



The Grainger College of Engineering

Center for Academic Resources in Engineering

MATH 241

Midterm 3 Review

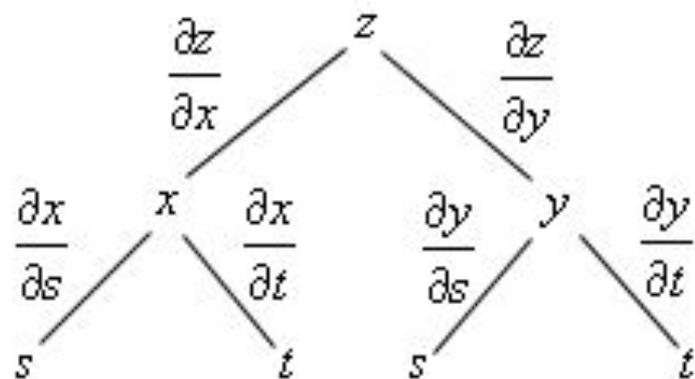
Keep in mind that this presentation was created by CARE tutors, and while it is thorough, it is not comprehensive.

QR Code to the Queue



The queue contains the worksheet and the solution to this review session

Chain Rule



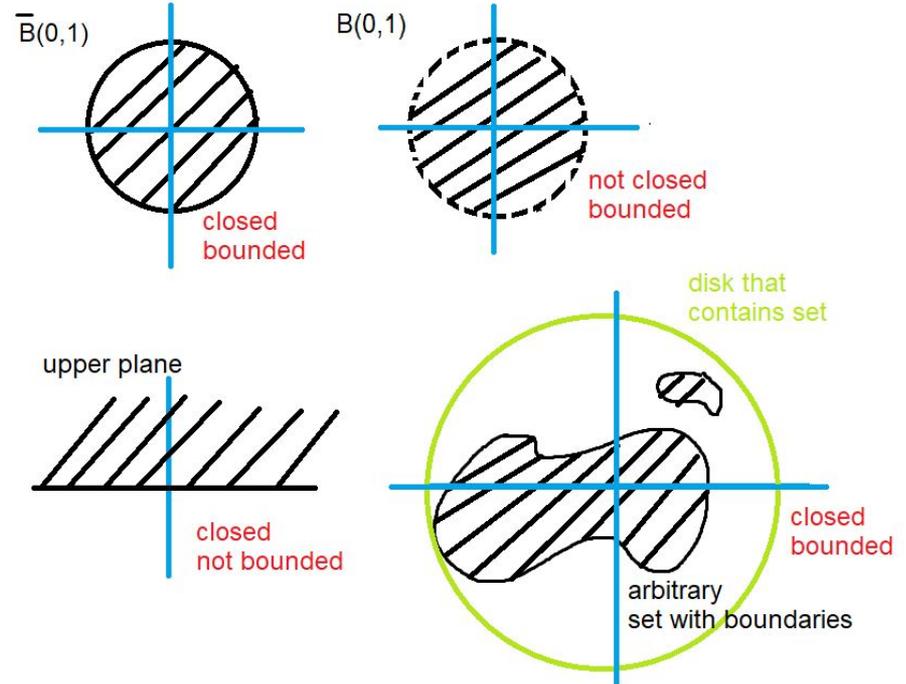
$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

change in z with respect to t z_x change in x with respect to t

Extreme Value Theorem

If $f(x,y)$ is continuous on a closed and bounded set D , then it is guaranteed that f has an absolute minimum and maximum value

- The absolute min and max will either occur at the critical points of f , or on the endpoints of the boundary D



Gradient and Directional Derivatives

$$\nabla f = \langle f_x, f_y, f_z \rangle = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

- The gradient will always point **perpendicular to the level curves/surfaces of f**
- $\nabla f = 0$ at a local minimum/maximum

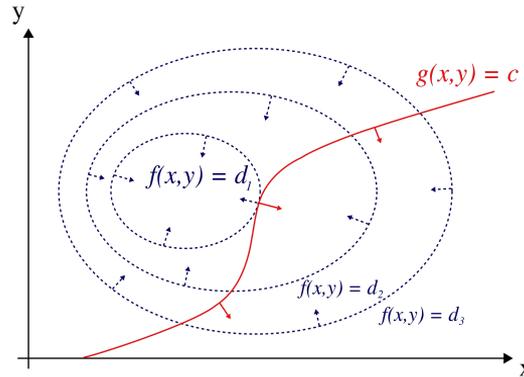
$$D_{\mathbf{u}} f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}$$

- Tells you how the function f changes along the vector \mathbf{u}

Lagrange Multiplier

- Solve the following system of equations for λ (Lagrange Multiplier)
 - Where f is the function, and g is the constraint

$$\begin{aligned}\nabla f(x, y, z) &= \lambda \nabla g(x, y, z) \\ g(x, y, z) &= k\end{aligned}$$



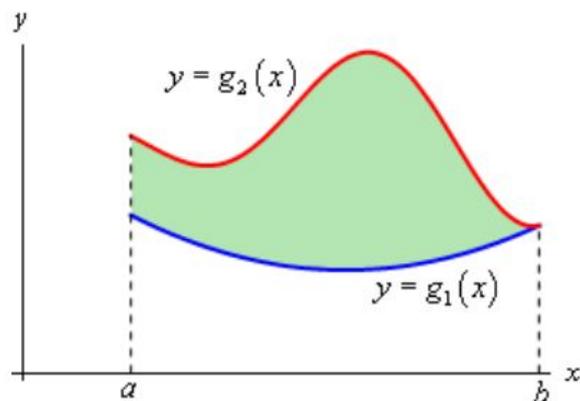
Double Integrals

$$V = \iint_R f(x, y) dA.$$

$$\iint_R f(x, y) dA = \iint_R f(x, y) dx dy = \iint_R f(x, y) dy dx.$$

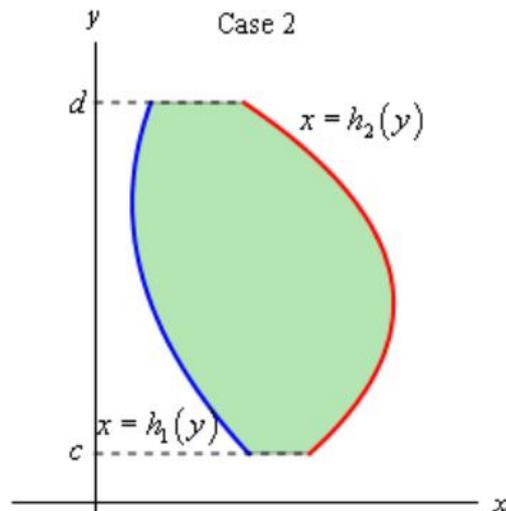
Double Integral Over General Regions

Case 1



$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx$$

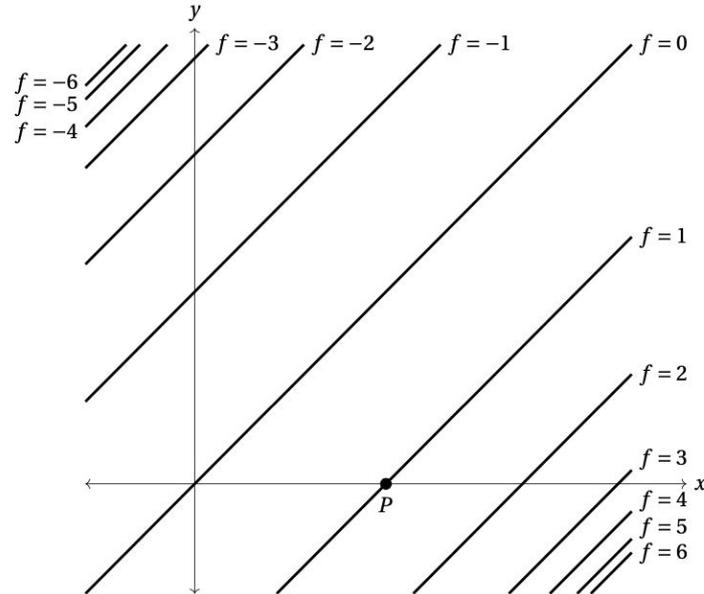
Case 2



$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx \, dy$$

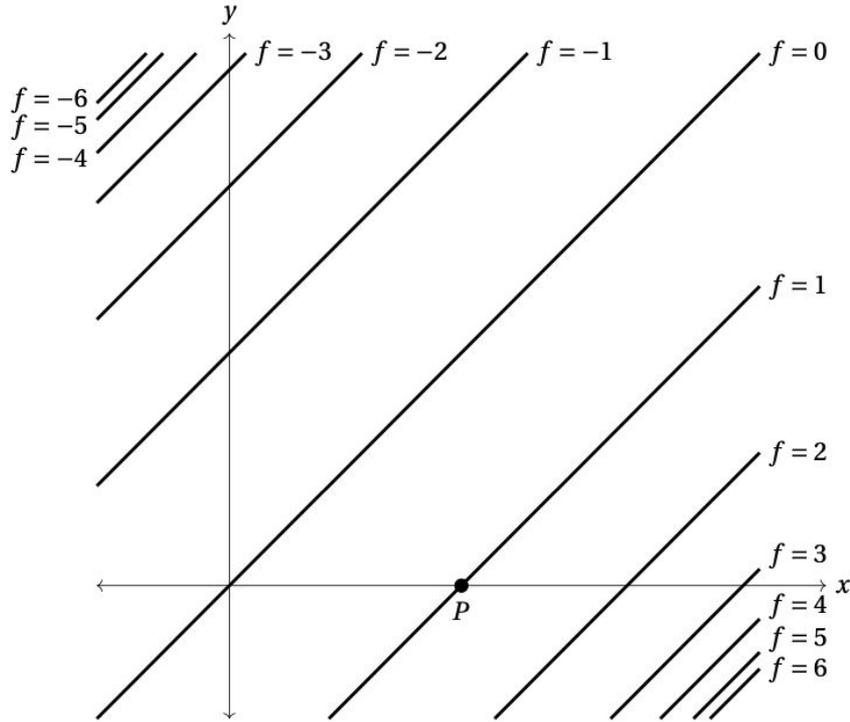
Example Question #1

- A contour map for a function f of x, y , and a point P in the plane are given below. Determine if the following quantities are negative, zero, or positive: $f_x(P)$, $f_{xx}(P)$, $f_{xy}(P)$



Example Solution #1

- $f_x(P)$: positive
- $f_{xx}(P)$: positive
- $f_{xy}(P)$: negative



Example Question #2

- Calculate the following derivative:

Find f_t for $f(x, y) = 2xy$, $x(s, t) = st$, $y(s, t) = s^2t^2$

Example Solution #2

Find f_t for $f(x, y) = 2xy$, $x(s, t) = st$, $y(s, t) = s^2t^2$

Applying Chain Rule:

$$f_t = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t}$$

$$\frac{\partial f}{\partial x} = 2y, \quad \frac{\partial x}{\partial t} = s, \quad \frac{\partial f}{\partial y} = 2x, \quad \frac{\partial y}{\partial t} = 2ts^2$$

$$f_t = 6s^3t^2$$

Example Question #3

- Consider the function $f = x^3 + y^3 + 3xy$. If the critical points of f are $(0, 0)$ and $(-1, -1)$, classify them into local mins, maxes, and saddle points.

Example Solution #3

- Consider the function $f = x^3 + y^3 + 3xy$. If the critical points of f are $(0, 0)$ and $(-1, -1)$, classify them into local mins, maxes, and saddle points.

$$f_x = 3x^2 + 3y \quad f_y = 3y^2 + 3x \quad f_{xx} = 6x \quad f_{yy} = 6y$$

$$f_{xy} = f_{yx} = 3$$

$$\text{At } (0, 0), D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} = -9 \rightarrow \text{Saddle Point}$$

$$\text{At } (-1, -1), D = \begin{vmatrix} -6 & 3 \\ 3 & -6 \end{vmatrix} = 27 \rightarrow \text{Because } f_{xx} = -6 < 0 \rightarrow \text{Local Max}$$