ILLINOIS

Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Phys 214 − University Physics: Quantum Physics

Final Exam Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Monday, 3/9, 7 - 8:50 pm CIF 4031 (Zaahi, Aparna, Liz)

Can't make it to a session? Here's our schedule by course:

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844

2. Click "New Question"

3. Add your NetID and Name

4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. A wave propagating through the ocean is measured by a sensor and can be described by the equation $f(x,t) = \cos(0.4x - 2t)$.

   (i) What is the wavelength, frequency and amplitude of the wave?

   (ii) In which direction is the wave traveling?

   a) $-x$ b) $+x$ c) The direction is time dependent

2. Three speakers lie on the perimeter of a circle. The sound intensity at each source is I_0 while the total intensity at the center of the circle is observed to be zero. Use phasors to determine the relative phase shift of each speaker such that this is possible.

3. An interferometer with equal arm lengths is sourced by a laser of wavelength 700 μm. If the length of one arm is increased by 0.12 mm, by what amount are the waves out of phase?

4. Continuing from the previous question, assuming that the intensity received at the detector was 4 W/m^2 when the arm lengths were equal, what is the new intensity?
   Values are given in W/m^2

a) 2.95

b) 0

c) 0.898

d) 1.21

e) 4

5. The distance to the first minimum of a circular diffraction pattern is found to be 0.012 cm from the center. Assuming the distance to the screen is 10 mm and the diameter of the opening is 200 μm, what is the wavelength of the light used? Values are given in μm

   a) 1.97
   b) 2.28
   c) 0.94

6. A single slit diffraction experiment is set up such that the central bright spot is 10 cm in width, and the screeen is 3 m away from the slit. Using light of 900 nanometers, calculate the slit spacing a.

7. A laser beam with $\lambda = 200$ nm and power $P = 2.3 \times 10^{-4}$ W is incident on a material with work function $\Phi = 3.4$.

a) Calculate the energy E_{e^-}, the maximum energy of each ejected electron.

b) Calculate N_γ, the number of photons hitting the material per second.

c) Say we have a device that detects the power of the ejected electrons. Calculate the maximum power P this device could measure (assuming every photon ejects an electron, and each electron has maximum energy).

8. **True or False**: The light intensity at any location on a screen is proportional to the probability that a photon arrives at that location, and that probability density is given by the square of the absolute value of the wavefunction.

9. A wavefunction given by $\Psi(x) = C_1\Psi_1 + C_2\Psi_2 + C_3\Psi_3$ is a superposition of three eigenstates. Let $C_1 = C_2 = 0.5$. What must C_3 be for $\Psi(x)$ to be normalized?

10. Compute the magnitude of the normalization constant for $\Psi(x) = Ne^{ikx}$ over the interval $0 \le x \le 3$. Assume the wavefunction equals zero for all other regions of space.

   a) 0.333

   b) 0.577

   c) 0.816

11. An electron ($m_{e^-} = 9.109 \times 10^{-31}$ kg) in an infinite square well is found to have energy $E = 0.002407$ eV. We are told that this is the $n = 4$ energy eigenstate.

   (a) Find the length of the well.

   (b) Determine the ground state energy in eV.

   (c) When the electron drops to a lower energy eigenstate, it releases its energy in the form of a photon. Determine all wavelengths of light this electron can produce from $n = 4$.

12. An electron is found to be in an infinite square well. From the ground state, we find two wavelengths of light that the electron is able to absorb: $\lambda_1 = 69.065$ nm and $\lambda_2 = 8.633$ nm. We are not given what these energy levels are, but we're told that they must be below $n = 7$.

   (a) Determine the energy levels the electron is brought to by each light absorption.
   Hint: Use the formula
   $$E_n - E_1 = E_1(n^2 - 1)$$
   coupled with the fact that n can only be natural numbers $(1, 2, 3\ldots)$.

   (b) Determine the ground state energy.

   (c) Determine the length of the well, L.

13. Suppose two particles in a quantum harmonic oscillator have different values of k (spring constant), but identical mass. Which graph corresponds to the particle with lower k (spring constant)? Orange or Blue?
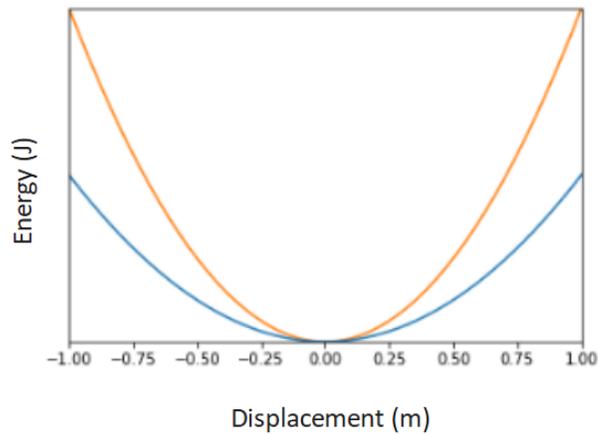
14. A particle is in the ground state of a quantum harmonic oscillator (ground state energy $= \frac{1}{2}\hbar\omega$), what would happen is some scientists shoot a photon with energy $\hbar\omega$ at it? What if they change the photon energy to $0.9\hbar\omega$? $1.1\hbar\omega$?

15. Consider the oscillator below:

———————————— 11 eV

———————————— 9 eV

———————————— 7 eV

———————————— 5 eV

———————————— 3 eV

———————————— 1 eV
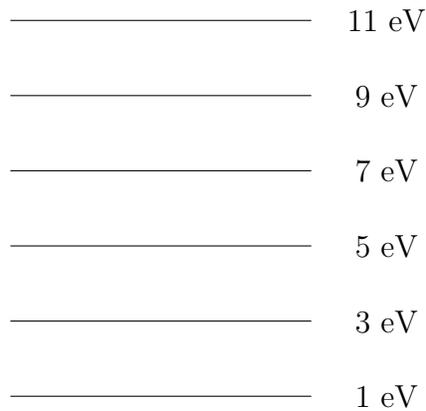
Six electrons are in a harmonic oscillator. Fill in the diagram to represent the oscillator's lowest excited state, using arrows to represent the spin of the electrons. What would the diagram look like if it is irradiated with light of energy 3 eV?

16. We have a material that is transparent to yellow light ($\lambda = 750$ nm), but opaque to cyan light ($\lambda = 510$ nm).

(a) Given this information, give an energy range for the possible value of the electronic gap.

(b) Suppose we are now told that the material is opaque to green light ($\lambda = 550$ nm). Write a new, smaller range for the possible electronic gap.

17. According to the band structure model, which of the following materials has the largest and smallest energy gap? Aluminum (good conductor), silicon (semiconductor), and silicon dioxide glass (good insulator).

18. Suppose we have an unpolarized beam of photons with intensity I_o. We have a horizontal filter, i.e. a polarizer that only allows photons in the Ψ_H state to pass through.

(a) If the beam is incident on the horizontal filter, determine the resultant intensity in terms of I_o as well as the wavefunction of photons exiting the filter.

(b) Now suppose we rotate the horizontal filter by $90°$, effectively creating a vertical filter. Write the resultant wave function of the exiting photons in this scenario.

(c) Next, we rotate the filter such that it makes a $45°$ angle with the horizontal. Once again, write the resultant wave function.

(d) In (a)-(c), we didn't add any additional filters; we just rotated the prexisting one. Do these rotations influence the resultant intensity?

(e) Keeping the filter at $45°$, we place another horizontal filter into the system, after the first filter. Determine the probability for a photon to pass through this second filter.

19. We have the following Stern-Gerlach experiment setup:
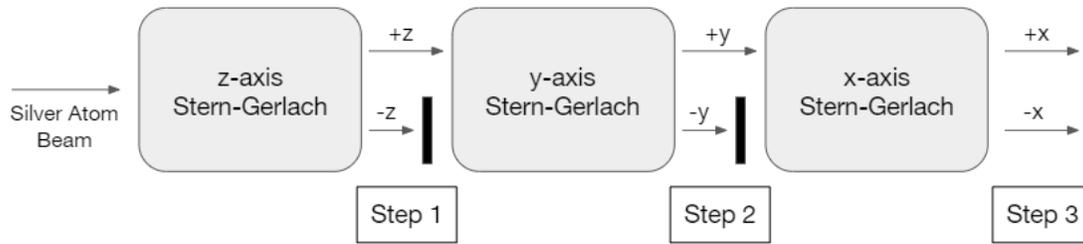
(a) Assuming the silver atom beam is initially unpolarized in spin (i.e. it has equal components up and down), determine the probability of a silver atom to reach the $+x$ path in step three.

(b) Now let's assume the silver atoms are described by the wave function

$$\Psi = \frac{1}{5}\left(4\Psi_\uparrow + 3\Psi_\downarrow\right)$$

Given this information, determine the new probability of a silver atom reaching the $+x$ path.

20. Consider the alpha decay of a radium-226 nucleus: $^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

    The decay releases a total energy of $Q = 4.87$ MeV, which goes into the kinetic energy of the α particle and the recoiling radon nucleus. Use the following constants:

    - Mass of α particle (He): $m_\alpha = 6.64 \times 10^{-27}$ kg
    - Mass of radon-222 nucleus: $M_{\text{Rn}} = 3.69 \times 10^{-25}$ kg
    - 1 MeV $= 1.602 \times 10^{-13}$ J

    *Assume the radium-226 nucleus is starting at rest.

    (a) Using conservation of momentum, calculate the kinetic energy of the α particle and the radon nucleus.

    (b) Determine the speed of the α particle and the recoiling radon nucleus (Assume p = mv).

    (c) Estimate the mass of the radium-226 nucleus.

21. A simple entanglement example problem. Consider the following state describing particle one and particle two.

$$|\Psi\rangle = a(\uparrow\uparrow) + b(\downarrow\downarrow)$$

    where the first arrow is the spin state of the first particle and likewise for the second particle. If the spin of particle one is measured to be up ($\uparrow$) what is the probability that particle two is:

    (a) Spin down?

    (b) Spin up?

22. Consider a β^+ decay, where a proton inside a nucleus converts into a neutron, emitting a positron (e^+) and an electron neutrino (ν_e):

$$p^+ \rightarrow n^0 + e^+ + \nu_e$$
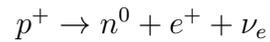
Suppose you measure the spins of the positron and proton and find that both are spin up ($\uparrow$). Assume the system is initially in a spin-conserving state.

(a) Determine the two possible states that are consistent with total spin conservation.

(b) Determine the probability that the neutrino has spin down ($\downarrow$).

(c) If you measure the neutrino spin and find it is up ($\uparrow$), what is the probability that the neutron has spin down ($\downarrow$)?