm{d}U}{\mathrm{d}T} = \frac{N_{\text{DOF}}}{2} Nk$$

$Nk = nR$

R = 8.314 J/(mol K)

R = 0.08206 L atm/(mol K)

Entropy

- Microstate vs Macrostate:
  - Microstate: individual, **specific** arrangement
  - Macrostate: property that arises from the microstates
  - **Many microstates can lead to the same macrostate**
    - Two people have the same weight (macroststate), but the distribution of the weight can be different (microstate)

Entropy (Cont.)

- Entropy (*S*) is a measure of the degree of 'diversity' associated with a macrostate
    - $S = k\ ln(\Omega)$, where Ω is the number of microstates
    - **Second Law of Thermodynamics:** $\Delta S \geq 0$

- Equilibrium
    - Occurs when the macrostate of the system ceases to change
    - The **most probable macrostate** is the one with the **highest entropy (most microstates)**
    - **Equilibrium** is achieved when **S, entropy, is maximized**

Binomial Coefficient

If I have ***N* coins** and I am looking for the macrostate with ***q* heads**, the number of microstates:

$$\binom{N}{q} = \frac{N!}{q!(N-q)!}$$

Example: If I have **20 coins**, how many microstates are associated with the macrostate of getting **7 heads**?

Answer:

Binomial Coefficient

If I have ***N* coins** and I am looking for the macrostate with ***q* heads**, the number of microstates:

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Example: If I have **20 coins**, how many microstates are associated with the macrostate of getting **7 heads**?

Answer:

$$\binom{20}{7} = \frac{20!}{7!\,(20-7)!} = 77520$$

Entropy (Cont.) - Dice Example

- **Microstate**:
- **Macrostate**:
- What is the most likely macrostate?
    -
    -
- What is the macrostate with the highest entropy?
    -

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

- **Entropy of a macrostate is simply a measure of the number of microstates associated with it**
- **More microstates → Higher Entropy, Higher Probability**

Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**:
- What is the most likely macrostate?
    -
    -
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Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
- What is the macrostate with the highest entropy?

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Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
  - 7: has the largest number of microstates associated with it
  - Probability = 6/36 = 0.167

- What is the macrostate with the highest entropy?
  -

- **Entropy of a macrostate is simply a measure of the number of microstates associated with it**
- **More microstates ⟶ Higher Entropy, Higher Probability**

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	3	4	5	6	7	8
	4	5	6	7	8	9
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Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
  - 7: has the largest number of microstates associated with it
  - Probability = 6/36 = 0.167
- What is the macrostate with the highest entropy?
  - Again, 7: has the largest number of microstates associated with it
- **Entropy of a macrostate is simply a measure of the number of microstates associated with it**
- **More microstates → Higher Entropy, Higher Probability**

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⚅	7	8	9	10	11	12

Important Equations

Start with the definition of temperature

$$\frac{1}{T} \equiv \left(\frac{\partial S}{\partial U}\right)_{V,N}$$

multiply by dU

$$\frac{1}{T} dU = dS$$

integrate

$$\Delta S = \int_{U_i}^{U_f} \frac{1}{T} dU$$

Or the definition of heat capacity

$$C_V = \left(\frac{dQ}{dT}\right)_{V,N}$$

at constant volume
$dQ = dU$

$$C_V = \left(\frac{dU}{dT}\right)_{V,N}$$

multiply by dT

$$C_V dT = dU$$

plug dU into ΔS

$$\Delta S = \int_{T_i}^{T_f} \frac{C_V(T)}{T} dT$$

multiply by dT

$$C_V dT = dQ$$

integrate

$$\Delta U = \int_{T_i}^{T_f} C_V(T) dT$$

Differential Manipulation

- Know these tricks!

- Heat Capacity is the link between *dU* and *dT*
    - **If you know the heat capacity and the temperature change, you can find the change in internal energy and the change in entropy**

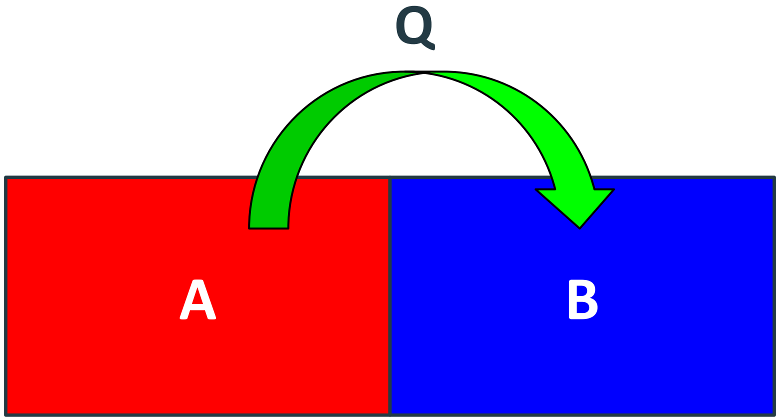
$$C = \frac{\partial U}{\partial T} \implies \Delta U = \int_{T_i}^{T_f} C \mathrm{d}T$$

$$\frac{\partial S}{\partial U} = \frac{1}{T} \implies \Delta S = \int \frac{\mathrm{d}U}{T} = \int_{T_i}^{T_f} \frac{C \mathrm{d}T}{T}$$

Entropy of Two Blocks in Thermal Contact - Example

Two blocks A and B, are in thermal contact and insulated from surroundings. Initially, block A is at a higher temperature than block B. **Each block has a temperature-dependent heat capacity given by $C = pT^2$.**

Determine the entropy change for each block



Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

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$$= \int_{T_i}^{T_f} pT\,dT$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

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$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

$$= \int_{T_i}^{T_f} pT\,dT$$

$$\boxed{= p\frac{{T_f}^2}{2} - p\frac{{T_i}^2}{2} = \frac{p}{2}\left[{T_f}^2 - {T_i}^2\right]}$$

Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

$$= \int_{T_i}^{T_f} pT\,dT$$

$$= p\frac{T_f^{\,2}}{2} - p\frac{T_i^{\,2}}{2} = \frac{p}{2}\left[T_f^{\,2} - T_i^{\,2}\right]$$

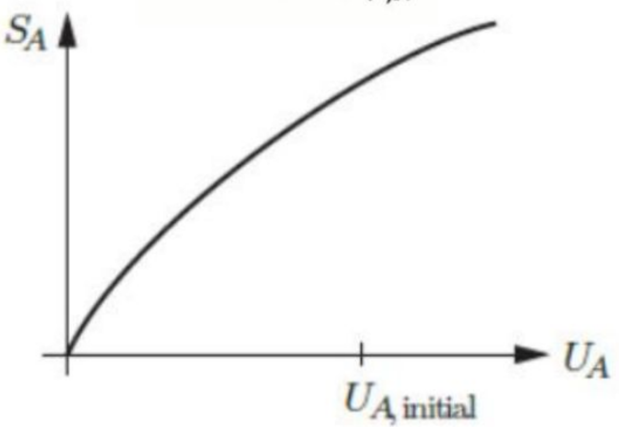
$$\Delta S_A = \frac{p}{2}\left[T_{A,f}^{\,2} - T_{A,i}^{\,2}\right]$$

$$\Delta S_B = \frac{p}{2}\left[T_{B,f}^{\,2} - T_{B,i}^{\,2}\right]$$

Entropy (S) vs. Internal Energy (U)

- Since slope is always positive, temperature is always positive
- **More energy = greater entropy**
- **Diminishing returns:** it gets harder and harder to increase the entropy as internal energy increases
- **Decreasing slope = increasing temperature**
    - **More energy means greater temperature**

$$\frac{1}{T} \equiv \left(\frac{dS}{dU}\right)_{V,N}$$



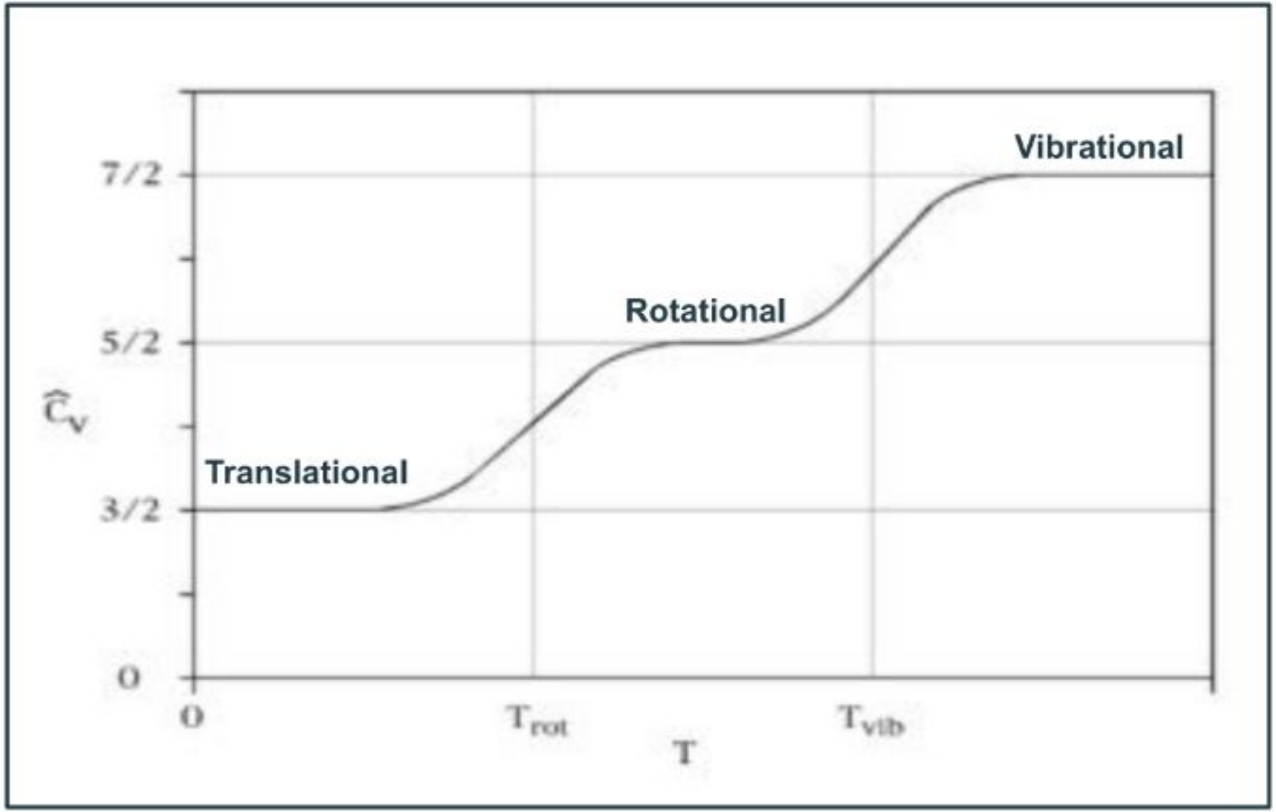
Units for the Exam

- Kinetic Theory of Ideal Gases
- Quasistatic Processes
- Thermodynamic Cycles
- Gibbs Free Energy

Ideal gas and Equipartition

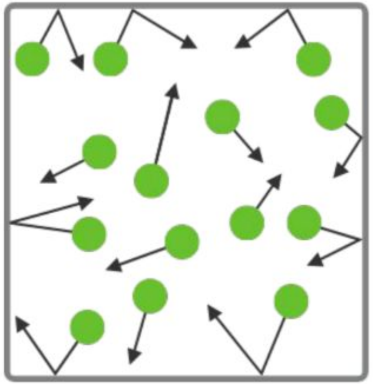
- Ideal Gas: Approximation of particles as points with no interactions:
  - Follows **ideal gas law**: $$pV = NkT = nRT$$

- **Equipartition:** each degree of freedom contributes ½ kT of energy
  - Internal energy in each particle:
    - $U = (N_{DOF}/2)kT$
  - Molar heat capacity:
    - $c_M = (N_{DOF}/2)kN_A$



Vibrational
Rotational
Translational
7/2
5/2
3/2
0
$\hat{C}_V$
0
T_{rot}
T_{vib}
T

Root-Mean-Square Velocity



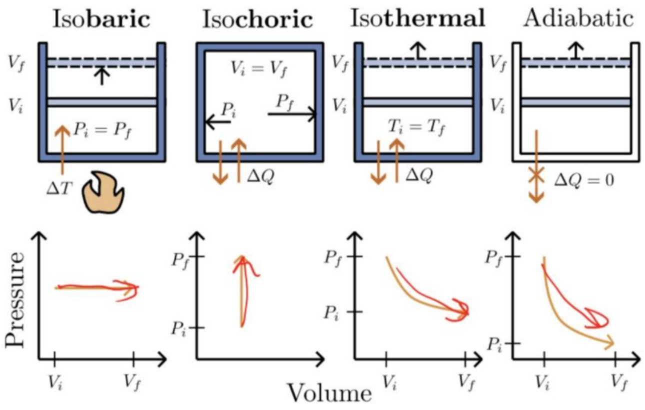
- ***v_{rms}*: Average (translational) velocity of gas particles**
- Translational Kinetic Energy: $KE_{translational} = 1/2\ m\ (v_{rms})^2$
- Relationship to temperature:

$$\frac{1}{2}mv_{rms}{}^2 = \frac{3}{2}kT$$

- This only applies to **ONE PARTICLE**
- **Notice:** this does **NOT** depend on the number of DOFs; it's **ALWAYS** (3/2)*kT*
  - Why? Translational KE only depends on the translational modes motion (there are only 3 translational modes: v_x, v_y, v_z)

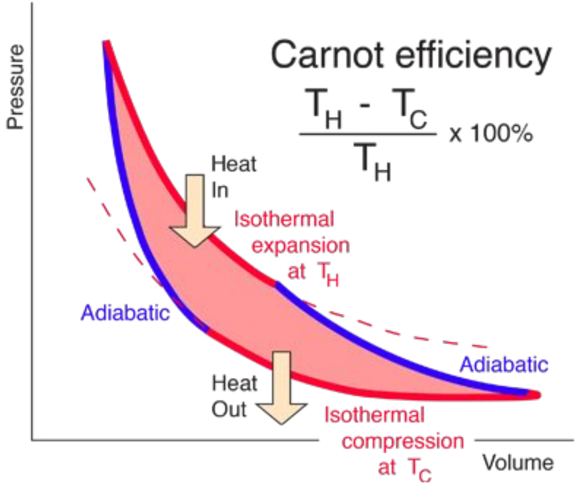
Thermodynamic Processes

- Isochoric or Isovolumetric
  - Constant VOLUME
- Isobaric
  - Constant PRESSURE
- Isothermal
  - Constant TEMPERATURE, REVERSIBLE $(\Delta S_{total} = 0, \Delta U = 0)$
- Adiabatic
  - Constant HEAT (dQ = 0), REVERSIBLE $(\Delta S_{total} = 0, \Delta Q = 0)$



Reversible Processes

- **Isothermal + Adiabatic processes are REVERSIBLE**
  - $\Delta S_{total} = 0$ (no change in entropy)
  - For **isothermal** processes:
    - PV = constant
  - For **adiabatic** processes:
    - PV^{γ} = constant
    - $\gamma = (2/N_{DOF}) + 1$ (given on equation sheet)



Example Problem - Adiabatic Process

- Assume we have a gas undergoing an adiabatic process, determine the work done given the following parameters:
  - $V_i = 10\ m^3$, $p_i = 10$ kPa
  - $V_f = 4\ m^3$, $N_{DOF} = 3$

1. Calculate γ

2. Find pV^γ

3. Calculate W

4. Calculate W

Example Problem - Adiabatic Process

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  - $V_i = 10\ m^3$, $p_i = 10\ kPa$
  - $V_f = 4\ m^3$, $N_{DOF} = 3$

$$\gamma = \frac{2}{N_{DOF}} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$$

2. Find pV^{γ}

3. Calculate *W*

4. Calculate *W*

Example Problem - Adiabatic Process

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$$\gamma = \frac{2}{N_{DOF}} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$$

$$pV^{\gamma} = Constant = p_i V_i^{\gamma} = (1000)(10)^{\frac{5}{3}} = 464158.88$$

3. Calculate *W*

4. Calculate *W*

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$$pV^{\gamma} = Constant = p_i V_i^{\gamma} = (1000)(10)^{\frac{5}{3}} = 464158.88$$

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{C}{V_i^{\gamma}}\,dV = \left[C\frac{V_f^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1}\right] - \left[C\frac{V_i^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1}\right]$$

4. Calculate *W*

Example Problem - Adiabatic Process

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    - $V_i = 10\ m^3$, $p_i = 10\ kPa$
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$$pV^{\gamma} = Constant = p_i V_i^{\gamma} = (1000)(10)^{\frac{5}{3}} = 464158.88$$

$$W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{C}{V_i^{\gamma}} dV = \left[C \frac{V_f^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right] - \left[C \frac{V_i^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right]$$

$$W = \left[(464158.88) \frac{4^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right] - \left[(464158.88) \frac{10^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right] \approx -126.30\ kJ$$

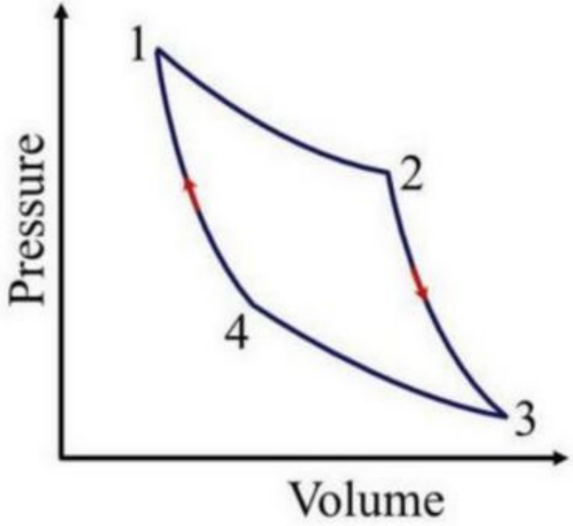
$$W \approx -126.30\ kJ$$

p-V Diagrams

$$W_{\text{by}} = \int_{V_i}^{V_f} p\,dV$$

- Used to visualize thermodynamic cycles

- **Area enclosed in the curve is equal to the work per cycle**
    - **Clockwise direction: work is positive** (engine did work)
    - **Counterclockwise direction: work is negative** (work done on engine)



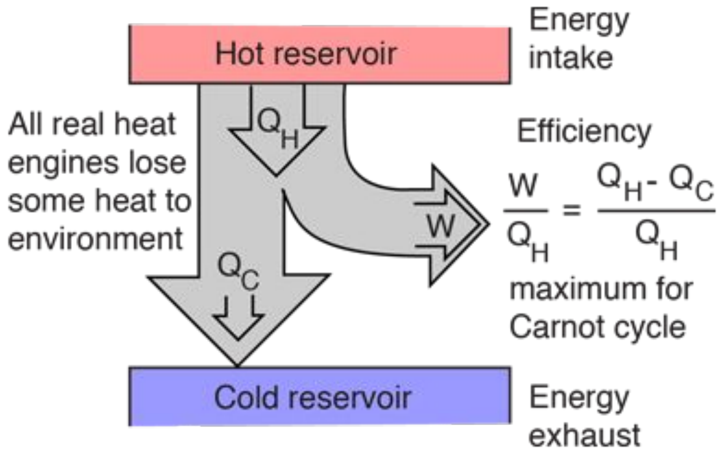
Heat Engines

- Cycles of Thermodynamic processes are used to make engines, heat pumps, and refrigerators

- **Efficiency of engines:** $\varepsilon = \frac{W_{by}}{Q_H} \leq 1 - \frac{T_C}{T_H}$

- **COP of pumps and refrigerators:**
  - Heat pump: $\mathrm{COP} = \frac{Q_H}{W_{on}} \leq \frac{1}{1 - \frac{T_C}{T_H}}$
  - Refrigerator: $\mathrm{COP} = \frac{Q_C}{W_{on}} \leq \frac{1}{\frac{T_H}{T_C} - 1}$

Efficiency/COP can be thought of as "what you get out" divided by "what you put in."



Example Problem: Engine/Heat Pump COP

Given a hot reservoir at a temperature of $T_h = 373K$ and a cold reservoir at a temperature of $T_c = 293K$, calculate the maximum efficiency ϵ of an engine and the maximum COP of a heat pump between the two reservoirs.

Heat Engine Efficiency:

Heat Pump COP:

Example Problem: Engine/Heat Pump COP

Given a hot reservoir at a temperature of $T_h = 373K$ and a cold reservoir at a temperature of $T_c = 293K$, calculate the maximum efficiency ϵ of an engine and the maximum COP of a heat pump between the two reservoirs.

Heat Engine Efficiency:

$$\epsilon = 1 - \frac{T_c}{T_h} \approx 0.21$$

Heat Pump COP:

Example Problem: Engine/Heat Pump COP

Given a hot reservoir at a temperature of $T_h = 373K$ and a cold reservoir at a temperature of $T_c = 293K$, calculate the maximum efficiency ϵ of an engine and the maximum COP of a heat pump between the two reservoirs.

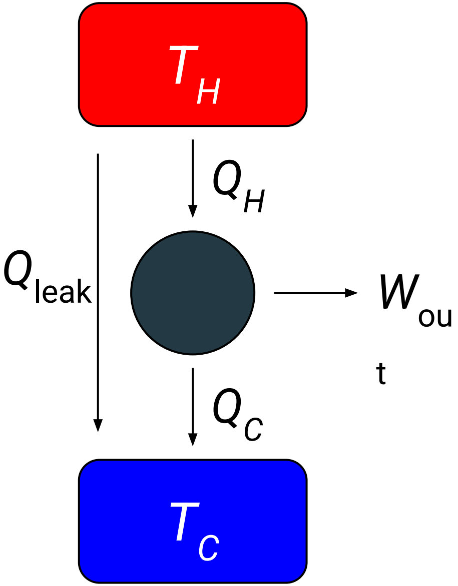
Heat Engine Efficiency:

$$\epsilon = 1 - \frac{T_c}{T_h} \approx 0.21$$

Heat Pump COP:

$$COP = \frac{1}{1 - T_c/T_h} \approx 4.7$$

Engine, Pump, and Refrigerator Diagrams



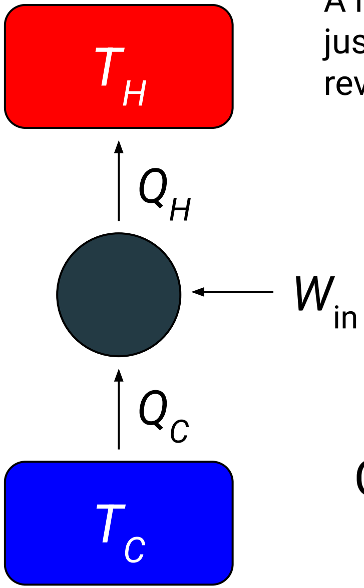
Heat Engine Diagram

In a leaky engine, the heat we put in is $Q_H + Q_{leak}$, which affects the efficiency.

$$Q_H = W_{out} + Q_C$$

$$\epsilon = W_{out}/(Q_H + Q_{leak})$$

$$\epsilon_C = 1 - (T_C/T_H)$$



Heat Pump/Refrigerator Diagram

A heat pump or fridge is just a heat engine run in reverse.

$$Q_H = W_{in} + Q_C$$

Gibbs Free Energy

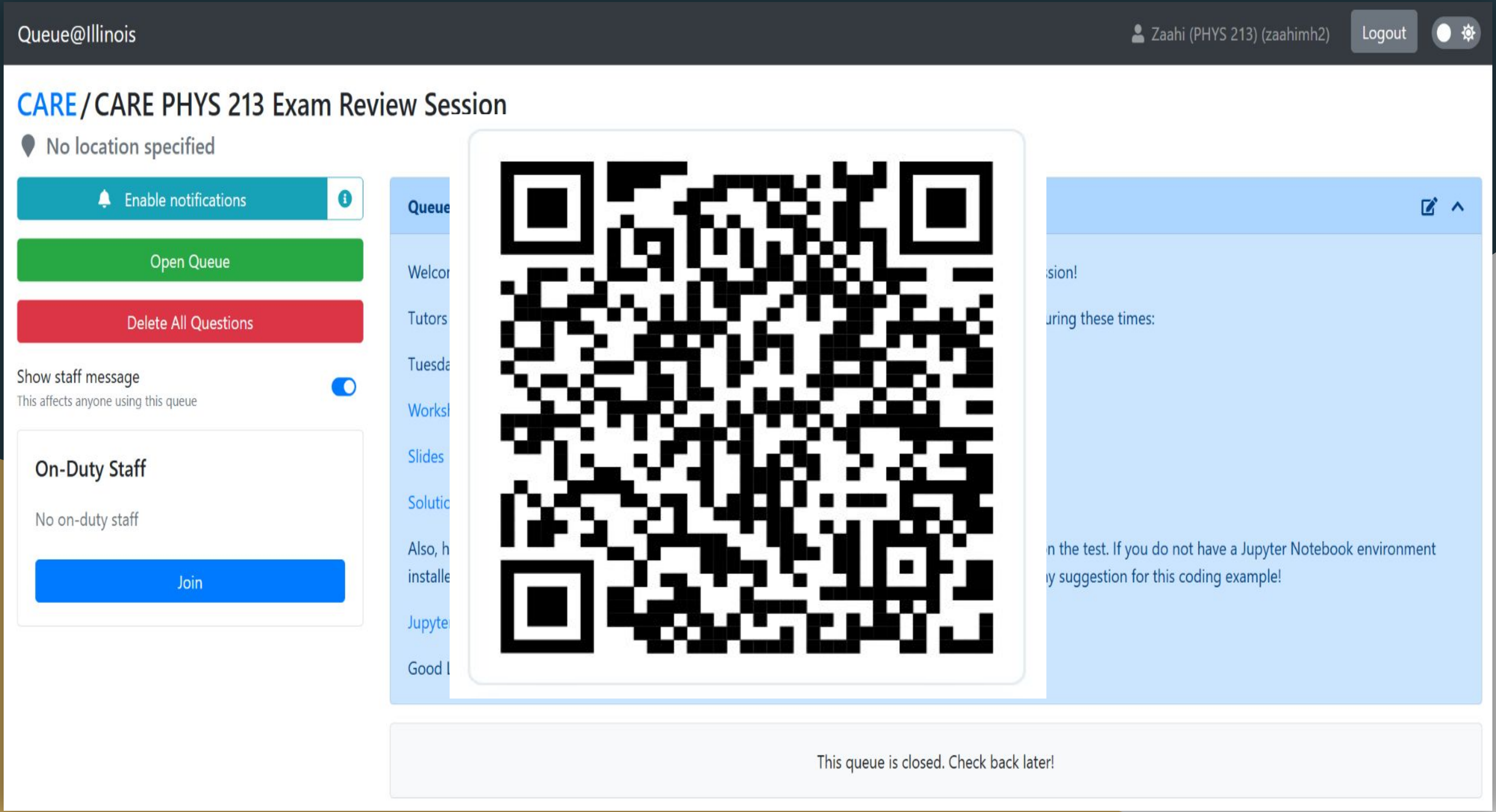
- Useful when **<u>temperature</u> and <u>pressure</u> are fixed**

$$G = U - T_{env}S + pV$$

- **Minimizing Gibbs of a system will maximize total (system + environment) Entropy**
  - As a system approaches equilibrium, free energy will decrease to a minimum

- **Fundamental Thermodynamic Relation in Equilibrium:**
  - $TdS = dU + pdV - \mu dN$
  - $\mu = (dG/dN) \rightarrow \mu N = G$ (at fixed temperature and pressure)
  - Equilibrium favors lowest μ

C.A.R.E. PHYS 213 Final Exam Review Session



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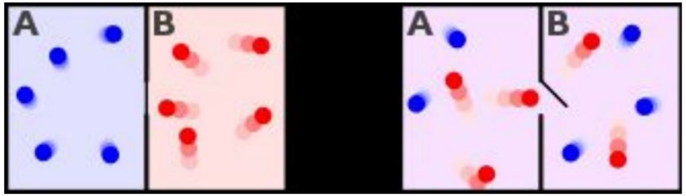
New Units for the Exam

- Chemical Potential and Phase Diagrams
- Ideal Solutions
- Boltzmann Factors
- Semiconductors

Chemical Potential

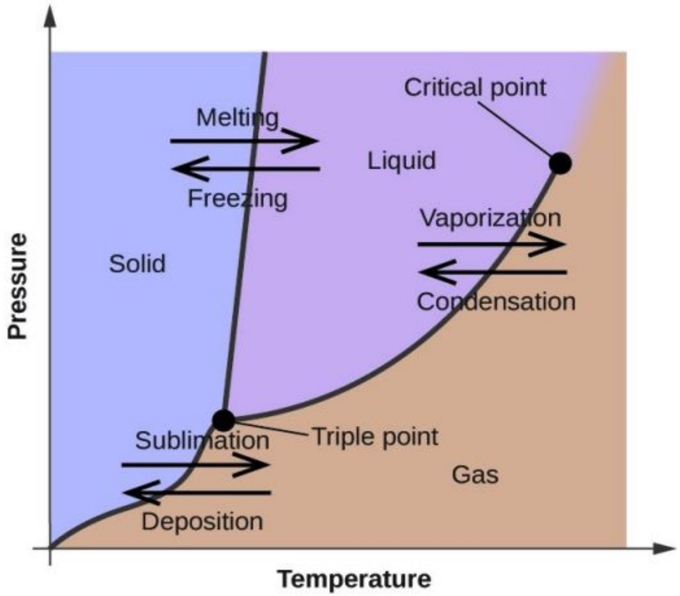
- **Chemical Potential μ describes which way particles in a system will go:**
  - Temperature is to heat as chemical potential is to number of particles
  - Particles go from **high μ to low μ**
  - When **μ's are equal,** the system is in **equilibrium**
- **$\mu = (dG/dN) \rightarrow \mu N = G$ (at fixed temperature and pressure)**
- **Equilibrium favors lowest μ**
  - Important concept with phase diagrams



Phases

- Phases coexist at **equal chemical potentials**
- **Higher pressure phase → Higher density**
- **Higher temperature phase → Higher entropy**

- **Using the phase diagram:**
    - Rank the phases from **highest to lowest density**
        -
    - Rank the phases from **highest to lowest entropy**
        -



Phases

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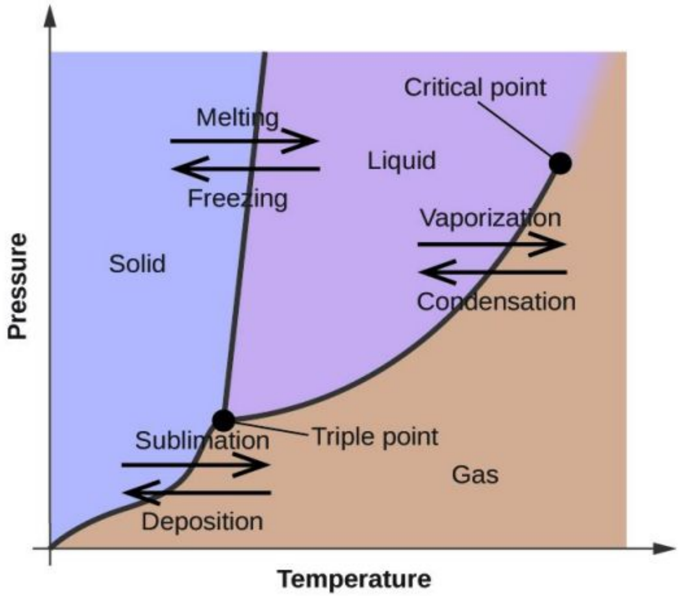
  - Rank the phases from **highest to lowest density**
    - Solid → Liquid → Gas
  - Rank the phases from **highest to lowest entropy**
    -



Phases

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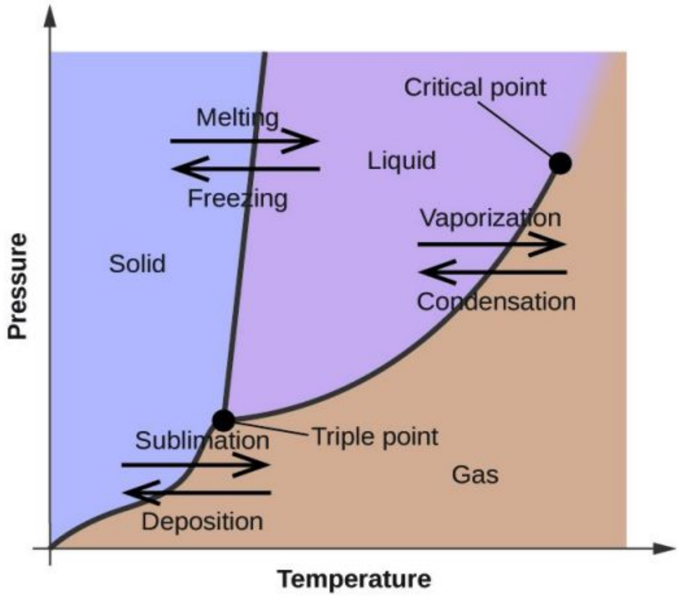
- **Using the phase diagram:**
    - Rank the phases from **highest to lowest density**
        - Solid → Liquid → Gas
    - Rank the phases from **highest to lowest entropy**
        - Gas → Liquid → Solid



Phases (Cont.) - Latent Heat

- **Latent Heat:** the amount of heat required to perform a phase change
  - Heat of vaporization: boiling/condensing
  - Heat of fusion: melting/freezing

- **Relation to Entropy:** $L = T\Delta S$

- **Relation to Enthalpy:** $L = \Delta H, \text{ where } H = U + pV$

  - **At constant pressure:** $L = \Delta U + p\Delta V$

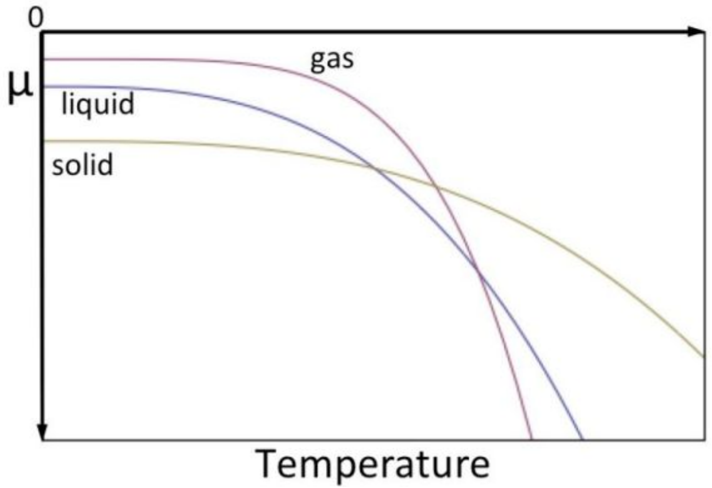
- WATCH YOUR UNITS!
  - *L* is often given in **units** of **J/kg or J/g**
  - Sometimes you will have to find the **change in entropy *per particle***, not per kg
  - Use **molar mass and/or Avogadro's Number** to perform unit conversions

Phases (Cont.) - μ vs. T Diagrams

- The substance exists in the phase with the **lowest chemical potential** at a particular temperature

- **Adding solute lowers the chemical potential curve of the liquid phase which:**
  - **Lowers the melting point**
  - **Raises the boiling point**

- **Applications to real life:**
  - Salting in icy road lowers the freezing point, causing the ice to melt
  - Adding salt to boiling water will cause the water to boil at a higher temperature (cooking your food faster)



Boltzmann Factors - Equations

- **Formula:** $$f_i = e^{\frac{-E_i}{kT}}$$

- **Represents a <u>likelihood</u> of being in a particular microstate with energy E**
  - **Lower energy states will <u>ALWAYS</u> have a greater likelihood**

- **Normalization** with the partition function (*Z*) gives us **probability:**

$$Z = \sum_i f_i$$

- **Probability of state i:** $$P(E_i) = \frac{e^{\frac{-E_i}{kT}}}{Z}$$

Boltzmann Factors - Equations (Cont.)

- To determine the ratio of probabilities:

$$\frac{P(E_1)}{P(E_2)} = \frac{e^{-E_1/kT}}{e^{-E_2/kT}} = e^{-(E_1-E_2)/kT}$$

*Notice how the partition function (Z) is not needed

Useful when not all f_i's are known

- To determine the average energy of the system:

$$E_{\text{avg}} = \sum_i E_i P(E_i)$$

Boltzmann Factors - Concepts

- When temperature is very low (close to 0K), ALL of the Boltzmann factors become very small
  - The ground state becomes the most likely, with a probability approaching 1

$$T \to 0 \implies e^{-E_i/kT} \to 0$$

$$P(\text{ground state}) \approx 1,\ P(\text{excited}) \approx 0$$

- When temperature is very large, all of the Boltzmann factors approach 1
  - All microstates become EQUALLY LIKELY (same probability)

$$T \to \infty \implies e^{-E_i/kT} \to 1$$

Likelihood of each state ≈ 1

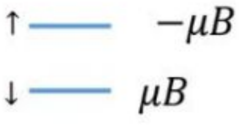
*Remember: Entropy from microstates

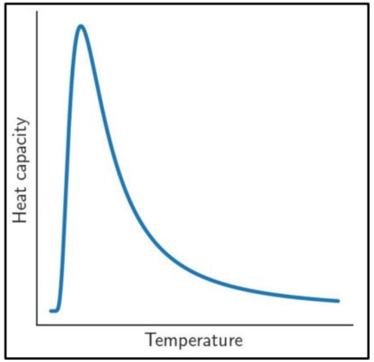
$$S = k ln(\Omega)$$

Boltzmann Factors - Quantum Systems

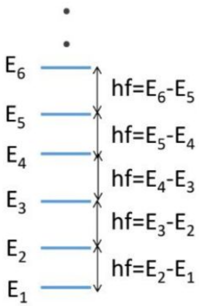
- **Looking at two specific quantum systems and their heat capacity:**

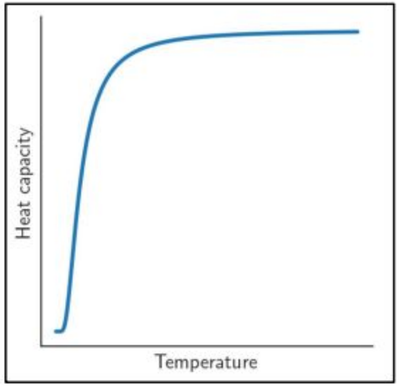
Two-State System:





Harmonic Oscillators:



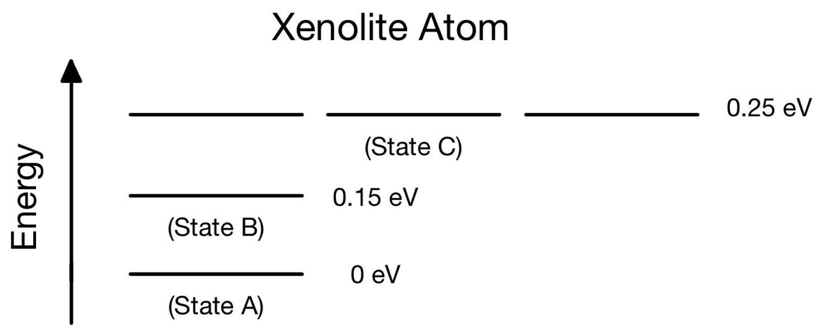


Boltzmann Factors - Example Problem

A fictional atom 'Xenolite' can occupy three different quantum states:

- The ground state **(State A)**, energy = 0 eV
- An excited state **(State B)**, energy = 0.15 eV
- And a second excited state with 3 levels **(State C)**, energy = 0.25 eV

- **At 1200 K, what is the probability that the atom is in State C? (use k in eV)**



Boltzmann Factors - Example Problem (Cont.)

1. Equation for Probability
2. Boltzmann Factor of f_A
3. Boltzmann Factor of f_B
4. Boltzmann Factor of f_C
5. Calculate Z
6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

2. Boltzmann Factor of f_A

3. Boltzmann Factor of f_B

4. Boltzmann Factor of f_C

5. Calculate Z

6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

$$f_A = e^{\frac{-E_A}{kT}} = e^{\frac{-0}{(8.617*10^{-5})*1200}} = 1$$

3. Boltzmann Factor of f_B

4. Boltzmann Factor of f_C

5. Calculate Z

6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

$$f_A = e^{\frac{-E_A}{kT}} = e^{\frac{-0}{(8.617*10^{-5})*1200}} = 1$$

$$f_B = e^{\frac{-E_B}{kT}} = e^{\frac{-0.15}{(8.617*10^{-5})*1200}} = 0.234$$

4. Boltzmann Factor of f_C

5. Calculate Z

6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

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$$f_B = e^{\frac{-E_B}{kT}} = e^{\frac{-0.15}{(8.617*10^{-5})*1200}} = 0.234$$

$$f_C = 3 * e^{\frac{-E_C}{kT}} = 3 * e^{\frac{-0.25}{(8.617*10^{-5})*1200}} = 0.267$$

5. Calculate *Z*

6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

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$$Z = f_A + f_B + f_C = 1 + 0.234 + 0.267 = 1.502$$

6. Calculate $P(E_C)$ using step 1

Boltzmann Factors - Example Problem (Cont.)

$$P(E_C) = \frac{f_C}{Z} = \frac{f_C}{f_A + f_B + f_C}$$

$$f_A = e^{\frac{-E_A}{kT}} = e^{\frac{-0}{(8.617*10^{-5})*1200}} = 1$$

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$$Z = f_A + f_B + f_C = 1 + 0.234 + 0.267 = 1.502$$

$$P(E_C) = \frac{0.267}{1.502} = 0.178$$

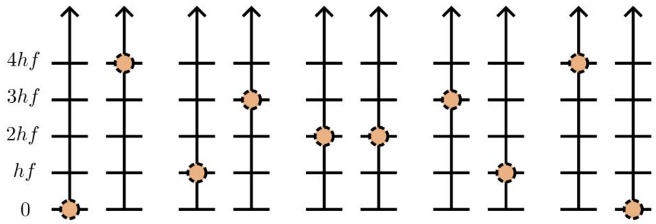
Quantum Harmonic Oscillators (QHO)

- A Quantum Harmonic Oscillator has **discrete energy levels**
  - **A collection of these oscillators have a finite amount of energy (quanta)**

For *N* oscillators with *q* quanta

$$\Omega = \binom{N-1+q}{q}$$

- For a single oscillator in an array:
  - Thermodynamically favorable to be in a **lower energy state**
  - Leads to **greater number of microstates** for the other QHOs and thus **greater entropy**



Semiconductors

- **Resistivity (ρ)** is directly proportional to **Resistance**

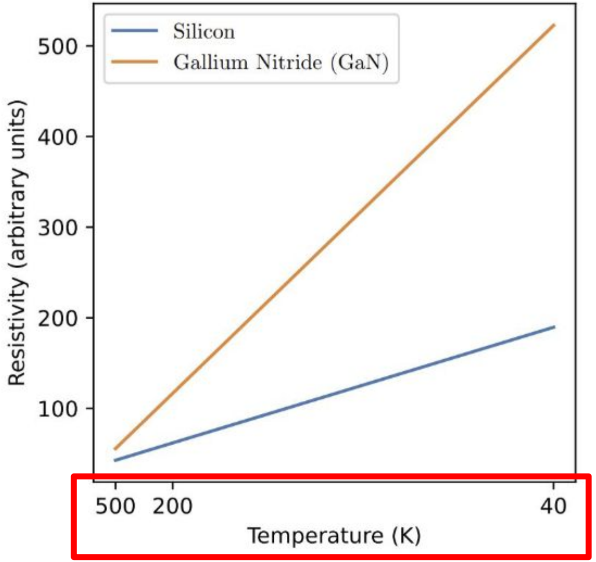
$$\rho = Ce^{\frac{\Delta}{2kT}}$$

- **Conductivity (σ):**

$$\sigma = \frac{1}{\rho} = Ce^{\frac{-\Delta}{2kT}}$$

- **Denominator is $2kT$ <u>NOT</u> kT**
- **Conductivity given on formula sheet**

Arrhenius Plot



$$\ln \rho = \ln C + \frac{\Delta}{2kT}$$

Larger Slope = Larger Energy Gap (Δ)

Good luck!

Feel free to ask any questions you may have.

You got this!