



UNIVERSITY OF  
**ILLINOIS**  
URBANA-CHAMPAIGN

# PHYS 211

## Exam 3 Prep



SCAN ME





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## CARE / CARE PHYS 211 Exam Review

New question ^

Net ID   
This allows you to add a question on behalf of a student.

Name   
Using a nickname is fine!

Topic

Location   
Please say whether you are waiting to meet **in person** or **online**. If your instructor requires it, also give the room number or online meeting URL.

# 1. Overview

Quick Bits of info to know

Teacher : what is newton's second law, class?

student:  $F_{net} = ma$

Rotational Dynamics:



# Rotational Kinematics

- Same concept as linear kinematics
- Linear-Rotation relations only apply for Rolling without Slipping

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2} at^2$	$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta \theta$

## Converting between the two:

$$v = \omega * r$$

$$a = \alpha * r$$

$$x = \theta * r$$

Variables:

$\omega$  = angular velocity

$\alpha$  = angular acceleration

$\theta$  = angular displacement

# Rotational Dynamics

- Moment of Inertia
  - The farther the distribution of mass from the Axis of Rotation, greater the Moment of Inertia
- Parallel Axis Theorem
  - Moment of Inertia about an axis other than Center of Mass

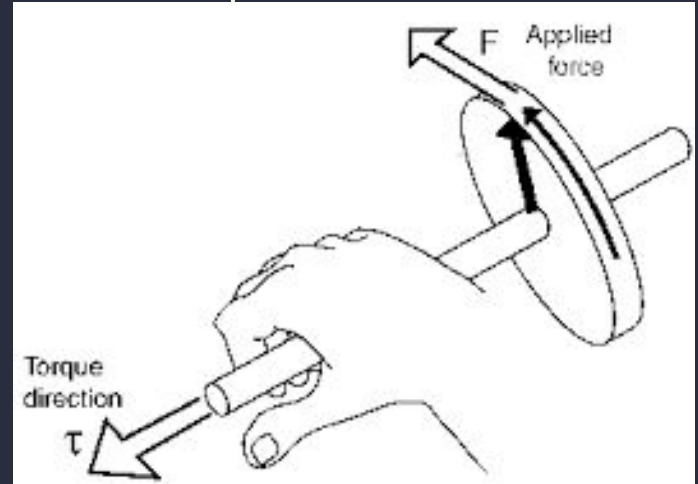
$$I_{\text{parallel}} = I_{\text{CM}} + MD^2$$

Many equivalencies

	<i>Linear Motion</i>	<i>Rotational Motion</i>
Newton's second law	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Work	$W = F\Delta x$ or $W = \int F \cdot dx$	$W = \tau\Delta\theta$ or $W = \int \tau \cdot d\theta$
Kinetic energy	$K \cdot E = \frac{1}{2} mv^2$	$K \cdot E = \frac{1}{2} I\omega^2$

# Torque

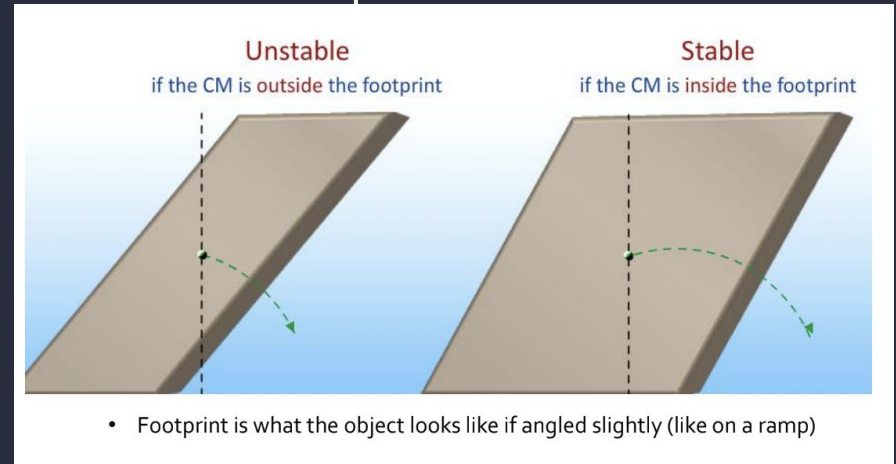
- Use right hand rule
  - Conventionally, thumb towards you is +, towards screen is -
- Visualize how force turns object
  - Clockwise (-)
  - Counterclockwise (+)
- Torque can either have  $\sin(\theta)$  or  $\cos(\theta) = \sin(90 - \theta)$  depending on the angle between  $r$  and  $F$



$$\tau = I\alpha \text{ (rotation about a fixed axis)}$$
$$\tau = \mathbf{r} \times \mathbf{F}, |\tau| = rF\sin\phi$$

# Statics

- During static equilibrium,
  - $\sum F_x = 0$
  - $\sum F_y = 0$
  - $\sum \tau = 0$  at all points on object
- Tipping
  - If Center of Mass is outside vertical footprint of object  $\rightarrow$  Will tip
  - If Center of Mass is inside vertical footprint of object  $\rightarrow$  Will slide



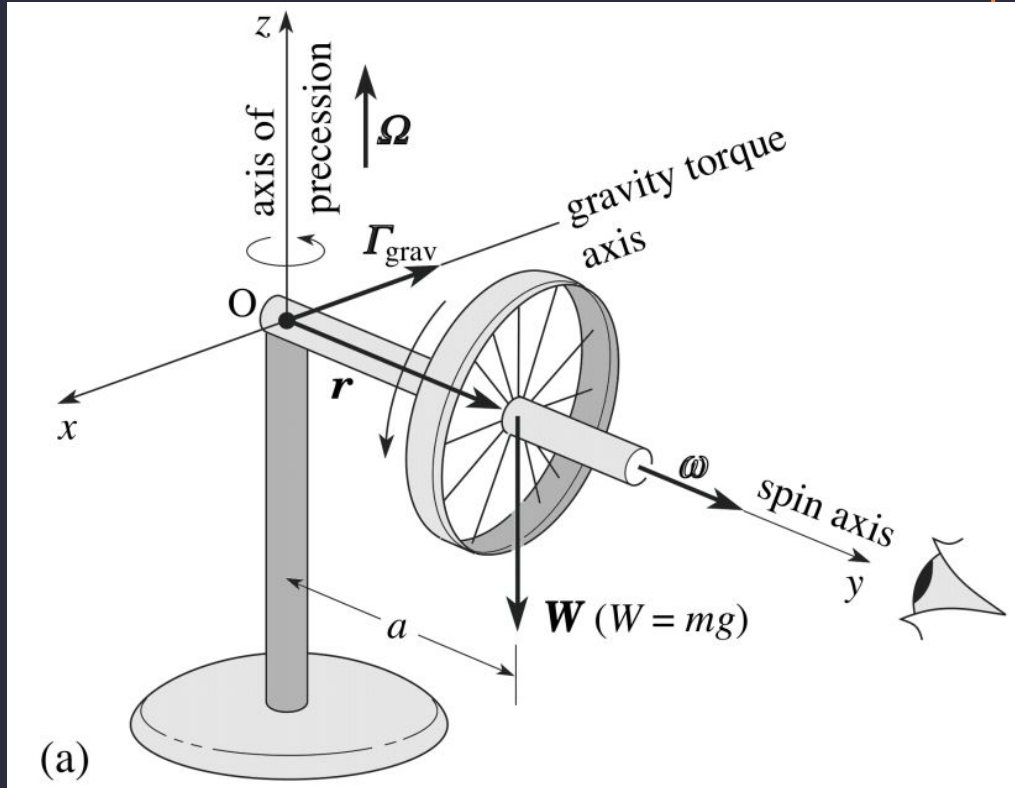


# Angular Momentum (L)

- Right hand rule to find Direction
  - Curl fingers in direction of rotation
- Conserved when net torque = 0.
- $\tau = dL/dt$
- From linear motion
  - $L = mvR \rightarrow R = \text{distance from axis of rotation}$
- Precession ( $\Omega$ ) = Torque ( $\tau$ ) / Angular Momentum (L)
  - Period =  $(2\pi)/\Omega$

$$L = I\omega = r \times p$$

# Direction of Precession

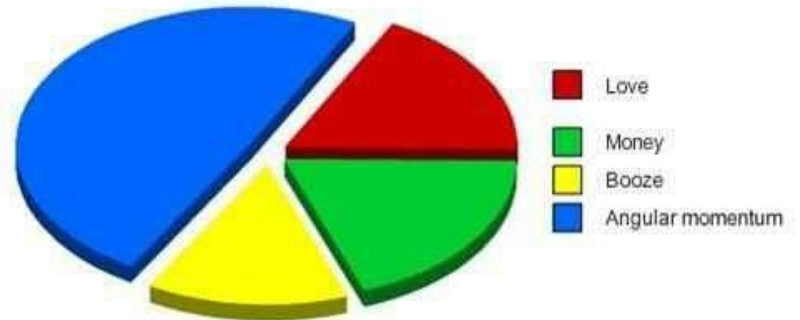


Tip: Curl fingers from rotation vector ( $\boldsymbol{\omega}$ ) to gravity torque vector

# 2. Problem Solving

Some Steps to Follow If You Are Lost

**What makes the world go round?**



# Rotational Kinematics

How to Identify:

- Has rotational variables ( $w, l, t$ )



# Rotational Kinematics

Treat just like linear kinematics!

- Write out all given variables and unknowns
- Choose equation that contains all given variables and unknowns
- Solve for unknowns
- Might need to convert between translational and rotational!

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2} at^2$	$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta \theta$

Converting between the two:

$$v = \omega * r$$

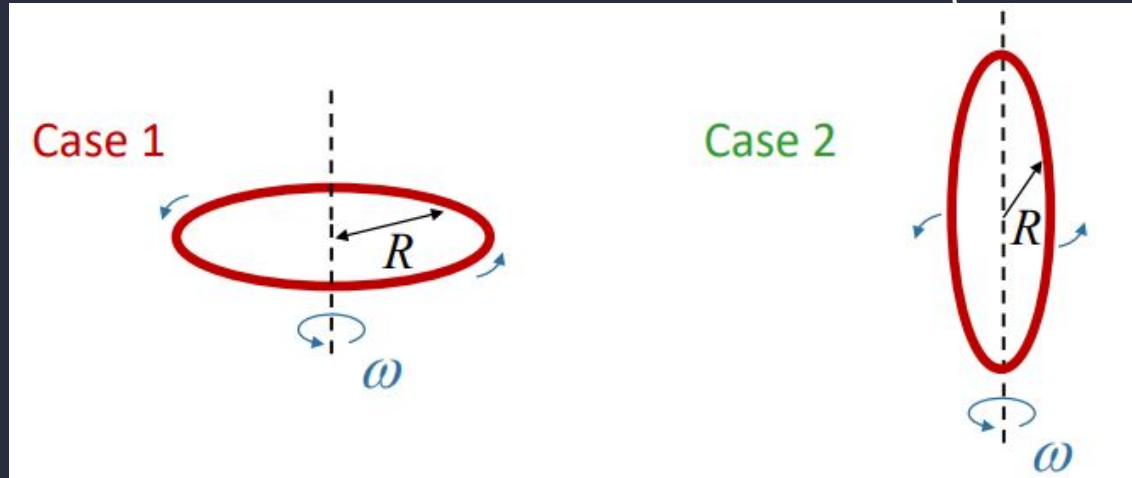
$$a = \alpha * r$$

$$x = \theta * r$$

# Rotational Kinematics Concept Question

In which case does the spinning hoop have the largest kinetic energy?

1. Case 1
2. Case 2



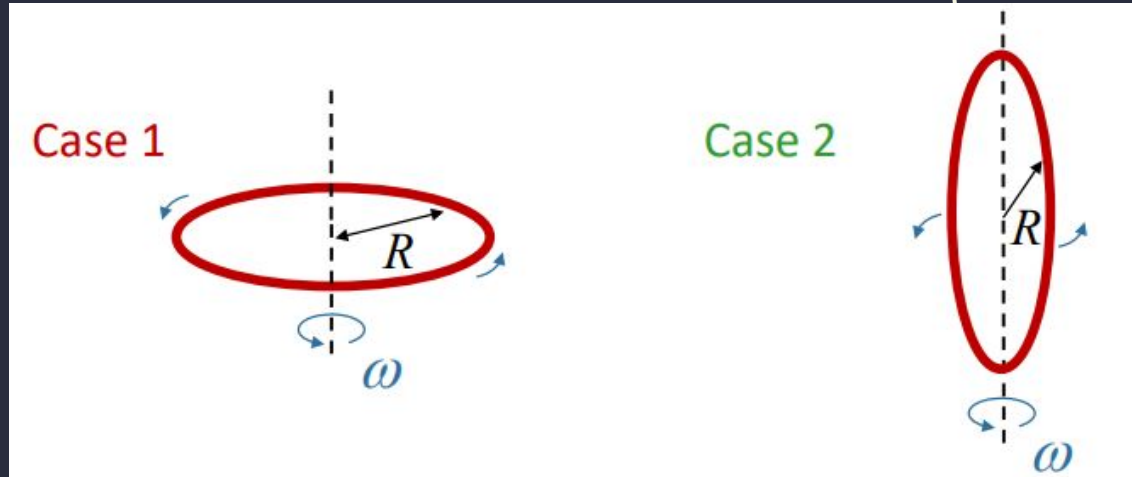
# Rotational Kinematics Concept Question

*Work & Energy*

$$K_{\text{rotation}} = \frac{1}{2}I\omega^2$$

In which case does the spinning hoop have the largest kinetic energy?

1. Case 1
  2. Case 2
- There is a larger contribution  $R$  further away from the center of mass -  $I$  is bigger**



# Rotational Dynamics

How to Identify:

- “Find moment of inertia about...”
- An object rotating around an axis



# Rotational Dynamics

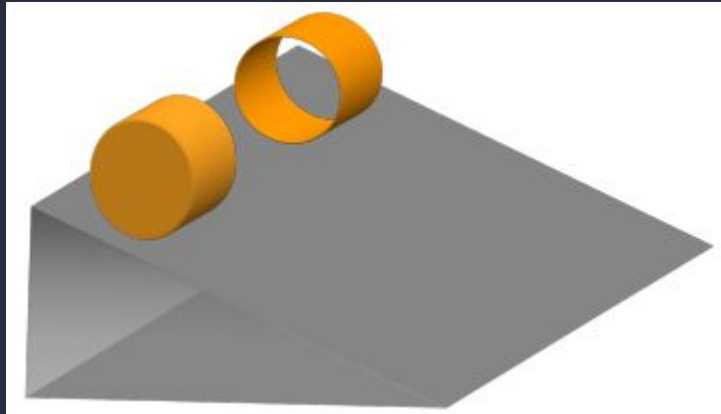
- Identify what kind of shape the object is and what axis it's rotating around
- Match with a Moment of Inertia equation & solve for moment of inertia
- If axis is not center of mass, use parallel axis theorem

## Rotational Dynamics Concept Question 1

A cylinder and a hoop are rolling down a ramp with the same mass and radius. Which reaches the bottom first?

1. Cylinder
2. Hoop

$$I_{\text{disk}} = I_{\text{cylinder}} = \frac{1}{2}MR^2$$
$$I_{\text{hoop}} = MR^2$$



## Rotational Dynamics Concept Question 1

A cylinder and a hoop are rolling down a ramp. Which reaches the bottom first? **1. The cylinder.** The hoop has a larger moment of inertia. If they both start with the same amount of potential energy, the hoop would have more rotational kinetic energy than the cylinder which means it is travelling slower.

$$K_{\text{rotation}} = \frac{1}{2}I\omega^2$$

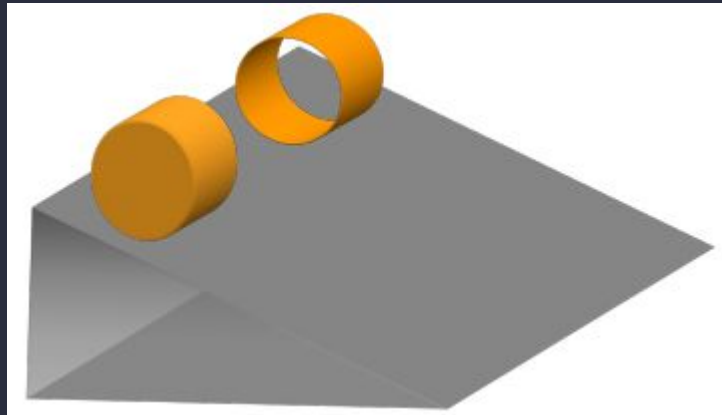
$$K_{\text{translation}} = \frac{1}{2}MV_{\text{cm}}^2$$

$$K_{\text{total}} = K_{\text{rotation}} + K_{\text{translation}}$$

## Rotational Dynamics Concept Question 2

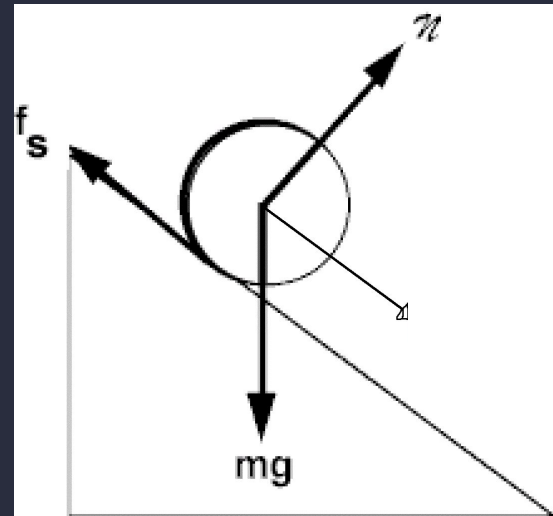
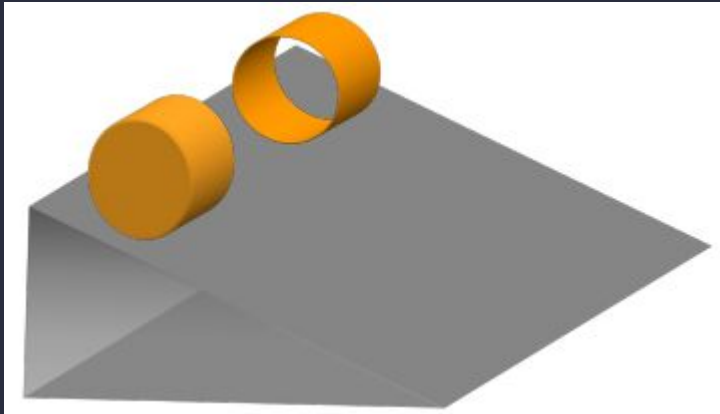
Which way does friction point on the wheels?

1. Down the Ramp
2. Out of the sides of the Ramp
3. Up the Ramp



## Rotational Dynamics Concept Question 2

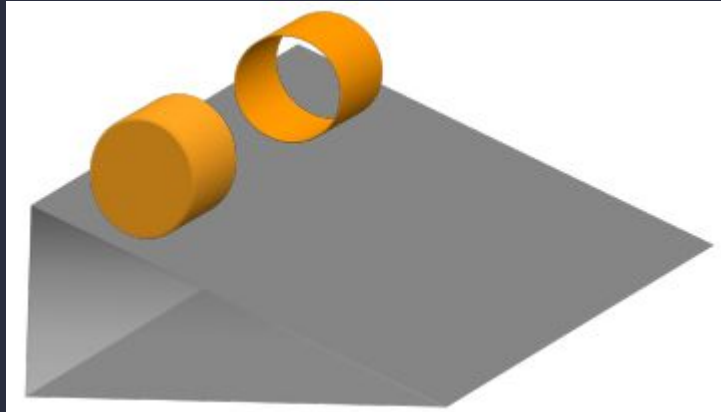
Friction **points up the ramp** to produce a speeding up torque, yet oppose the translational motion.



## Rotational Dynamics Concept Question 3

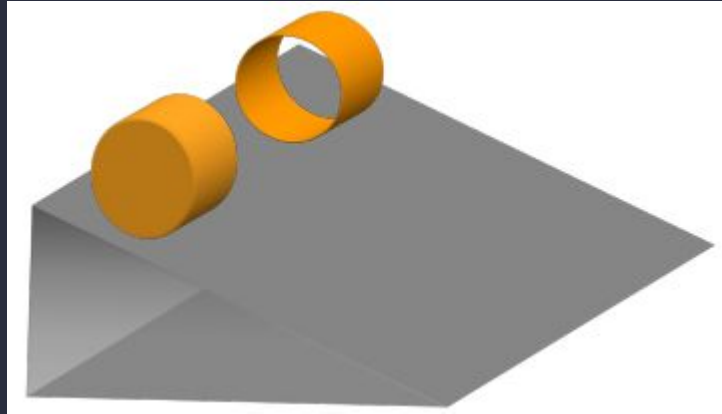
Is the friction static or dynamic? (they are not sliding)

1. Static
2. Dynamic



## Rotational Dynamics Concept Question 3

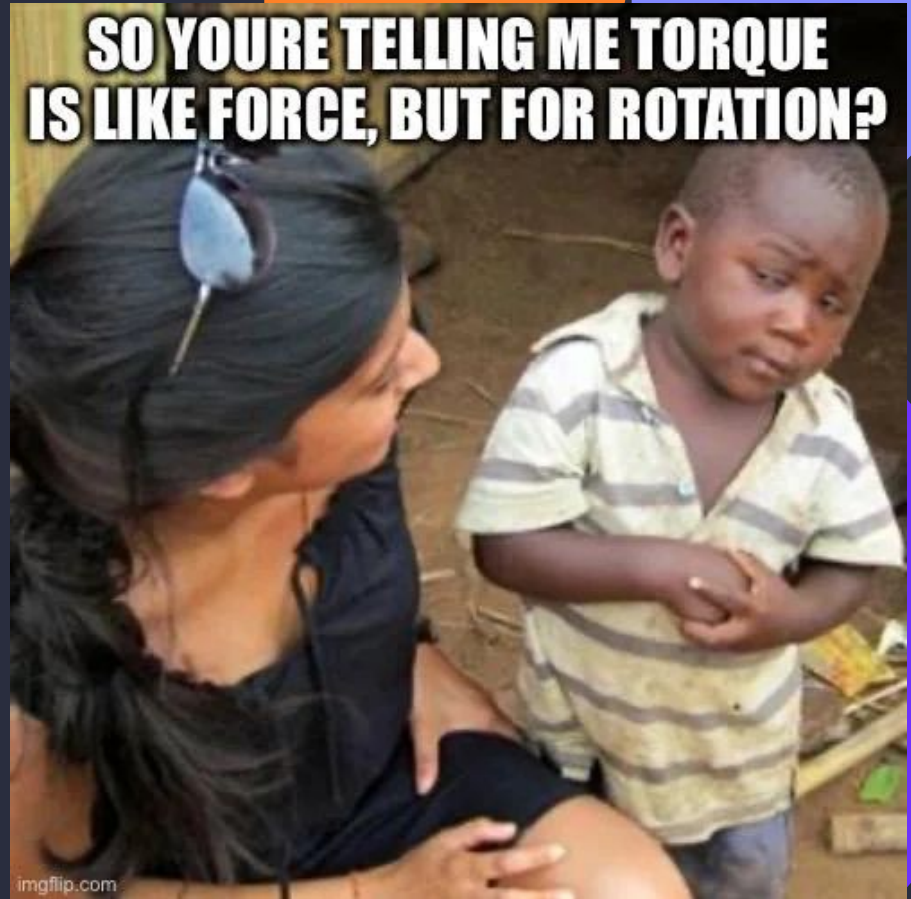
The friction is **static** because they aren't sliding.



# Torque

How to Identify:

- “Find sum of torques”
- Static problems



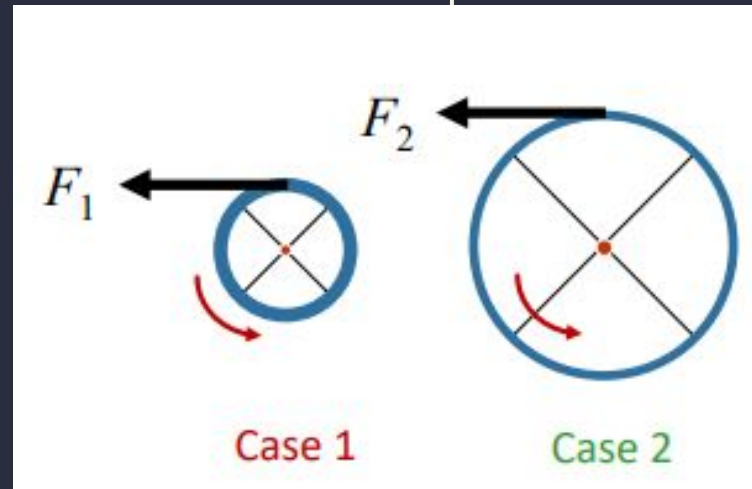
# Torque

- Choose a point of rotation to take torques about
  - Helpful to choose point that cancels out reaction forces
- Determine if object is moving ( $\Sigma\tau = I\alpha$ ) or is still ( $\Sigma\tau = 0$ )
- Use equation to find missing variable or quantity

## Torque Concept Question

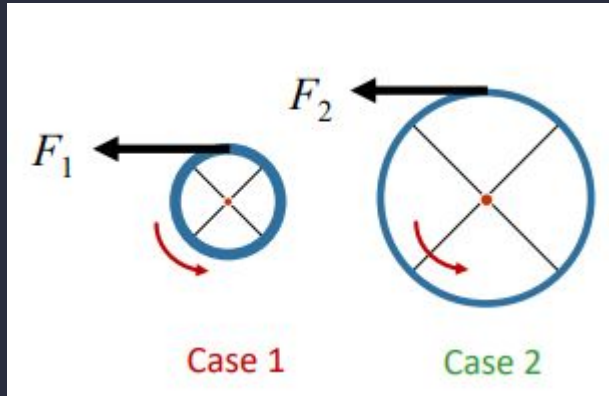
If the hoops below have the same mass and the same force, case 1 has radius  $R$  and case 2 has radius  $2R$ , how does the **angular acceleration** relate?

1. Case 1  $>$  Case 2
2. Case 1  $<$  Case 2
3. Case 1 = Case 2



## Torque Concept Question

If the hoops below have the same mass and the same force, case 1 has radius  $R$  and case 2 has radius  $2R$ , how does the angular acceleration relate? **Answer: (1) Case 1 > Case 2**



Formulas:

$$\tau = I\alpha, \tau = R \times F, I = MR^2$$

$$\rightarrow R \times F = MR^2 \alpha, R \times F \text{ at } 90^\circ$$

$$\alpha = \frac{F}{MR}$$

Case 1

$$\alpha_1 = \frac{F}{MR}$$

$$\alpha_1 = 2\alpha_2$$

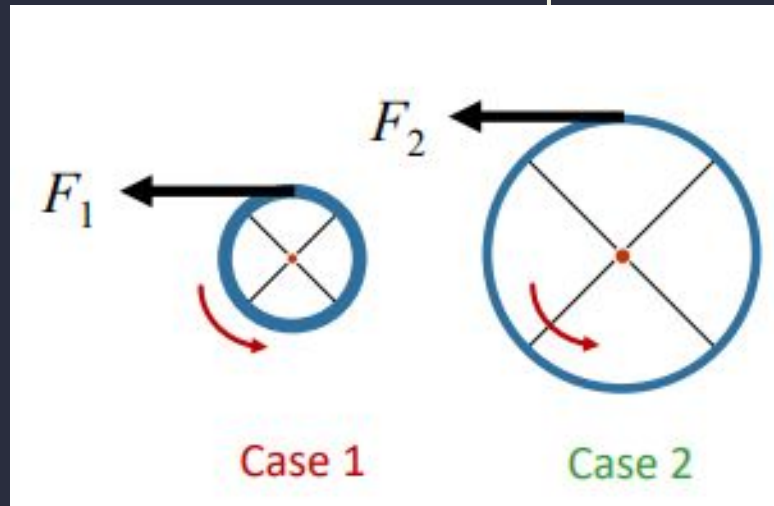
Case 2

$$\alpha_2 = \frac{F}{M \cdot 2R}$$

## Torque Concept Question

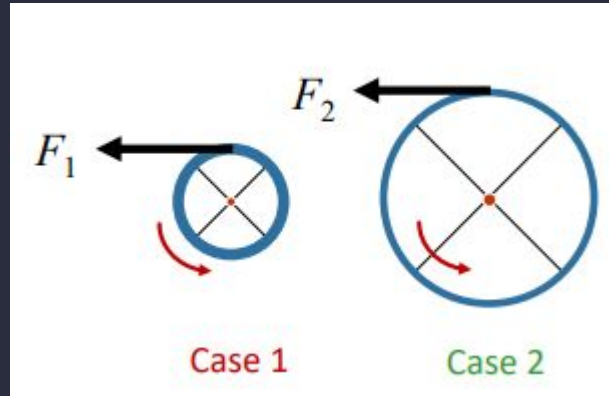
If the hoops below have the same mass and the same force, case 1 has radius  $R$  and case 2 has radius  $2R$ . What is the direction of torque?

1. Into Page
2. Out of Page



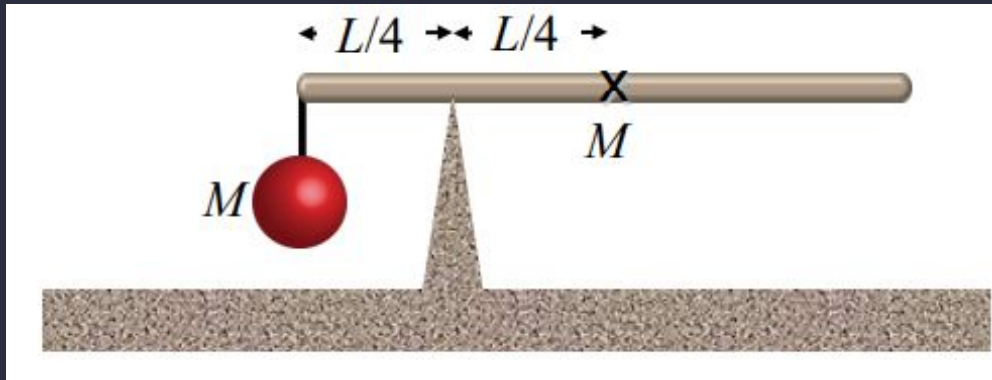
## Torque Concept Question

Torque is out of the page and positive



## Static Equilibrium Concept Question

An object is made by hanging a ball of mass  $M$  from one end of a plank and having the same mass and length  $L$ . The object is then pivoted at a point a distance  $L/4$  from the end of the plank supporting the ball. Is the system balanced?

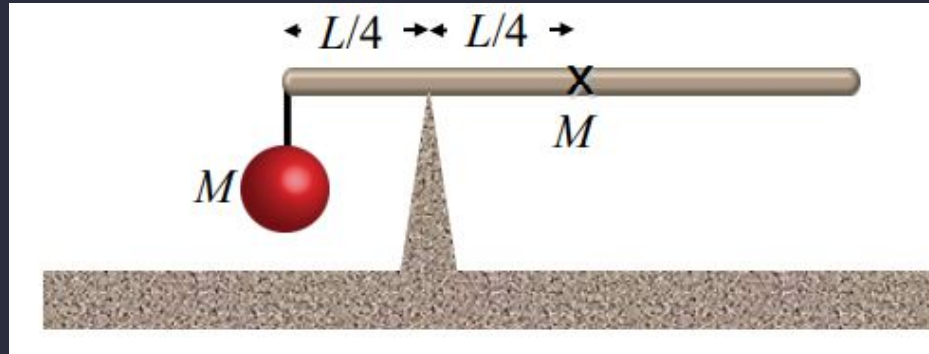


1. Yes
2. No

## Static Equilibrium Concept Question

An object is made by hanging a ball of mass  $M$  from one end of a plank and having the same mass and length  $L$ . The object is then pivoted at a point a distance  $L/4$  from the end of the plank supporting the ball. Is the system balanced? **Yes. The pivot is positioned at the center of mass of the system meaning the system is at static equilibrium.**

1. Yes
2. No



# Angular Momentum

How to Identify:

- Gyroscopes
- Precession
- Rotating object

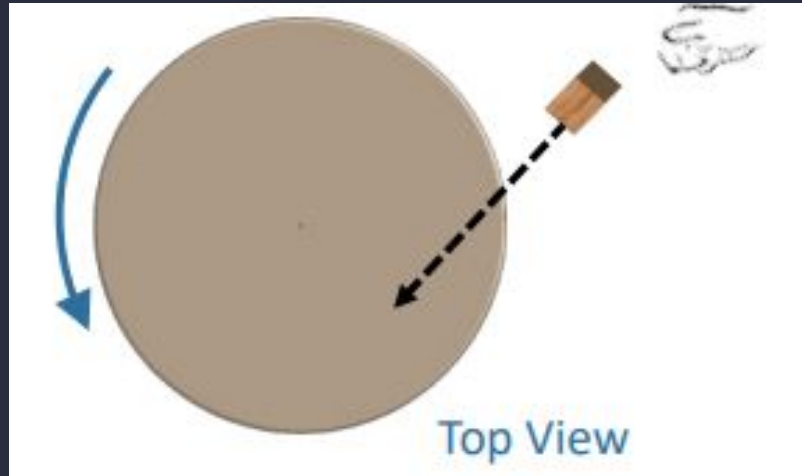


# Angular Momentum

- Identify what kind of motion, moment of inertia, angular velocity, etc
  - If linear motion, use  $L = pR = mvR$
  - If rotational motion, use  $L = I\omega$
  - Use correct moment of inertia
- Use right hand rules accordingly
- Angular momentum is conserved if no external torques
  - usually conserved in this class
  - $L_i = L_f$

## Angular Momentum Concept Question

The initial magnitude of the angular momentum of a freely rotating disk is  $L$ . You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. Is the total angular momentum of the disk-block system conserved during this event (after the block has left your hand)?

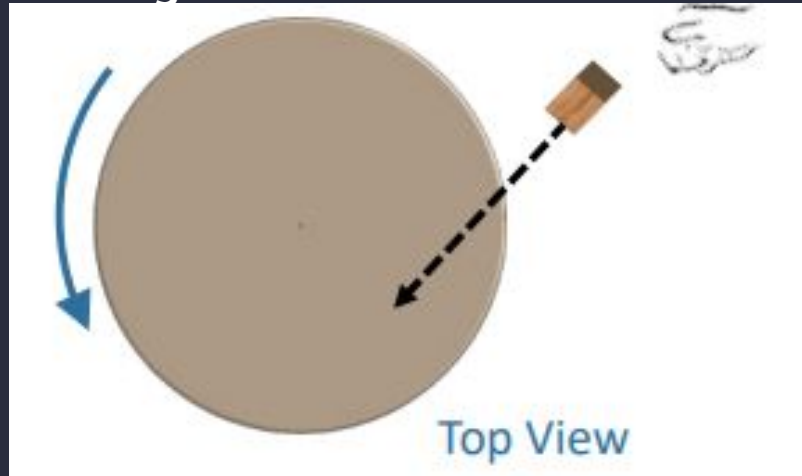


1. Yes
2. No

## Angular Momentum Concept Question

The initial magnitude of the angular momentum of a freely rotating disk is  $L$ . You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. Is the total angular momentum of the disk-block system conserved during this event (after the block has left your hand)?

**Angular momentum is conserved as there are no external torques.**



1. Yes
2. No



## Worksheet Time!

Enter Queue with your name and net ID:  
By entering the queue, you help us:

- Reserve a big enough space at the next review session

- Assign enough tutors for everyone to have access to help

Thank you!

