Math 115 Exam 3 Review Session

Rational Functions

What is a rational function?

A function *f* is called a function when

$$f(x) = \frac{P(x)}{Q(x)}$$

Both P and Q are polynomial functions

Polynomial Review

- Degree (highest power of the polynomial function) must be a positive integer, including 0
- All polynomial functions are continuous at all points, unless it is Q(x) where its denominator cannot be 0.

Domains of Rational Functions

Since both the numerator and denominator are both polynomials, we mostly worry about the denominator of the function when determining where we may have discontinuities (we will discuss this in the next slide)

$$
\begin{aligned}
f_1(x) &= \frac{1}{x^2}, \\
f_2(x) &= \frac{x+1}{x-1}, \\
f_3(x) &= \frac{2x^2-3x-1}{x},
\end{aligned}
$$

Discontinuities of Rational Functions

We have learned about different discontinuities for exponential functions, logramtic, etc., which we will use to evaluate long term behavior. The types are as listed below

Horizontal Asymptote: A horizontal line that defines where a function approaches a finite y-value but will never touch nor cross it. End behavior determined as f(x) goes to -∞ and ∞.

Vertical Asymptote: A vertical line that defines where a function approaches a fintie x-value but will never touch nor cross it, End behavior determined by the root of the denominator

.

Hole(s): This factor is determined when the root of the denominator causes both the numerator and denominator to equal 0/0. Determined by a empty circle on a function's path

Sketching Rational Functions

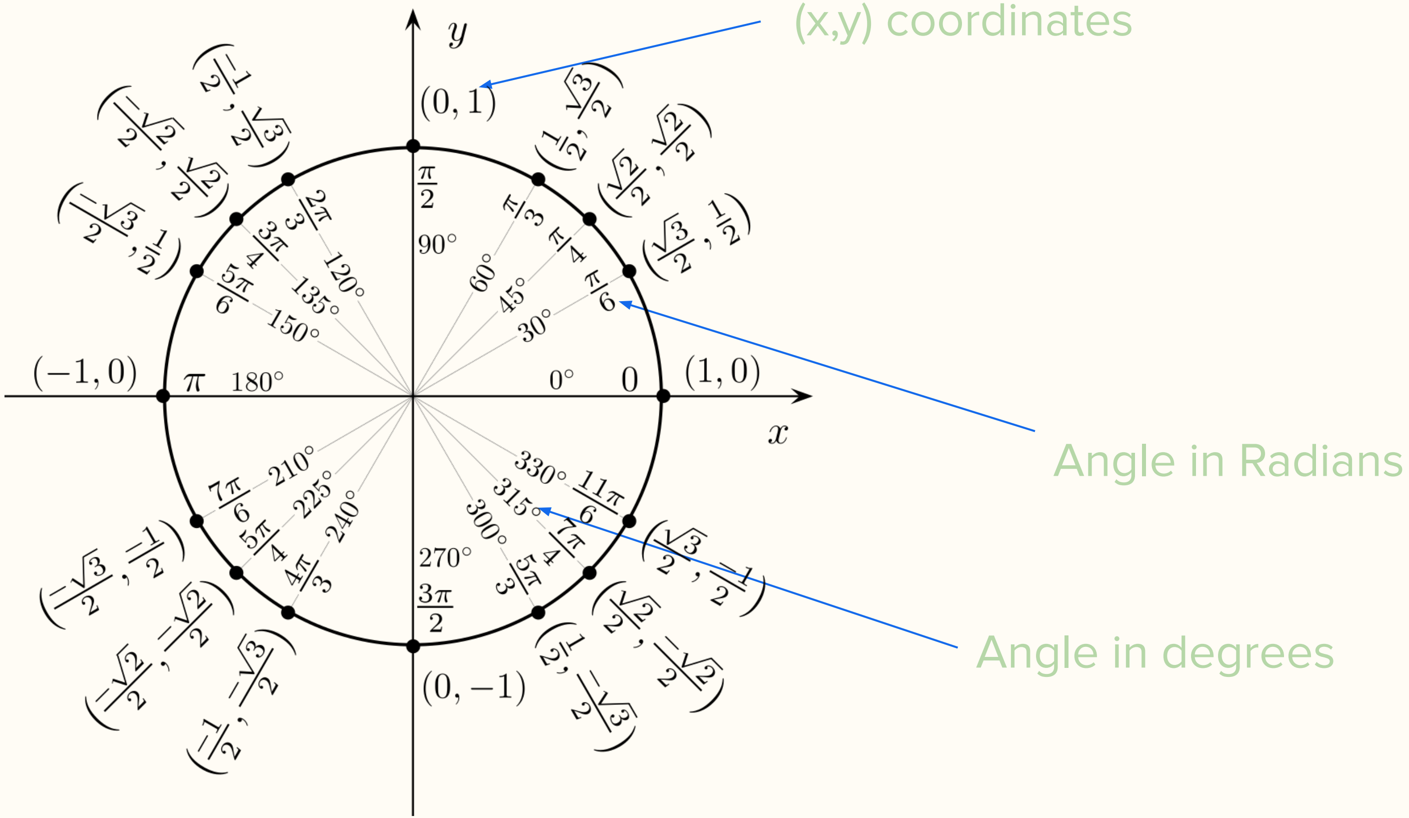
Steps to sketch rational functions(take it one step at a time)

1. Use the Zero Product Property to find the points that the denominator equals zero and take those through vertical asymptotes
2. Find y intercepts by plugging 0 into all x values
3. Find x intercepts by plugging 0 into y
4. Find the horizontal asymptotes by determining the long run behavior of the function
5. Find the vertical asymptotes through the points that we previously found in step 1
6. Graph!!

Trigonometry

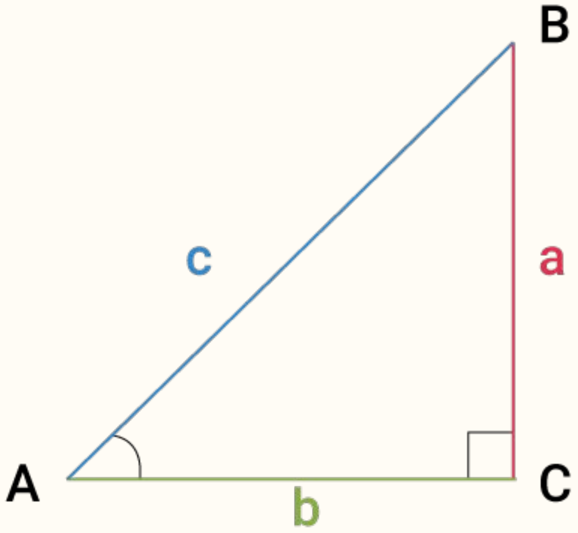
Breaking down the Unit Circle

Radius of 1



SOH-CAH-TOA

SOH – CAH - TOA

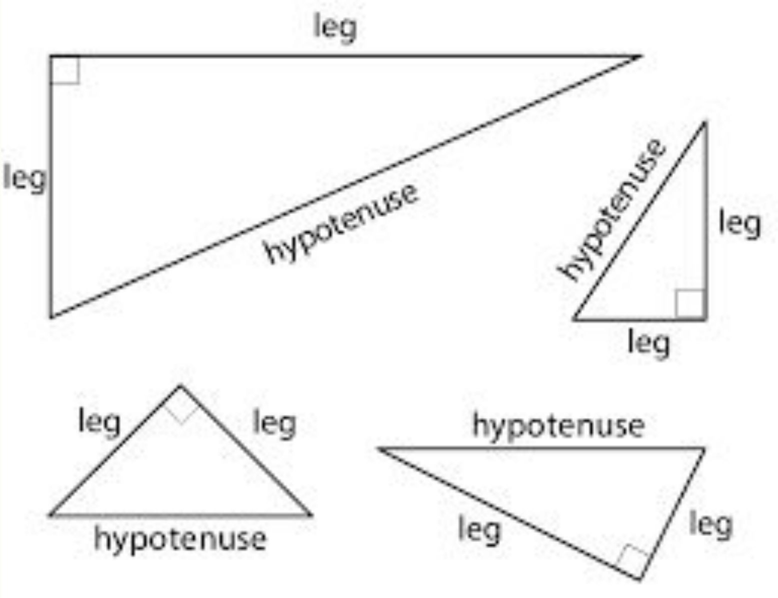


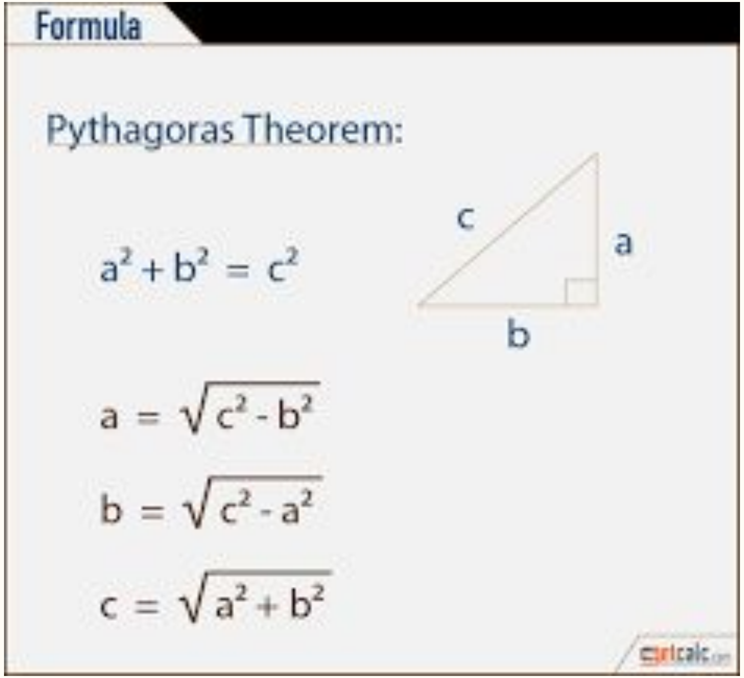
$$\text{sine } of\ \angle A = \sin A = \frac{Opposite}{Hypotenuse} = \frac{a}{c}$$

$$\text{cosine } of\ \angle A = \cos A = \frac{Adjacent}{Hypotenuse} = \frac{b}{c}$$

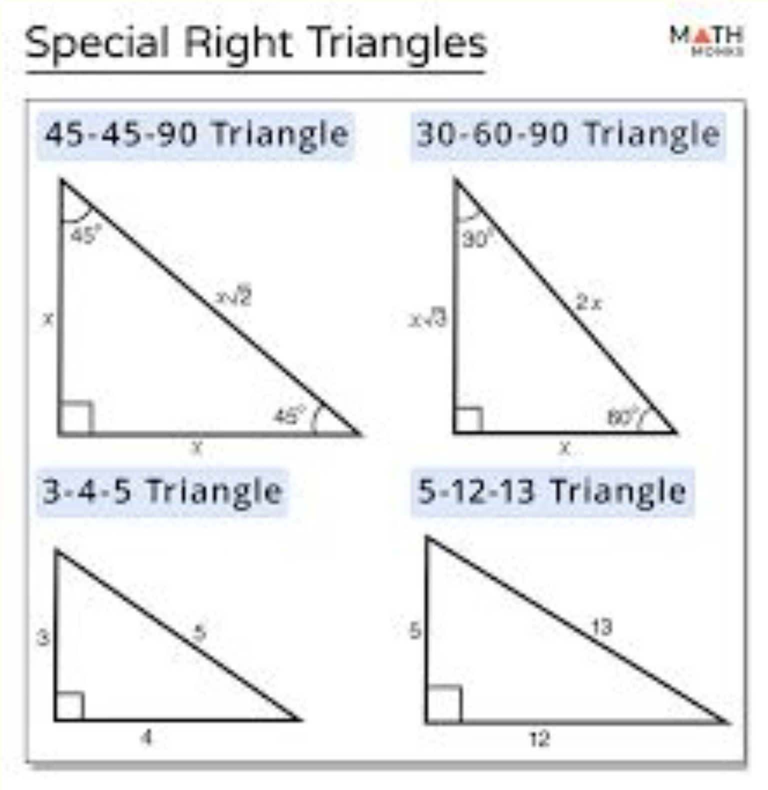
$$\text{tangent } of\ \angle A = \tan A = \frac{Opposite}{Adjacent} = \frac{a}{b}$$

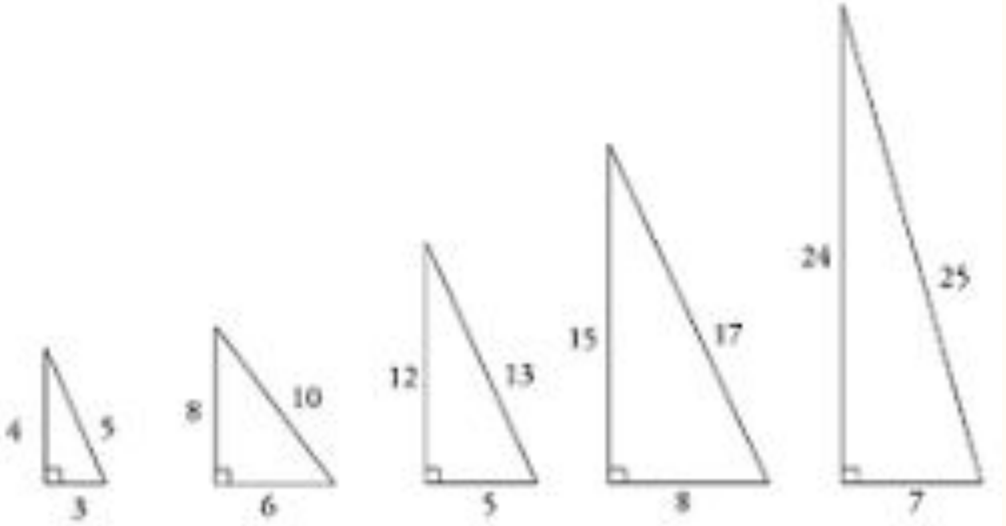
Pythagorean Theorem



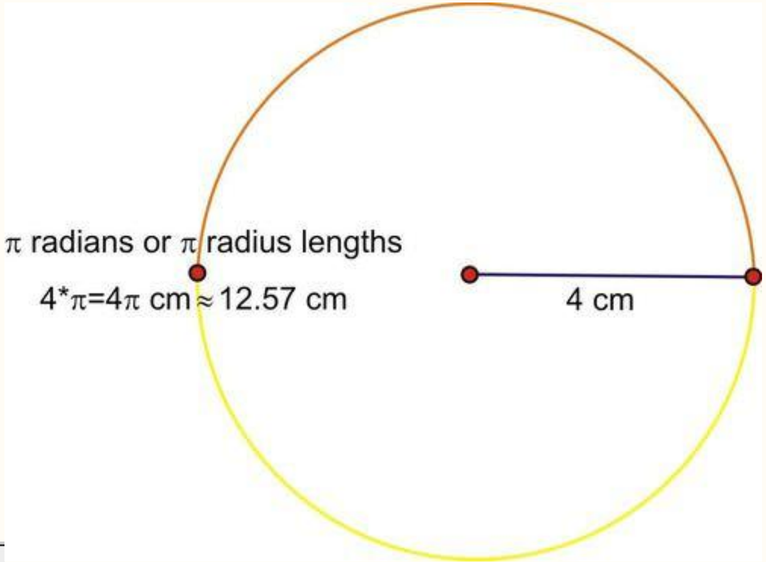


Common Right Triangles





Arc Length using Unit Circle



$$s = r\theta,$$

where s is the length of the arc, r is the radius, and θ is the measure of the angle in radians.

Graph of cos(x) and its properties

Points to graph

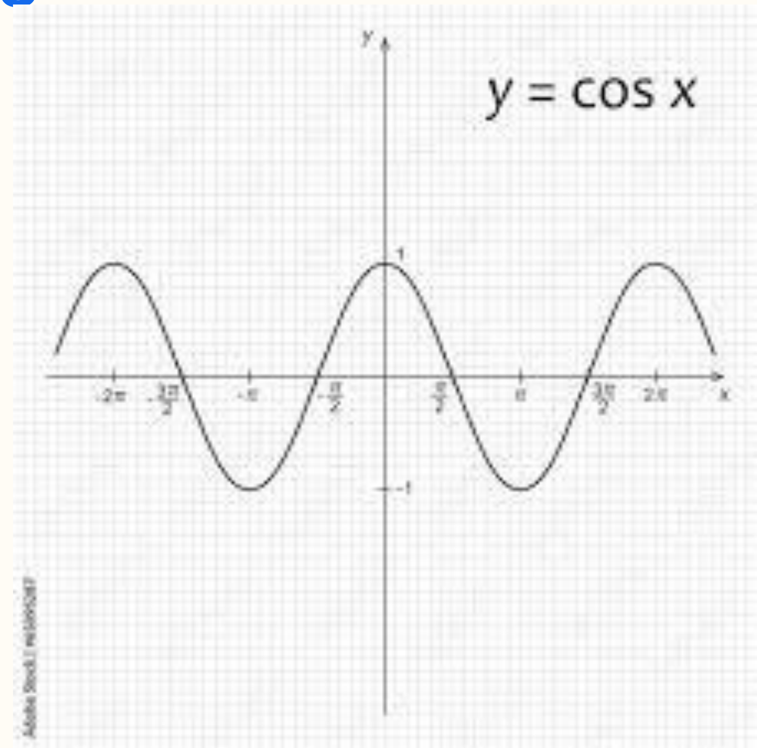
(0,1)

(π/2, 0)

(π, -1)

(3π/2, 0)

(2π, 1)



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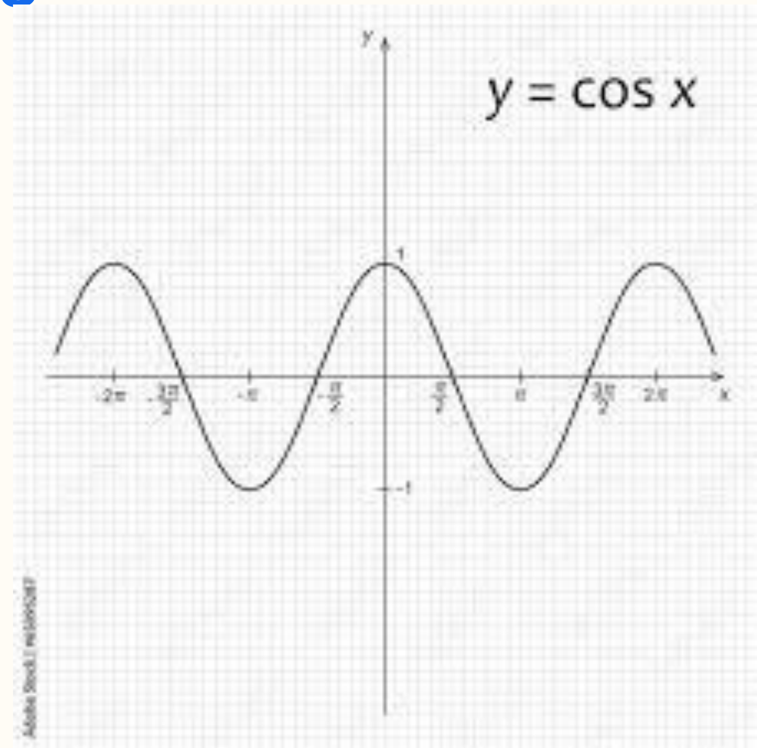
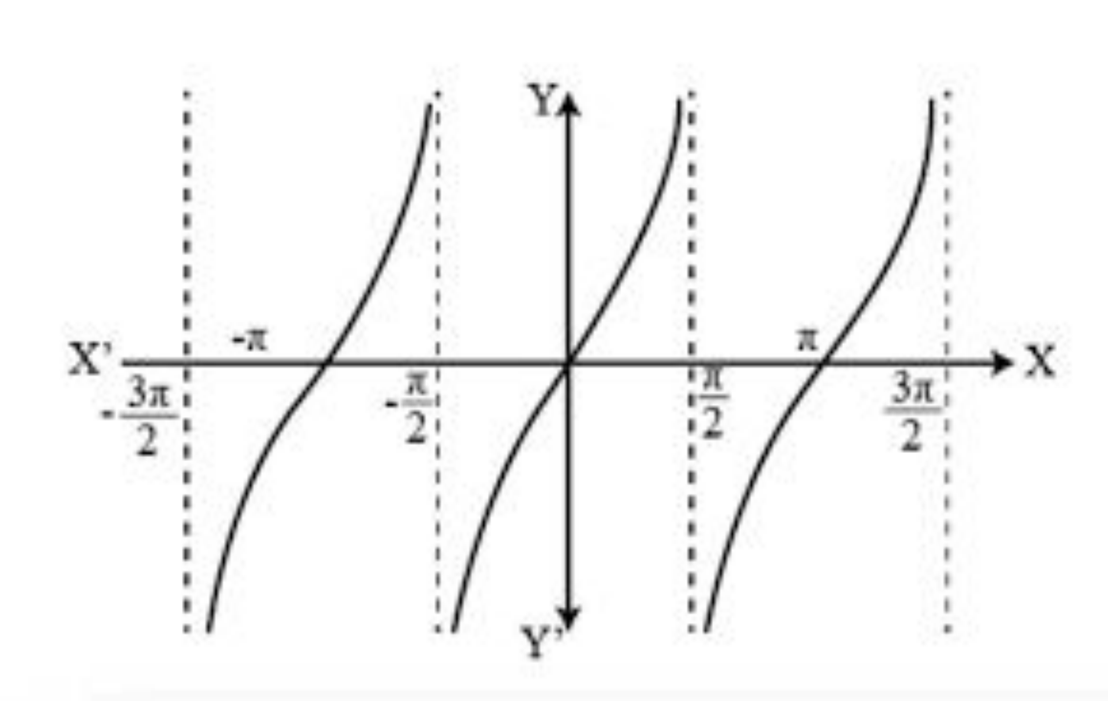


Graph of tan(x) and its properties

Can you tell me why we have horizontal asymptote?



What are inverse Trig Functions?

On these restricted domains, we can define the inverse trigonometric functions.

- The inverse sine function $y = \sin^{-1} x$ means $x = \sin\ y$. The inverse sine function is sometimes called the **arcsine** function, and notated $\arcsin\ x$.

$$y = \sin^{-1} x \text{ has domain } [-1, 1] \text{ and range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- The inverse cosine function $y = \cos^{-1} x$ means $x = \cos\ y$. The inverse cosine function is sometimes called the **arccosine** function, and notated $\arccos\ x$.

$$y = \cos^{-1} x \text{ has domain } [-1, 1] \text{ and range } [0, \pi]$$

- The inverse tangent function $y = \tan^{-1} x$ means $x = \tan\ y$. The inverse tangent function is sometimes called the **arctangent** function, and notated $\arctan\ x$.

$$y = \tan^{-1} x \text{ has domain } (-\infty, \infty) \text{ and range } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

How to Use Trig Functions to Find Exact Values with pi/6, pi/4, and pi/3

Evaluate each of the following.

1. $\sin^{-1}(-1)$
2. $\tan^{-1}(-1)$
3. $\cos^{-1}(-1)$
4. $\cos^{-1}\left(\frac{1}{2}\right)$

Inside the parentheses, we are working with a coordinate and trying to pull a reverse function to find the angle.

Determining exact values of Inverse trig and trig functions combined

Step 1. Evaluate the inverse trig function

$$\cos\left[\sin^{-1}\left(\frac{3}{5}\right)\right]$$

Step 2. Evaluate the regular trig function

Solving Equations using Trig

Step 1. You will isolate the trig function first

Step 2. Use your inverse trig functions to isolate x

Step 3. Find the possible solution(s) while remembering the bounds

Example from Written HW:

Solve the equation: $2\sin(x) = -\sqrt{3}$