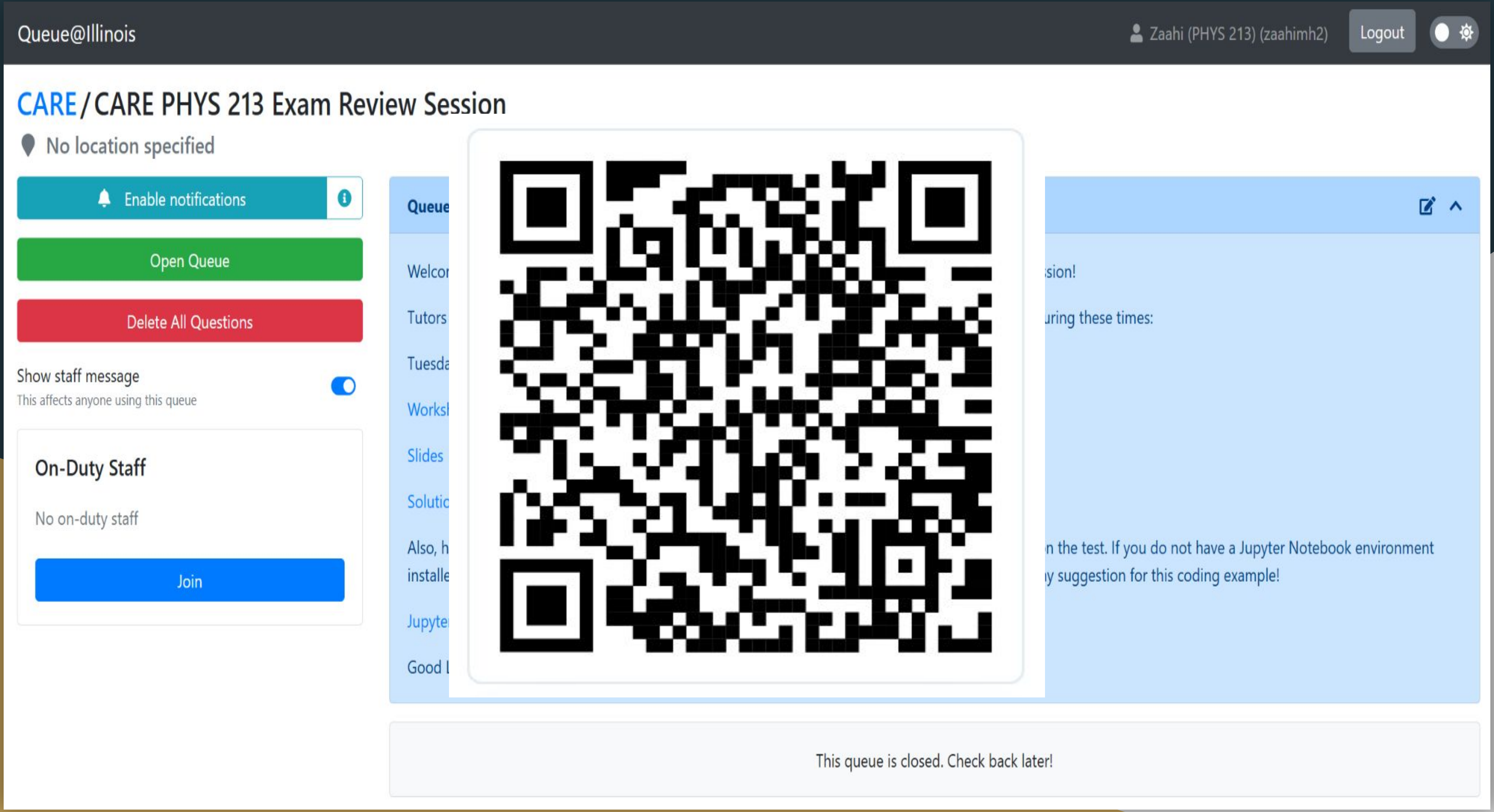
# C.A.R.E. PHYS 213 Quiz 2 Review Session



CARE / CARE PHYS 213 Exam Review Session

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Also, h n the test. If you do not have a Jupyter Notebook environment installe y suggestion for this coding example!

Jupyte

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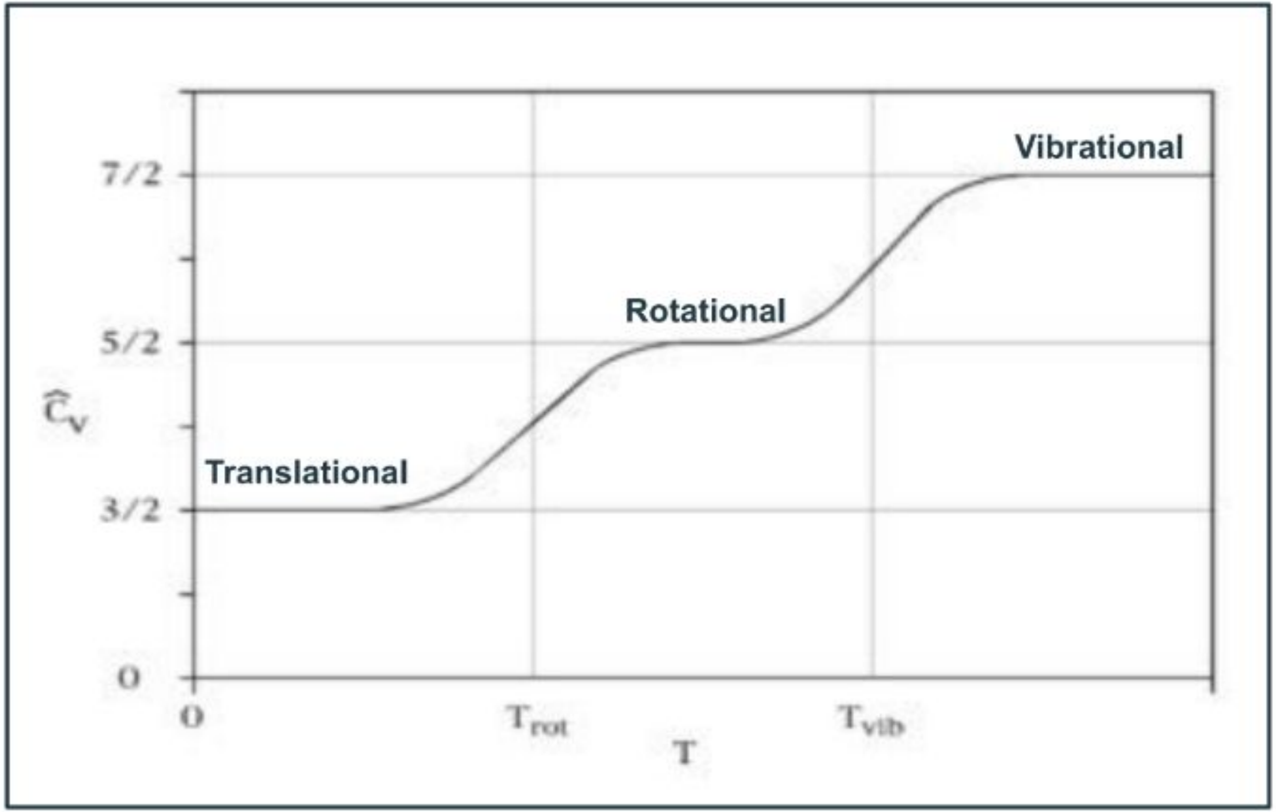
# Units for the Exam

- Kinetic Theory of Ideal Gases
- Quasistatic Processes
- Thermodynamic Cycles
- Gibbs Free Energy

# Ideal gas and Equipartition

- Ideal Gas: Approximation of particles as points with no interactions:
    - Follows **ideal gas law**: $\boxed{pV = NkT = nRT}$
- **Equipartition:** each degree of freedom contributes ½ $kT$ of energy
    - Internal energy in each particle:
        - $U = (N_{DOF}/2)kT$
    - Molar heat capacity:
        - $c_M = (N_{DOF}/2)kN_A$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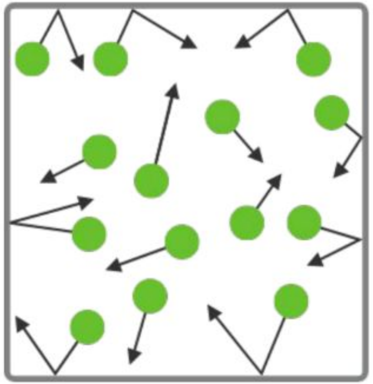
# Root-Mean-Square Velocity

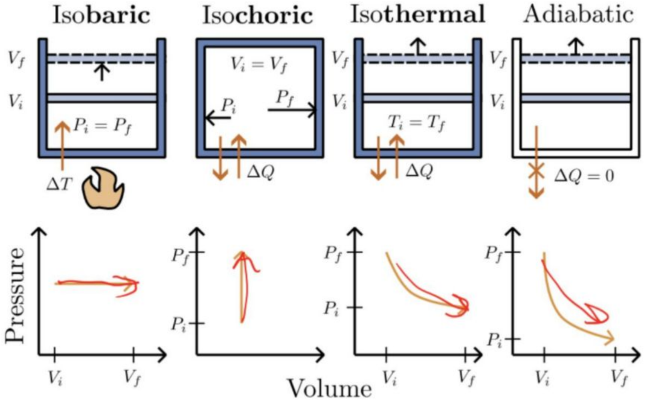


- $v_{rms}$: Average **(translational)** velocity of gas particles
- Translational Kinetic Energy: $KE_{translational} = 1/2\ m\ (v_{rms})^2$
- Relationship to temperature:

$$\boxed{\frac{1}{2} m {v_{rms}}^2 = \frac{3}{2} kT}$$

- This only applies to **ONE PARTICLE**
- **Notice:** this does **<u>NOT</u>** depend on the number of DOFs; it's **<u>ALWAYS</u>** (3/2)*kT*
  - Why? Translational KE only depends on the translational modes motion (there are only 3 translational modes: $v_x$, $v_y$, $v_z$)

# Thermodynamic Processes



- Isochoric or Isovolumetric
    - Constant VOLUME
- Isobaric
    - Constant PRESSURE
- Isothermal
    - Constant TEMPERATURE, **REVERSIBLE**

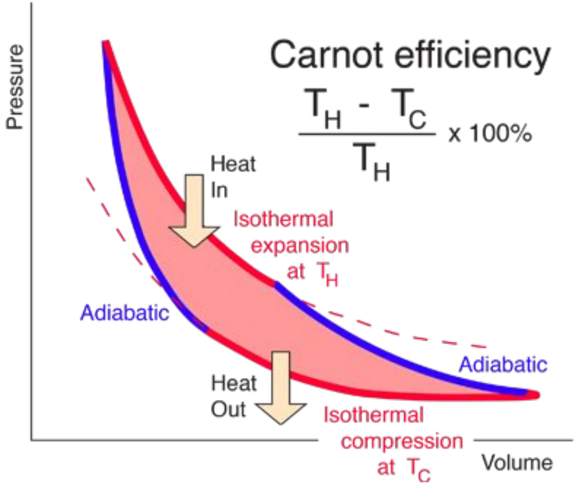
$$(\Delta S_{total} = 0, \Delta U = 0)$$

- Adiabatic
    - Constant HEAT **(dQ = 0)**, **REVERSIBLE**

$$(\Delta S_{total} = 0, \Delta Q = 0)$$

# Reversible Processes

- **Isothermal** + **Adiabatic** **processes** are **REVERSIBLE**
    - $\Delta S_{total} = 0$ (no change in entropy)
    - For **isothermal** processes:
        - $PV$ = constant
    - For **adiabatic** processes:
        - $PV^{\gamma}$ = constant
        - $\gamma = (2/N_{DOF}) + 1$ (given on equation sheet)



# Example Problem - Adiabatic Process

- Assume we have a gas undergoing an adiabatic process, determine the work done given the following parameters:
    - $V_i = 10\ \text{m}^3$, $p_i = 10\ \text{kPa}$
    - $V_f = 4\ \text{m}^3$, $N_{DOF} = 3$

1. Calculate $\gamma$

2. Find $pV^{\gamma}$

3. Calculate $W$

4. Calculate $W$

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$$\gamma = \frac{2}{N_{DOF}} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$$

2. Find $pV^{\gamma}$

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$$\gamma = \frac{2}{N_{DOF}} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$$

$$pV^{\gamma} = Constant = p_i V_i^{\gamma} = (1000)(10)^{\frac{5}{3}} = 464158.88$$

3. Calculate *W*

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$$W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{C}{V_i^{\gamma}}\, dV = \left[ C \frac{V_f^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right] - \left[ C \frac{V_i^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1} \right]$$

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# Example Problem - Adiabatic Process

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$$W = \left[(464158.88) \frac{4^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1}\right] - \left[(464158.88) \frac{10^{\left(-\frac{5}{3}+1\right)}}{-\frac{5}{3}+1}\right] \approx -126.30\ kJ$$

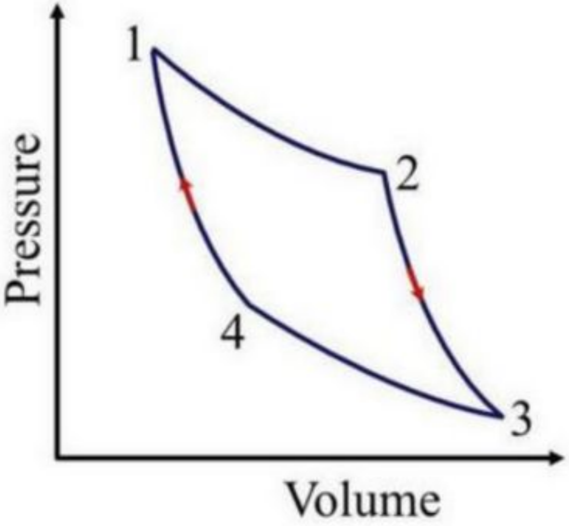
$$W \approx -126.30\ kJ$$

# p-V Diagrams

$$W_{\mathrm{by}} = \int_{V_i}^{V_f} p \, \mathrm{d}V$$

- Used to visualize thermodynamic cycles

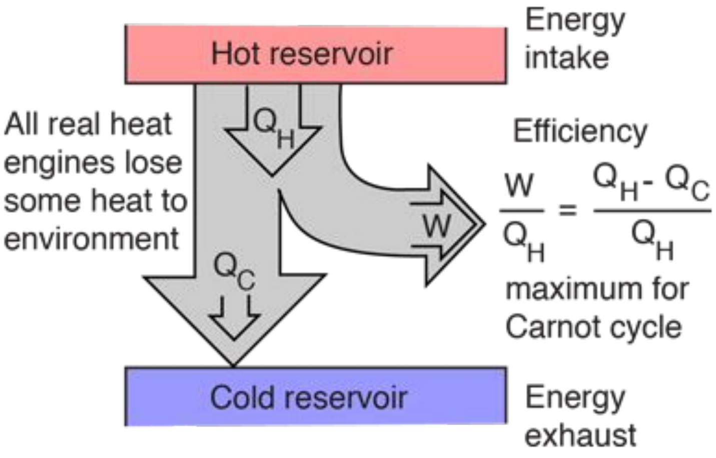
- **Area enclosed in the curve is equal to the work per cycle**
  - **Clockwise direction: work is positive** (engine did work)
  - **Counterclockwise direction: work is negative** (work done on engine)



# Heat Engines

**Efficiency/COP can be thought of as "what you get out" divided by "what you put in."**

- Cycles of Thermodynamic processes are used to make engines, heat pumps, and refrigerators
- **Efficiency of engines:** $\varepsilon = \frac{W_{by}}{Q_H} \leq 1 - \frac{T_C}{T_H}$
- **COP of pumps and refrigerators:**
    - Heat pump: $\text{COP} = \frac{Q_H}{W_{on}} \leq \frac{1}{1 - \frac{T_C}{T_H}}$
    - Refrigerator: $\text{COP} = \frac{Q_C}{W_{on}} \leq \frac{1}{\frac{T_H}{T_C} - 1}$



# Example Problem: Engine/Heat Pump COP

Given a hot reservoir at a temperature of $T_h = 373K$ and a cold reservoir at a temperature of $T_c = 293K$, calculate the maximum efficiency $\epsilon$ of an engine and the maximum COP of a heat pump between the two reservoirs.

Heat Engine Efficiency:

Heat Pump COP:

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Heat Engine Efficiency:

$$\epsilon = 1 - \frac{T_c}{T_h} \approx 0.21$$

Heat Pump COP:

[redacted]

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Given a hot reservoir at a temperature of $T_h = 373K$ and a cold reservoir at a temperature of $T_c = 293K$, calculate the maximum efficiency $\epsilon$ of an engine and the maximum COP of a heat pump between the two reservoirs.

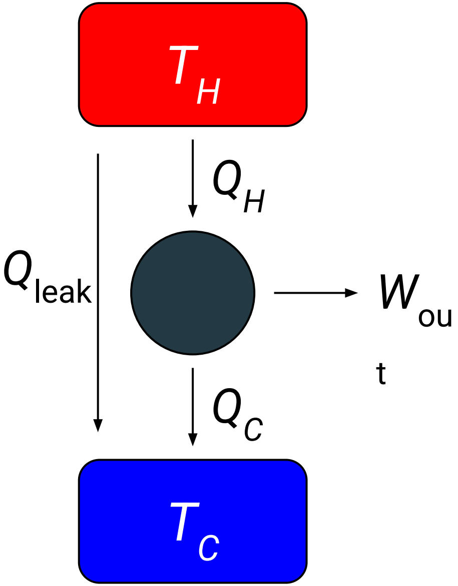
Heat Engine Efficiency:

$$\epsilon = 1 - \frac{T_c}{T_h} \approx 0.21$$

Heat Pump COP:

$$COP = \frac{1}{1 - T_c/T_h} \approx 4.7$$

# Engine, Pump, and Refrigerator Diagrams



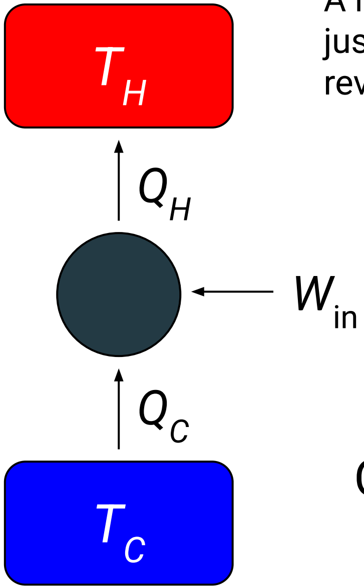
**Heat Engine Diagram**

In a leaky engine, the heat we put in is $Q_H + Q_{leak}$, which affects the efficiency.

$$Q_H = W_{out} + Q_C$$

$$\epsilon = W_{out}/(Q_H + Q_{leak})$$

$$\epsilon_C = 1 - (T_C/T_H)$$



**Heat Pump/Refrigerator Diagram**

A heat pump or fridge is just a heat engine run in reverse.

$$Q_H = W_{in} + Q_C$$

# Gibbs Free Energy

- Useful when **temperature** and **pressure** are fixed

$$\boxed{\mathrm{G} = \mathrm{U} - T_{env}S + pV}$$

- **Minimizing Gibbs of a system will maximize total (system + environment) Entropy**
  - As a system approaches equilibrium, free energy will decrease to a minimum

- **Fundamental Thermodynamic Relation in Equilibrium:**
  - $TdS = dU + pdV - \mu dN$
  - $\mu = (dG/dN) \rightarrow \mu N = G$ (at fixed temperature and pressure)
  - Equilibrium favors lowest $\mu$

# Good luck!

Feel free to ask any questions you may have.

**You got this!**