



## Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 241 – Calculus III

### CBTF Midterm 3 Worksheet

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*The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.*

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Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 23, 6-8pm Meredith, Gabe

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/845>
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

**Please be sure to follow the above steps to add yourself to the Queue.**

Good luck with your exam!

1. Compute the double integral over the indicated rectangle. Confirm your answer by switching the order of integration and recomputing.

$$\iint_R 2x - 4y^3 \, dA \quad R = [-5, 4] \times [0, 3]$$

2. Calculate the following derivatives:

(a) Find  $\frac{df}{dt}$  for  $f(x, y) = xe^{xy}$ ,  $x(t) = t^2$ ,  $y(t) = \frac{1}{t}$

(b) Find  $f_t$  for  $f(x, y) = 2xy$ ,  $x(s, t) = st$ ,  $y(s, t) = s^2t^2$

3. Let  $f(x, y)$  be a differentiable function on the disk  $\{D : x^2 + y^2 \leq 400\}$ , where:

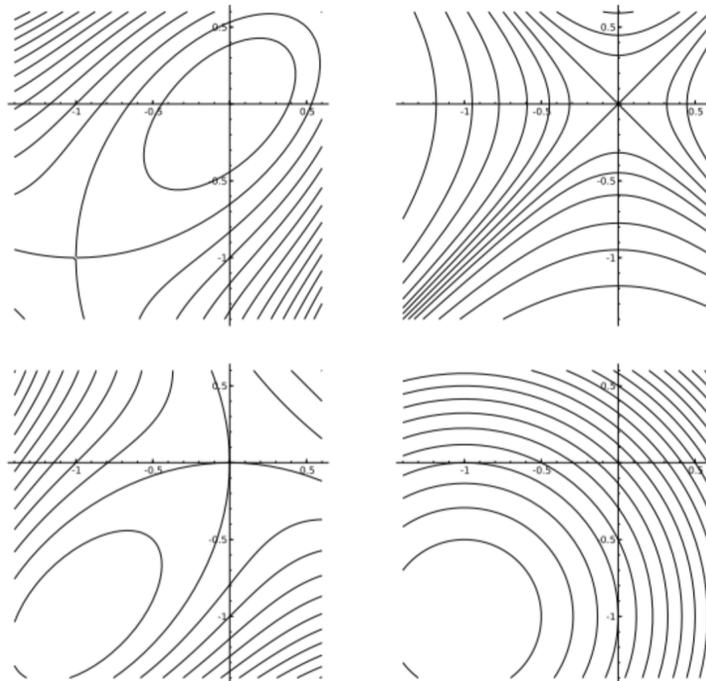
- (I)  $f(x, y) = 19$  for every point on the boundary of the disk  $x^2 + y^2 = 400$
- (II)  $f(0, 0) = 7$
- (III)  $f(x, y)$  has only one critical point which is at  $(-1, 2)$

Decide which statement is true:

- A)  $f(-1, 2) > 7$
- B)  $f(-1, 2) < 7$
- C)  $f(-1, 2) = 7$
- D) Not enough information is given

4. Consider the function  $f(x, y) = x^3 + y^3 + 3xy$

- (a) The critical points of  $f$  are  $(0, 0)$  and  $(-1, -1)$ . Classify them into local minima, local maxima and/or saddle points
- (b) Based on your answer in (a), identify the correct contour diagram of  $f$



5. Find min/max of  $f(x,y,z) = 3x^2 + 8y^2 + z^2 - 2z$  defined on the domain  $x^2 + 4y^2 + 2z \leq 8$  and  $z \geq 0$
- (a) The domain is (select all that apply)
- I) open
  - II) closed
  - III) bounded
  - IV) unbounded
- (b) Where are the critical points inside the domain? Evaluate the function value on these points.
- (c) What is the minimum and maximum on  $x^2 + 4y^2 + 2z = 8$ ?
- (d) What is the minimum and maximum on  $z = 0$ ?
- (e) What is the global minimum and maximum of the whole domain?

6. If  $f(x, y, z) = xye^z$ , find the gradient of  $f$  and the directional derivative at  $(2, 5, 0)$  in the direction of  $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$ .

7. Compute the double integral over the indicated rectangle. Confirm your answer by switching the order of integration and recomputing.

$$\iint_R 2x - 4y^3 \, dA \quad R = [-5, 4] \times [0, 3]$$