MATH 231 Exam Review

Midterm 03

Power Series

- Can be defined by the form:

$$\sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$$

  - C_n are the coefficients
  - Series is a function of x
- Can be centered at any number:

$$\sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1(x-a) + C_2(x-a)^2 + C_3(x-a)^3 + \cdots$$

Power Series

- Domain of Convergence: For what values will the series converge?
  - Use tests to find out what values of x satisfies convergence criteria.

Theorem 3.1. For any power series

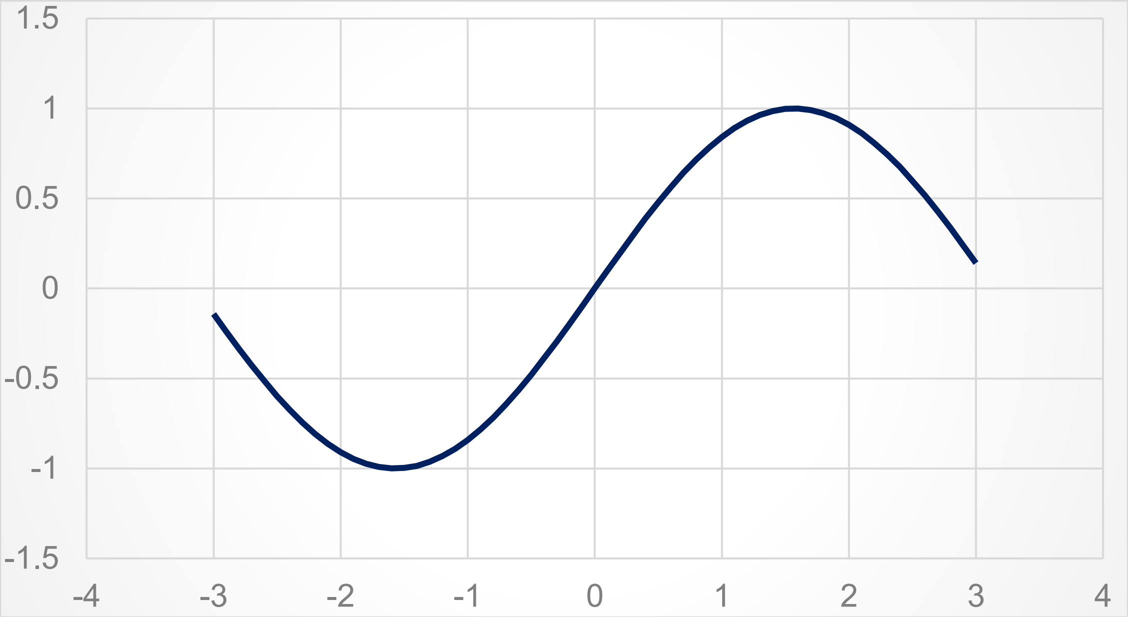
$$\sum_{n=0}^{\infty} c_n (x-a)^n,$$

there are exactly three possibilities for the domain of convergence (DOC) and radius of convergence (ROC).

1. Converges only at $x = a$, or
   DOC = $\{a\}$, ROC = 0;
2. Converges for all x, or
   DOC = $(-\infty, \infty))$, ROC = ∞;
3. There is an R such that the power series converges for $|x-a| < R$ and diverges for $|x-a| > R$,
   ROC = R.

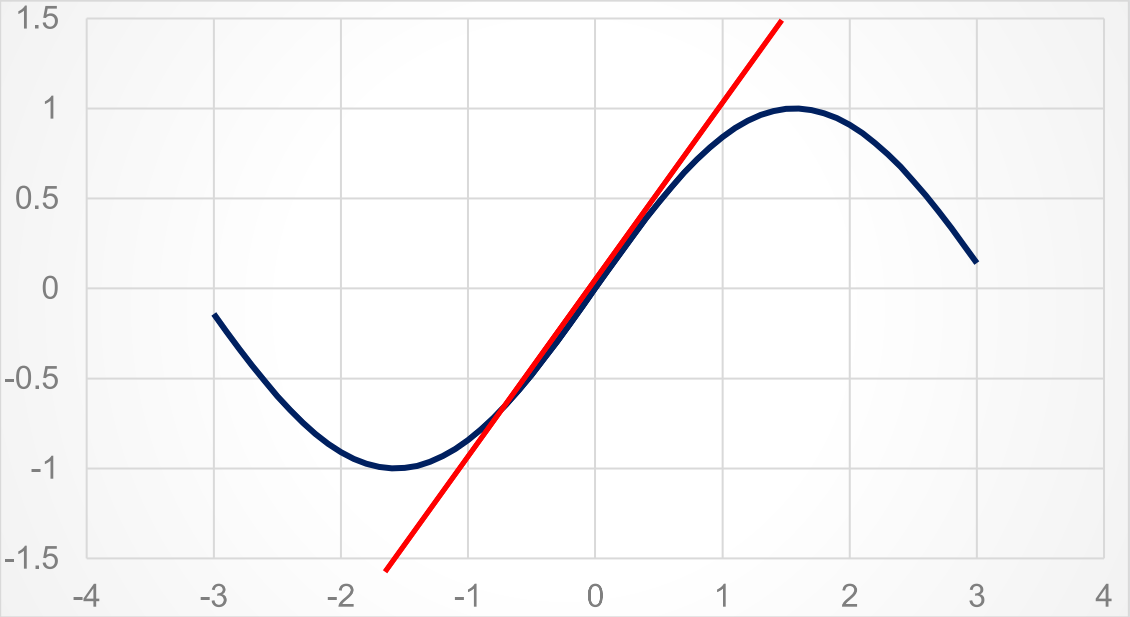
Remark 3.2. In the case with a radius of convergence R with $0 < R < \infty$, we have to check the endpoints "by hand".

Taylor Series



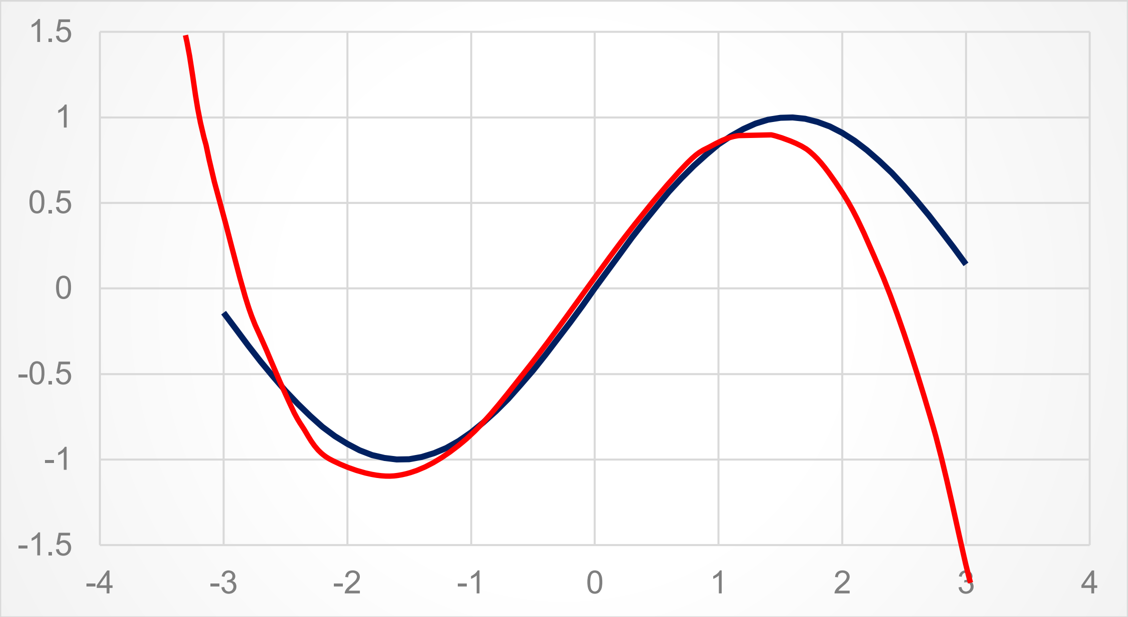
$\sin(x)$

Taylor Series



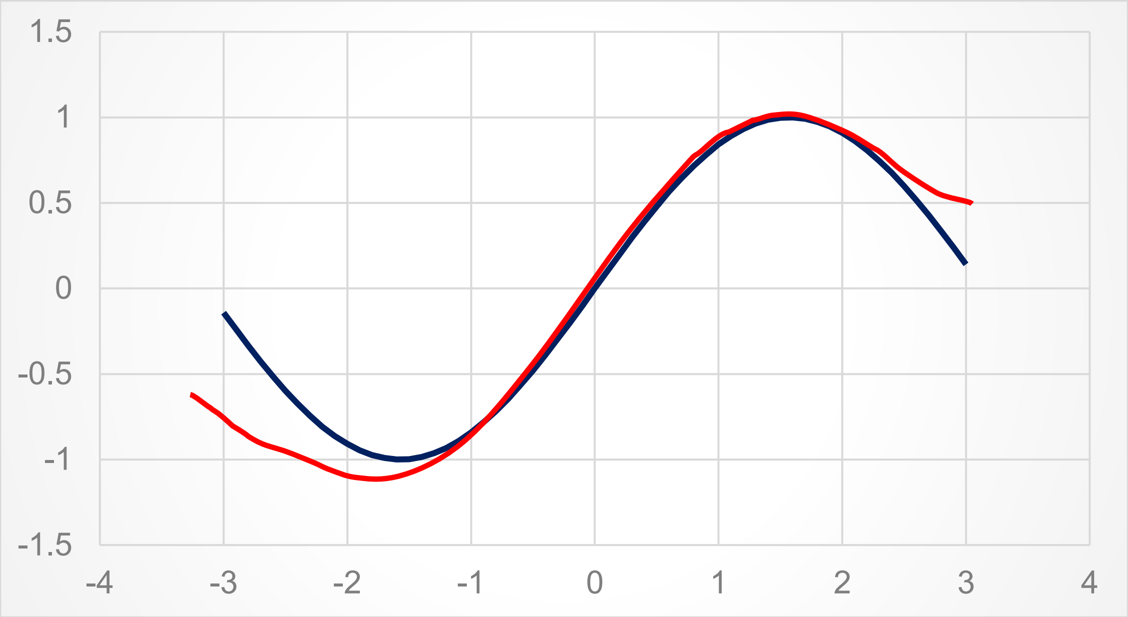
$\sin(x) \quad = x$

Taylor Series



$$\sin(x) = x - \frac{x^3}{3!}$$

Taylor Series



$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Taylor Series

Essentially a way to estimate a function about a point!

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$$

Big O Notation

"Everything else" Anything past a point **will not affect** the function much.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + O(x^4)$$

If a = 0, it's called a Maclaurin Series for f(x)

$$f(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \cdots + \frac{f^n(0)}{n!}(x)^n$$

Some Common Taylor Series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\ln(1 + x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\frac{1}{1 - x} = 1 - x + x^2 - x^3 + x^4 + \cdots$$

$$\frac{1}{x} = 1 - (x - 1) + (x - 1)^2 - (x - 1)^3 + (x - 1)^4 + \cdots$$

Parametric Curves

In most of your math classes, you have only dealt with curves that have functions in terms of one other variable such as f(x) = y. What if a graph doesn't pass the vertical line test?

Can we still graph it?

- Yes!
- Let's introduce a new variable: t
- The variable x and y can now be put into terms of t:
  - $x = g(t)$
  - $y = h(t)$

Parametric Curves, but with Calculus

- Derivatives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d(dy/dx)}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right)}{\frac{dx}{dt}}$$

- Area:

$$\int_a^b y\,dx = \int_c^d g(t)f'(t)\,dt.$$

$$x = f(t), \quad y = g(t), \quad t \in [c, d],$$

- Arc Length:

$$\int_c^d \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$$

Polar Coordinates

Define coordinates relative to the origin:

“r” – the distance from the origin

“θ” – the angle in between the line and the x axis

Transformation from regular Cartesian coordinates (x & y).

$x = r \cos \theta$

$y = r \sin \theta$

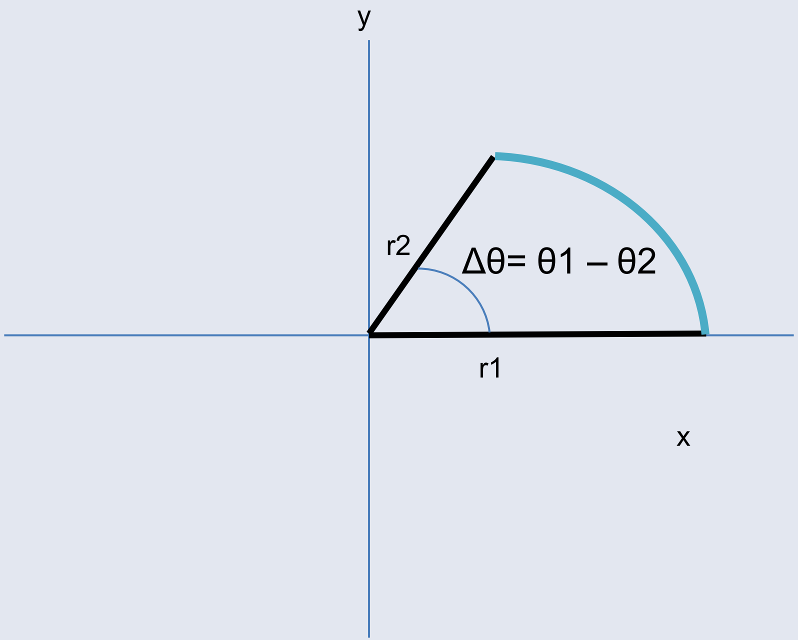
$$r^2 = x^2 + y^2$$

Not all polar coordinates are unique:

Ex. Repeating after 2π

Polar Curves

- How to plot?



- Exact path depends on function

Polar Coordinates, but with Calculus

- Area

$$A = \frac{1}{2}\int f(\theta)^2 d\theta = \frac{1}{2}\int r^2 d\theta$$

- Arc Length

$$AL = \int \sqrt{\left(f(\theta)\right)^2 + \left(f'(\theta)\right)^2} d\theta = \frac{1}{2}\int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$