PHYS 214 Final Review

Units for the Exam (from past exams)

- Waves
- Interference
- Diffraction
- Photons & The Photoelectric Effect
- Probability & Complex Numbers
- The Wave Function
- Momentum & Position

Units for the Exam (New Material)

- Spin & Polarization
- Energy in Quantum Mechanics
- Atomic Vibrations
- Multiple Electrons
- Nuclear Physics
- Band Structure

Units 1-3

Wave Equation

General Wave Propagation: $y(x, t) = A\cos(kx - \omega t + \phi)$

k = wave number (how the wave repeats in <u>SPACE</u>) [m^{-1}]

ω = angular frequency (how the wave repeats in <u>TIME</u>) [rad/s]

ϕ = phase shift (the starting phase of the wave) [rad]

$kx - \omega t + \phi$ = phase

Properties of Waves

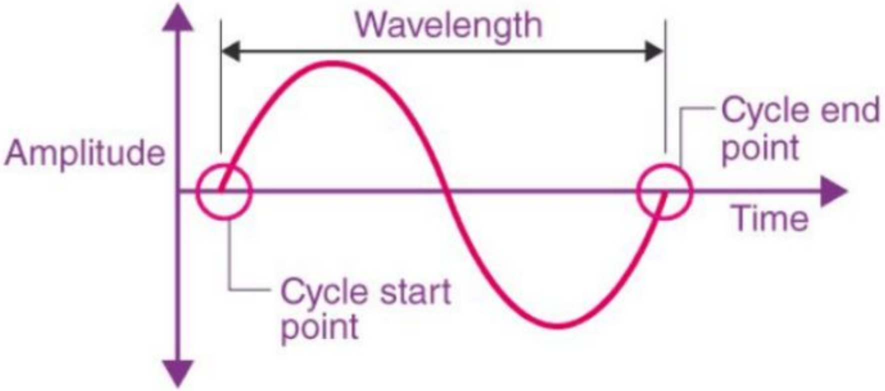
$\lambda = 2\pi/k\,;\quad f = \omega/\,2\pi$

$v = \omega/k \qquad v = \lambda f$

Intensity: $I(x,t) = |y(x,t)|^2$

$I_{\text{average}} = |A|^2/2$

$f = 1\,/\,T$



Interference

Superposition (adding): A fancy way of saying that when two waves interact, the resulting wave is the **SUM** of the two individual waves

$y_1(x, t) = A_1\cos(k_1x - \omega_1t + \phi_1)$

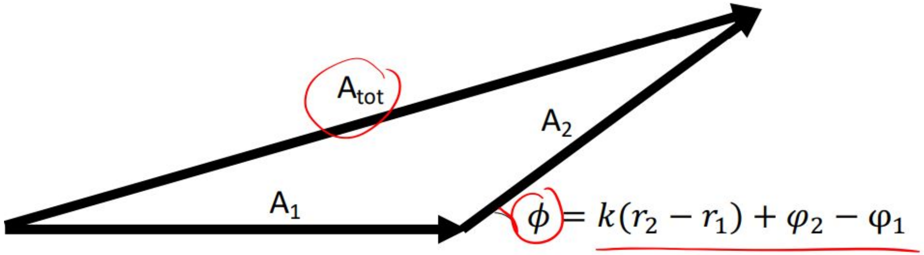
$y_2(x, t) = A_2\cos(k_2x - \omega_2t + \phi_2)$

$y_{tot}(x, t) = y_1(x, t) + y_2(x, t) = A_1\cos(k_1x - \omega_1t + \phi_1) + A_2\cos(k_2x - \omega_2t + \phi_2)$

If $\phi_1 = \phi_2$, the angular frequencies (ω) are the SAME, and the distance is the SAME, then the waves are IN PHASE

Phasors and Law of Cosines

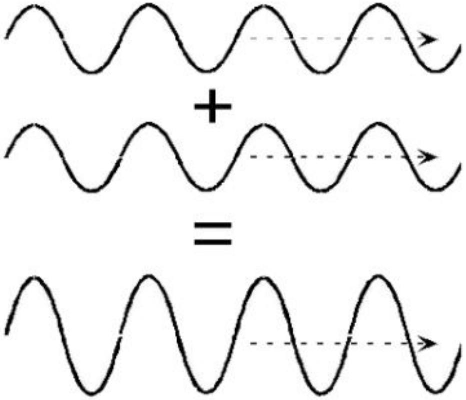
$$A_{tot}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)$$



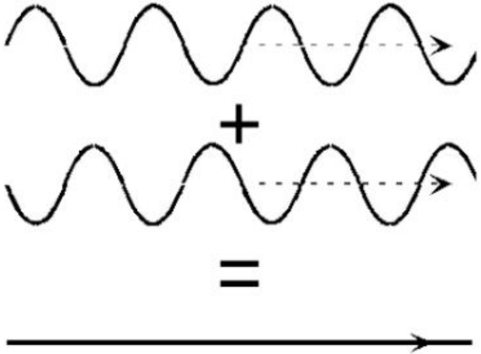
$$\phi = k(r_2 - r_1) + \varphi_2 - \varphi_1$$

Interference (Cont.)

Phase difference = $k(r_2 - r_1) = \phi$ for a two source system at different distances



Constructive



Destructive

In general, for two sources with the same amplitude/intensity:

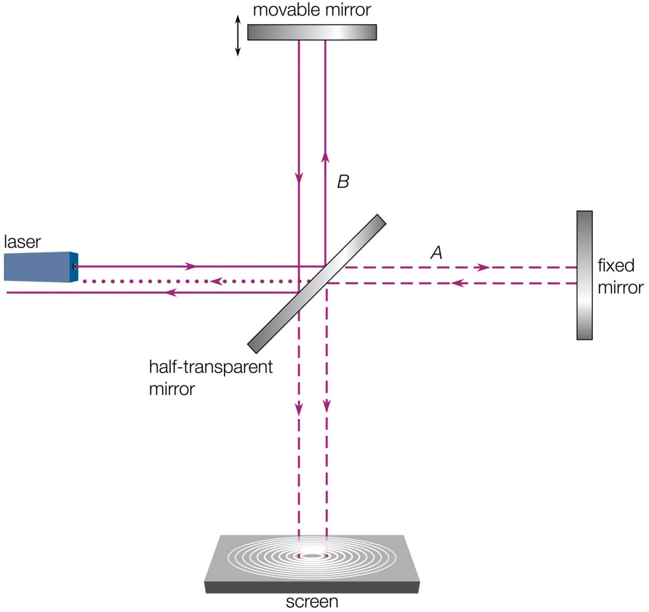
$$I_{\text{tot}} = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

In your equation sheet, this is written as:

$$I_{\text{total}} = 2A^2 \cos^2\left(\frac{kr_1 + \phi_1 - kr_2 - \phi_2}{2}\right)$$

Example Problem - Interferometer

A Michelson interferometer is illuminated by a laser of power **P = 5 mW** and wavelength **λ = 632.8 nm**



You want to adjust one of the mirrors to get a **new power of 2 mW**.

How far do you have to move the mirror to achieve this new intensity?

Example Problem - Interferometer (Cont.)

$$I_0 = \frac{P}{4} = \frac{5}{4} = 1.25\ mW$$

Example Problem - Interferometer (Cont.)

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Example Problem - Interferometer (Cont.)

$$I_0 = \frac{P}{4} = \frac{5}{4} = 1.25\ mW$$

$$I_{new} = 4I_0\ cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\phi = 2\cos^{-1}\left(\sqrt{\frac{I_{new}}{4I_0}}\right) = \ 2\cos^{-1}\left(\sqrt{\frac{2}{4(1.25)}}\right) = \ 1.77\ rad$$

Example Problem - Interferometer (Cont.)

$$I_0 = \frac{P}{4} = \frac{5}{4} = 1.25\, mW$$

$$\Delta\phi = \frac{2\pi}{\lambda}(L_2 - L_1) = \frac{2\pi}{\lambda}(2\Delta x)$$

$$I_{new} = 4I_0\, cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\phi = 2\cos^{-1}\left(\sqrt{\frac{I_{new}}{4I_0}}\right) = 2\cos^{-1}\left(\sqrt{\frac{2}{4(1.25)}}\right) = 1.77\, rad$$

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$$I_{new} = 4I_0\ cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta x = \frac{\Delta\phi\lambda}{4\pi} = \frac{1.77 * (600 * 10^{-9})}{4\pi} = 84.6\ nm$$

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Diffraction

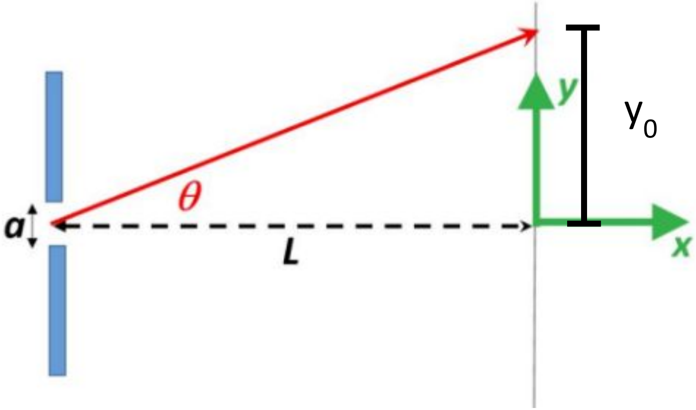
- Single slit diffraction:
    - a = slit width
    - θ_o = angle of first **minimum**
    - λ = wavelength

- **Small a → Large θ_o**

- Small angle approximation:
    - $\theta \cong \sin(\theta) \cong \tan(\theta) \cong y_o/L$

- Spot size:
    - Radius → $y_0 = L*\tan(\theta) \cong L*\theta$
    - Width → $2y_o = 2L*\tan(\theta) \cong 2*L*\theta$

$$a\sin(\theta_\circ) = \lambda$$



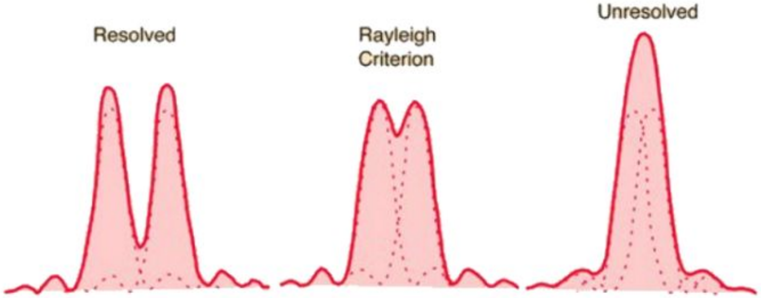
Diffraction (Cont.)

- Circular aperture diffraction
  - Similar to single slit; 1.22 factor

- Rayleigh Criterion:
  - Center of the diffraction maximum from the first object falls onto the diffraction minimum from the second object
  - ie. $\theta_o \leq \theta_{objects}$
    - θ_o = angle of first minimum of central bright spot
    - For multiple objects θ_o is the minimum angle required to distinguish the two objects
    - $\theta_{objects}$ = angle between two bright spots

D - Diameter of lens

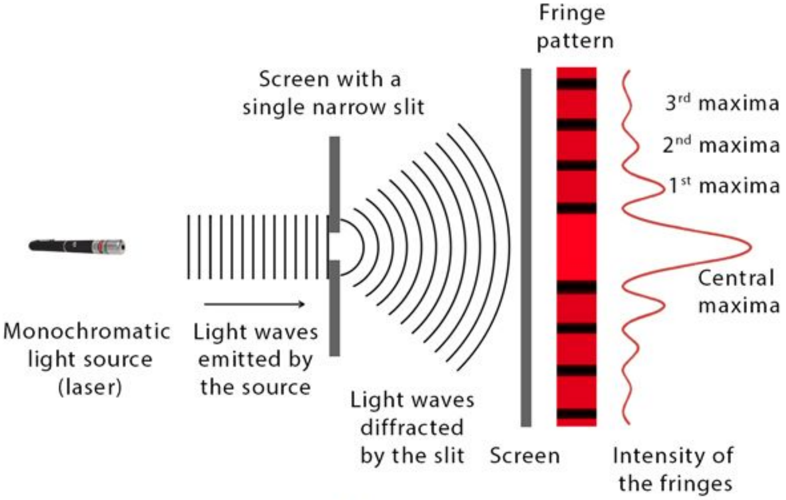
$$D\sin(\theta_\circ) = 1.22\lambda$$



Example Problem - Diffraction

A laser with wavelength **λ = 500 nm** illuminates a single slit width of ***a* = 0.1 mm**. A screen is placed ***L* = 2.00 m** from the slit.

Find the **vertical position *y_4*** on the screen of the fourth minimum measured from y = 0 (center of first maximum)



Example Problem - Diffraction (Cont.)

$$a \sin(\theta_0) = m\lambda$$

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Example Problem - Diffraction (Cont.)

$$a\sin(\theta_0) = m\lambda$$

$$a\theta_0 = m\lambda$$

$$\theta_0 = \frac{m\lambda}{a} = \frac{4(500 * 10^{-9})}{0.1 * 10^{-3}} = 0.02\,rad$$

Example Problem - Diffraction (Cont.)

$$a \sin(\theta_0) = m\lambda$$

$$y_4 = L\theta_0 = 2 * 0.02 = 0.04\, m$$

$$a\theta_0 = m\lambda$$

$$\theta_0 = \frac{m\lambda}{a} = \frac{4(500 * 10^{-9})}{0.1 * 10^{-3}} = 0.02\, rad$$

Example Problem - Diffraction (Cont.)

$$a\sin(\theta_0) = m\lambda$$

$$a\theta_0 = m\lambda$$

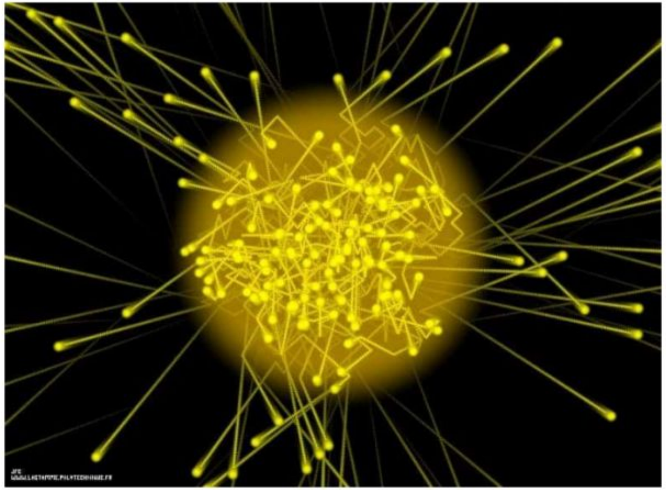
$$\theta_0 = \frac{m\lambda}{a} = \frac{4(500 * 10^{-9})}{0.1 * 10^{-3}} = 0.02\,rad$$

$$y_4 = L\theta_0 = 2 * 0.02 = 0.04\,m$$

$$y_4 = 0.04\,m = 4\,cm$$

Units 4-7

Photons



Photons: the quantized bits of light (particles of light)

- Energy of a single photon with frequency *f*:

$$E = hf = \hbar\omega = \frac{1240 \text{ eV nm}}{\lambda}$$

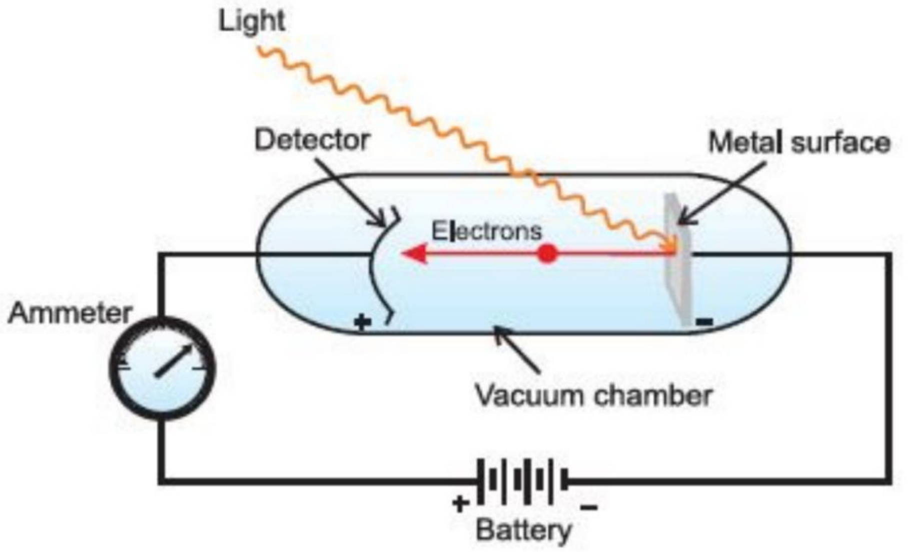
- Momentum of a single photon with wavelength λ:

$$p = \hbar k = h/\lambda$$

- $h = 6.626 \times 10^{-34}$ J · s
- $\hbar$ (‘h-bar’) = $h/2\pi$

- Photons have a velocity of c:
  - ○ $c = 3*10^8$ m/s
  - ○ $c = \lambda f \longrightarrow E = (hc)/\lambda$

The Photoelectric Effect

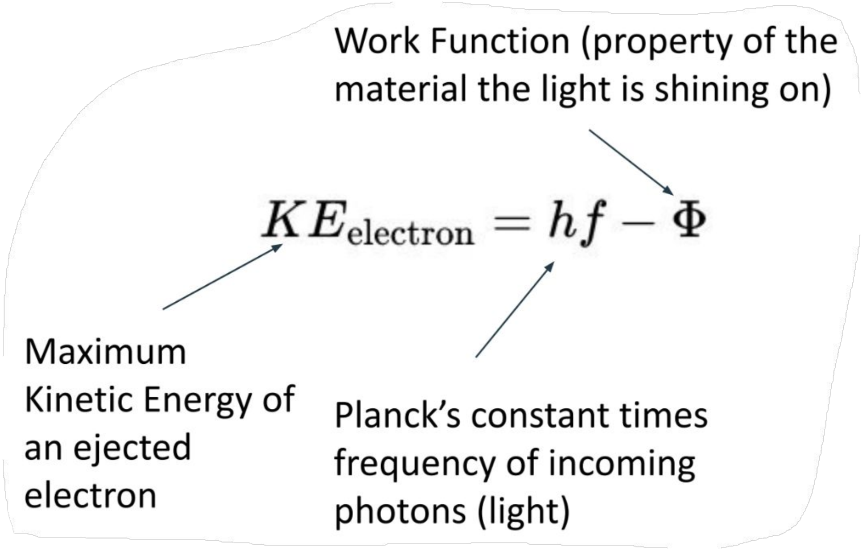


The Photoelectric (Cont.)

This experiment proves the existence of photons and that light can be **BOTH** a particle and a wave

$$KE_{\text{electron}} = eV_{\text{stop}}$$

Stopping Potential: Voltage applied to stop electrons from flowing between the two plates



Work Function (property of the material the light is shining on)

$$KE_{\text{electron}} = hf - \Phi$$

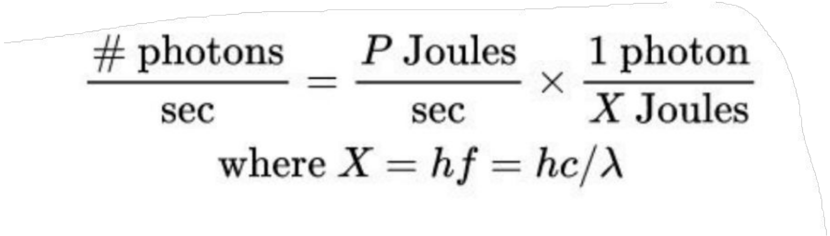
Maximum Kinetic Energy of an ejected electron

Planck's constant times frequency of incoming photons (light)

Effect of Power Source

Increasing the power of a photon source will <u>NOT</u> increase photon energy!

It will only increase photon flux, since frequency/wavelength is what determines photon energy



$$\frac{\#\text{ photons}}{\text{sec}} = \frac{P\text{ Joules}}{\text{sec}} \times \frac{1\text{ photon}}{X\text{ Joules}}$$

$$\text{where } X = hf = hc/\lambda$$

Force from Photons

$$F = \frac{dp}{dt} = \frac{momentum}{second}$$

$$\frac{momentum}{second} = \boxed{\frac{P\ joules}{second} * \frac{1\ photon}{hf\ joules}} * \frac{\frac{h}{\lambda}\ momentum}{1\ photon}$$

$$\downarrow$$

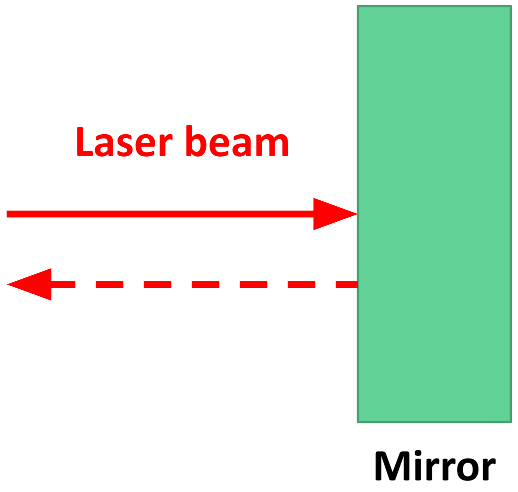
photons/second

Simplified → F = P/(λf) = P/c

Example Problem - Photons & Mirrors

A perfectly reflecting mirror is illuminated by a laser beam with a power of **10 mW** and wavelength of **500 nm**.

Determine the force exerted on the mirror by the photons



Example Problem - Photons & Mirrors (Cont.)

$$F = \frac{dp}{dt}$$

Example Problem - Photons & Mirrors (Cont.)

$$F = \frac{dp}{dt}$$

$$F = \frac{2P}{c} = \frac{2(10 * 10^{-3})}{3 * 10^{8}} = 66.67\ pN$$

Example Problem - Photons & Mirrors (Cont.)

$$F = \frac{dp}{dt}$$

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$$F = 66.67\ pN$$

Probability and Complex Numbers

Probability

- Probability Density Function:

$$\rho(x)$$

- Probability between two points:

$$P(a \leq x \leq b) = \int_a^b \rho(x)\,\mathrm{d}x$$

- Normalization:

$$\int_{-\infty}^{\infty} \rho(x)\,\mathrm{d}x = 1$$

Complex Numbers

- Two forms:

$$z = a + bi = |z|e^{i\theta}$$

- Conversion:

$$\theta = \arctan(b/a) \quad z = |z|(\cos\theta + i\sin\theta)$$

- Magnitude Squared:

$$|z|^2 = (z^*)(z)$$

- Identities:

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

The Wave Function

- Wave function is notated as **Ψ(x)**
    - Contains **ALL** information about the properties of a quantum particle

- Properties of **Ψ(x):**

$$\rho(x) = \psi^*(x)\psi(x) = |\psi(x)|^2$$

Ψ*(x) is the complex conjugate of Ψ(x)

$$P(a \leq x \leq b) = \int_a^b (\Psi^*)(\Psi)\,\mathrm{d}x$$

Probability of detecting a particle between points a and b

Example Problem - Detection Probability

Given the following normalized wavefunction, what is the integral that gives the chance of measuring the particle in the range |x| ≤ 1?

$$\Psi(x) = \frac{1}{\sqrt{2}} e^{-|x|/2}$$

Example Problem - Detection Probability (Cont.)

$$P(\,|x|\le 1) = \int_{-1}^{1} (\frac{1}{\sqrt{2}} e^{-|x|/2})^2 dx = \int_{0}^{1} e^{-x} dx \approx 0.63$$

Momentum and Position

- For particles with momentum **$p = \hbar k$**, they are described by this wave function:

$$\psi(x) = Ae^{ikx} \quad \longrightarrow$$

Momentum eigenstate
(1 possible momentum)

- Some particles' wave functions are a superposition of momentum eigenstates:

$$\psi(x) = \underbrace{\boxed{Ae^{ik_1x}}}_{\text{1st eigenstate}} + \underbrace{\boxed{Be^{ik_2x}}}_{\text{2nd eigenstate}}$$

Momentum and Position (Cont.)

- Probability of each momentum is given by:

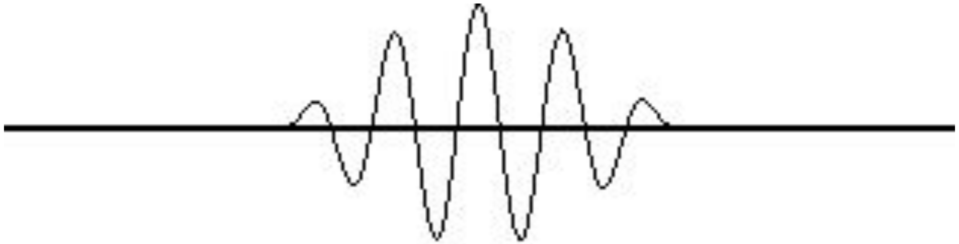
$$P(\hbar k_1) = |A|^2/(|A|^2 + |B|^2)$$
$$P(\hbar k_2) = |B|^2/(|A|^2 + |B|^2)$$

- Where A and B are the coefficients seen in the wave function: $\psi(x) = \underline{A}e^{ik_1x} + \underline{B}e^{ik_2x}$

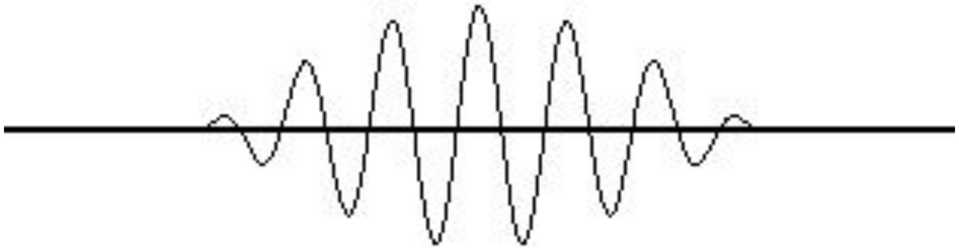
- Overall uncertainty is given by **Heisenberg's Uncertainty Principle:**
- The **fewer** momentum eigenstates, the more certain we are for momentum
- The **more** momentum eigenstates, the more certain we are for position

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Momentum and Position (cont.)



Momentum (→ wavelength → colour)



Position

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Example Problem - Probability of $\hbar k_1$

Determine the probability of the momentum given by $\hbar k_1$:

$$\psi(x) = 2ie^{ik_1x} + 3e^{ik_2x}$$

Example Problem - Probability of $\hbar k_1$ (Cont.)

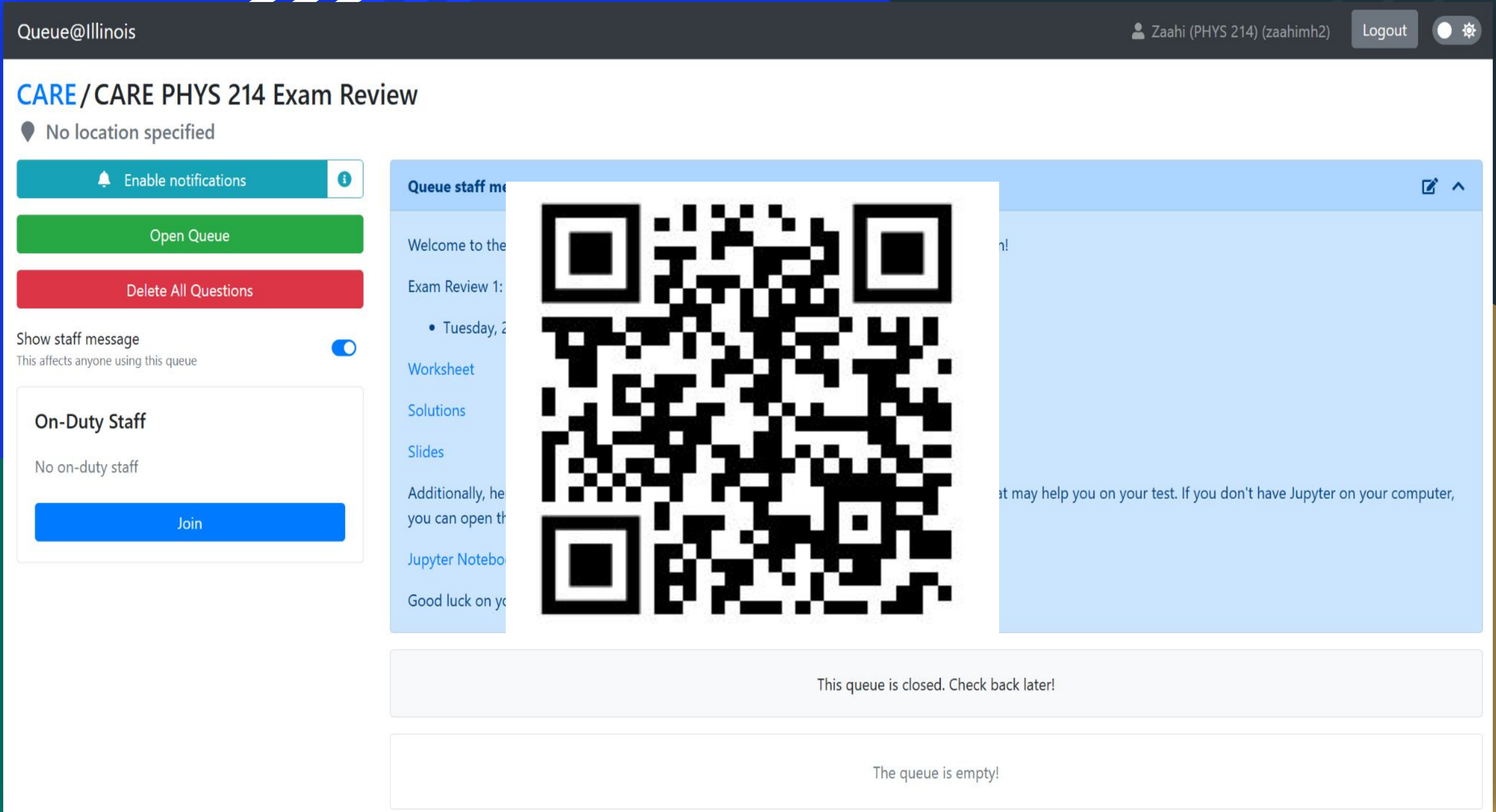
$$P(\hbar k_1) = \frac{|2i|^2}{|2i|^2 + |3|^2} = \frac{4}{4+9} = \frac{4}{13}$$

PHYS 214 Final Review

Units 8-13
(focus of the final!)

Units for the Exam (New Material)

- Spin & Polarization
- Energy in Quantum Mechanics
- Atomic Vibrations
- Multiple Electrons
- Nuclear Physics
- Band Structure



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Exam Review 1:

- Tuesday, 2

Worksheet

Solutions

Slides

Additionally, he at may help you on your test. If you don't have Jupyter on your computer, you can open th

Jupyter Notebo

Good luck on yo

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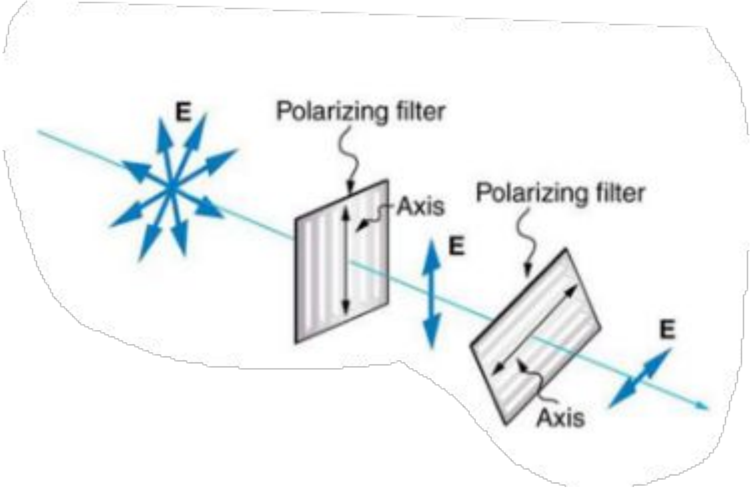
Polarization

- Polarization = direction of the electric field vector in an EM wave
- Polarizers enforce a polarization direction:
  - e.g. a horizontal polarizer will extract only the horizontal component of the light
  - Similar to taking the dot product to extract a horizontal component
- **In quantum physics, polarization is represented by *polarization states*:**

$$\Psi = a\Psi_v + b\Psi_h$$



Polarization (Cont.)

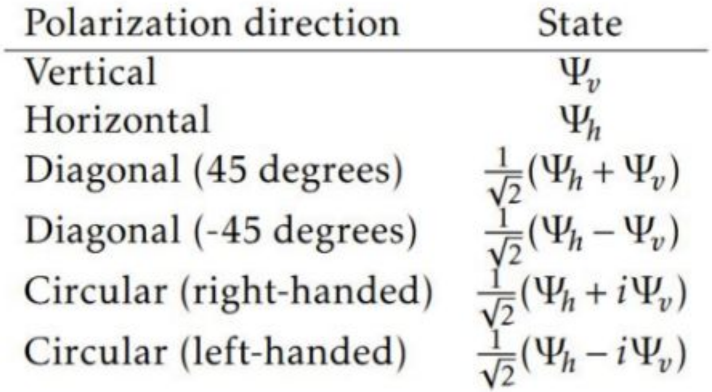
- Light can have polarization states:
    - Horizontal polarization: Ψ_h
    - Vertical polarization: Ψ_v
- Wave function can be a superposition of horizontal and vertical polarization:

$$\Psi = a\Psi_v + b\Psi_h$$

- In this case, the probability of passing through a vertical filter is:

$$P(\text{vertical}) = \frac{|a|^2}{|a|^2 + |b|^2}$$

Note: This formula **ALWAYS** works regardless of whether the wave function is normalized or unnormalized



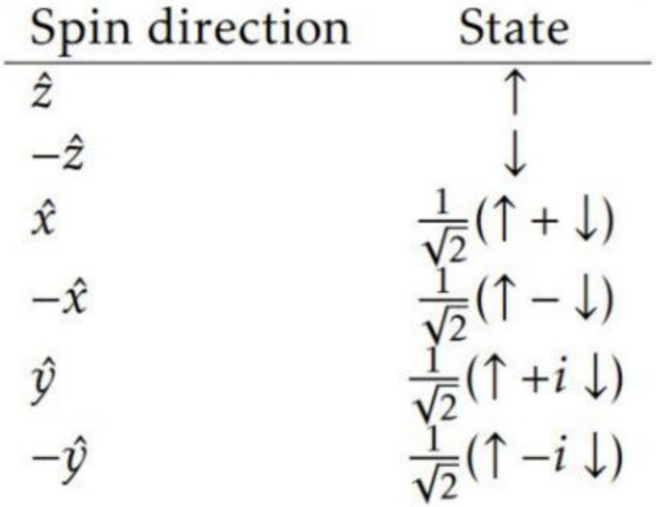
Polarization direction	State
Vertical	Ψ_v
Horizontal	Ψ_h
Diagonal (45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h + \Psi_v)$
Diagonal (-45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h - \Psi_v)$
Circular (right-handed)	$\frac{1}{\sqrt{2}}(\Psi_h + i\Psi_v)$
Circular (left-handed)	$\frac{1}{\sqrt{2}}(\Psi_h - i\Psi_v)$

Spin

- Spin is similar in idea, just like horizontal and vertical in light, we have up and down spin for electrons:

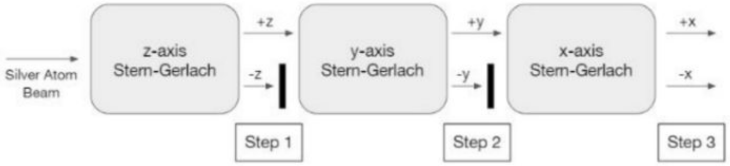
- Spin up: ↑
- Spin down: ↓

- Any electron can be the superposition of those two spins (±x̂, ±ŷ)



Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

Stern-Gerlach setup: the “polarizer” for spin states:



Analogy to Vector Dot Products

- If a system is in the Ψ state and we want to find the probability of observing the S state:

$$\mathrm{P}(S) = |S^* \cdot \Psi|^2$$

dot product

Note: This formula **<u>ONLY</u>** works if the wave function is normalized

- Recall **vector dot products:**

$$\hat{x} \cdot \hat{x} = 1, \;\; \hat{y} \cdot \hat{y} = 1$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\vec{a} = a_x\hat{x} + a_y\hat{y}, \;\; \vec{b} = b_x\hat{x} + b_y\hat{y}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

- This dot product method works for **any S or Ψ**

In quantum:

$$\Psi_h, \Psi_v \text{ are like } \hat{x}, \hat{y}$$

$$\uparrow, \downarrow \text{ are like } \hat{x}, \hat{y}$$

$$\Psi_h^* \cdot \Psi_h = \Psi_v^* \cdot \Psi_v = 1$$

$$\uparrow^* \cdot \uparrow \;=\; \downarrow^* \cdot \downarrow \;= 1$$

$$\Psi_h^* \cdot \Psi_v = \Psi_v^* \cdot \Psi_h = 0$$

$$\uparrow^* \cdot \downarrow \;=\; \downarrow^* \cdot \uparrow \;= 0$$

Polarization and Spin

Polarization:

- Unit States: Ψ_h, Ψ_v
- State filter: **polarizers**
- **Probability of observing opposite states:**

$$\Psi_h^* \cdot \Psi_v = \Psi_v^* \cdot \Psi_h = 0$$

- **Probability of** observing a “table” state from unpolarized photons = 1/2

Spin:

- Unit States: ↑, ↓
- State filter: **Stern-Gerlach experiment**
- **Probability of observing opposite states:**

$$\uparrow^* \cdot \downarrow \; = \; \downarrow^* \cdot \uparrow \; = 0$$

- **Probability of** observing a “table” state from ‘unpolarized’ particle beam = 1/2

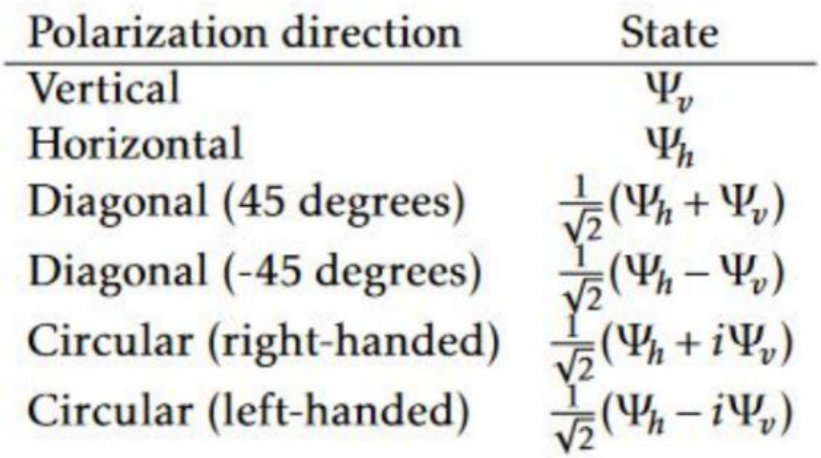
For BOTH Polarization and Spin

Probability of Observing State S from Ψ:

$$\mathrm{P}(S) = |S^* \cdot \Psi|^2$$

Only used for NORMALIZED Wavefunctions

Table States



Polarization direction	State
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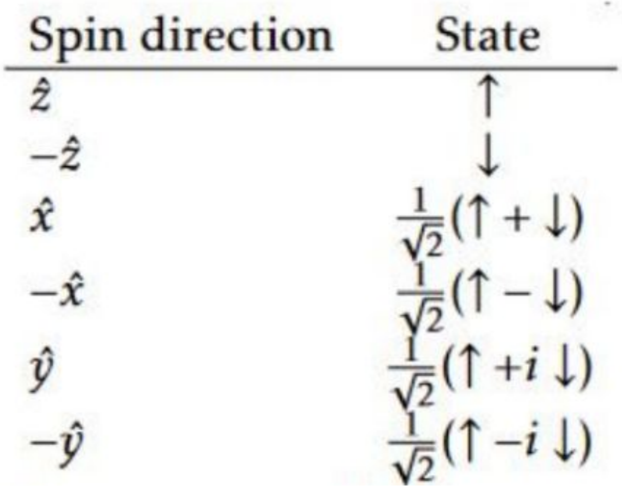

Spin direction	State
$\hat{z}$	$\uparrow$
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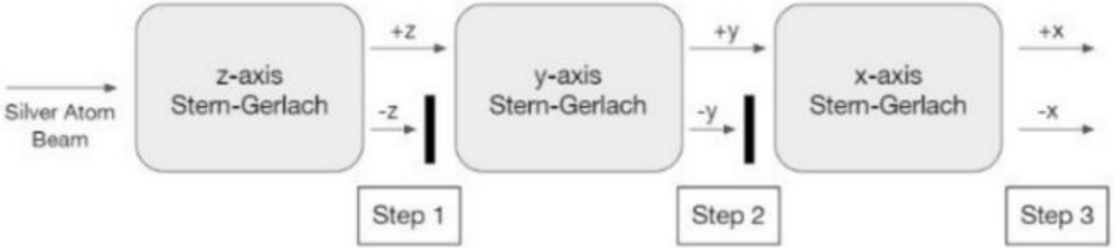
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

Example Problem: Find the Probability P(+x)

Assume a single silver atom starts in an unpolarized state $\hat{u}$ and is passed through the Stern-Gerlach filters shown below. What is the probability of observing the atom with spin state $+\hat{x}$ after the three filters?

Stern-Gerlach setup: the "polarizer" for spin states:



Example Problem: Find the Probability P(+x)

Probability of transmission given (normalized) input state $\hat{\psi}$ and output state $\hat{S}$:

$$P(S) = |\hat{S}* \cdot \hat{\psi}|^2$$

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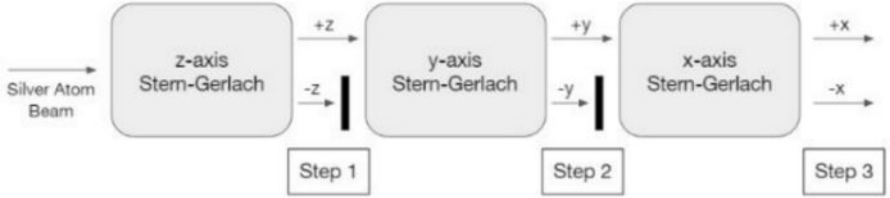

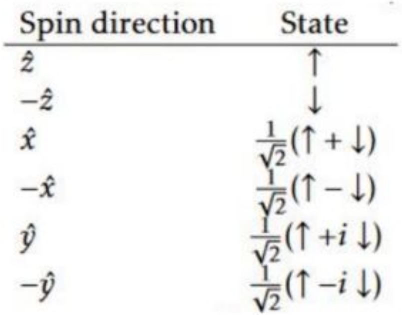

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
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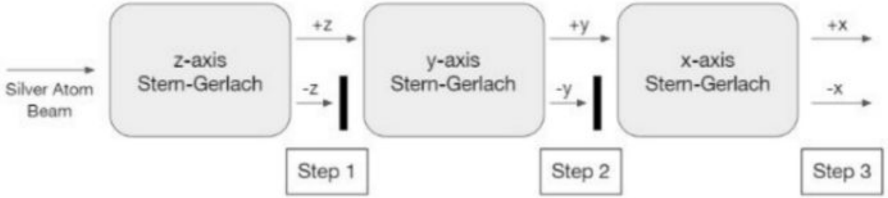

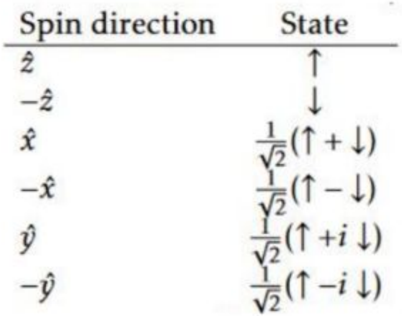

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Probability of transmission is ½ for each polarizer, then probability of observing spin +x: P(+x)=½*½*½=⅛

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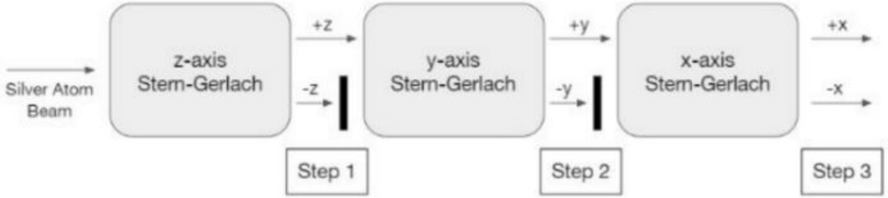

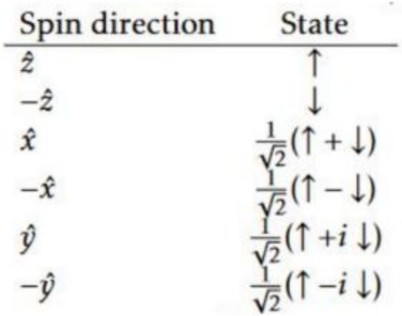

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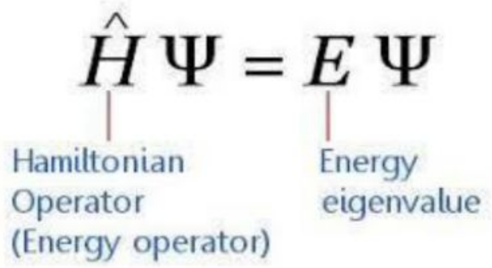
Energy Eigenstates

- Quantum particles can sometimes be in two energy states at once!

- If we are in a single energy eigenstate, we know with certainty that the particle has **one possible energy**

- This means that if we measure the energy we will get a **definite value**



$$\hat{H}\Psi = E\Psi$$

Energy Eigenstate Example

- **Infinite Square Well:**

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}) & \text{if } 0 < x < L \\ 0 & \text{otherwise,} \end{cases}$$

Wave Function

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Energy

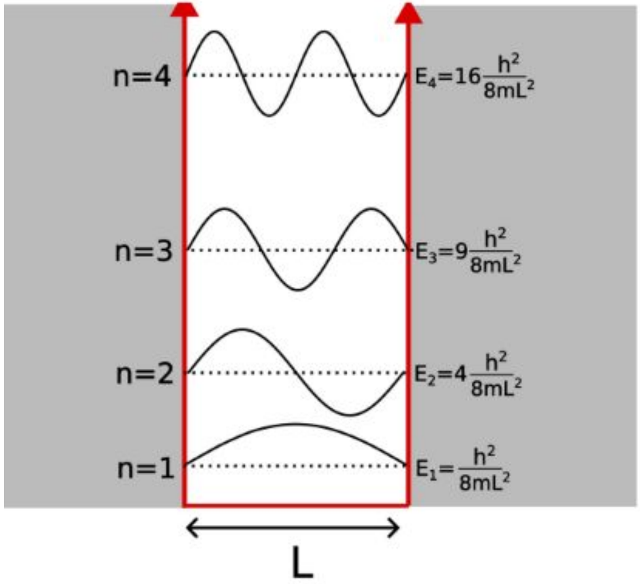
$$\begin{aligned} E_n - E_m &= \frac{\hbar^2 \pi^2}{2mL^2}(n^2 - m^2) \\ &= E_1 \left(n^2 - m^2\right) \end{aligned}$$

Energy Difference

- The energy levels of the particle are discrete and are indexed by n, **starting at n = 1**

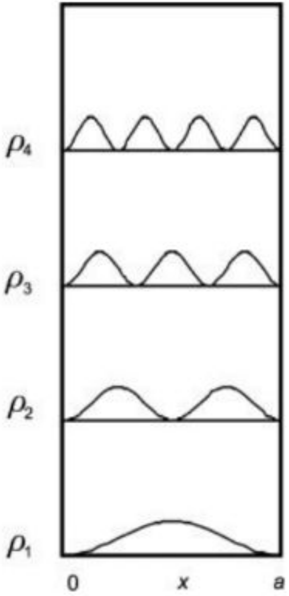
- A particle can jump between energy levels, absorbing or emitting energy as it goes

Recognize Symmetries!



Eigenstates

$|\Psi|^2$



Probability Distributions

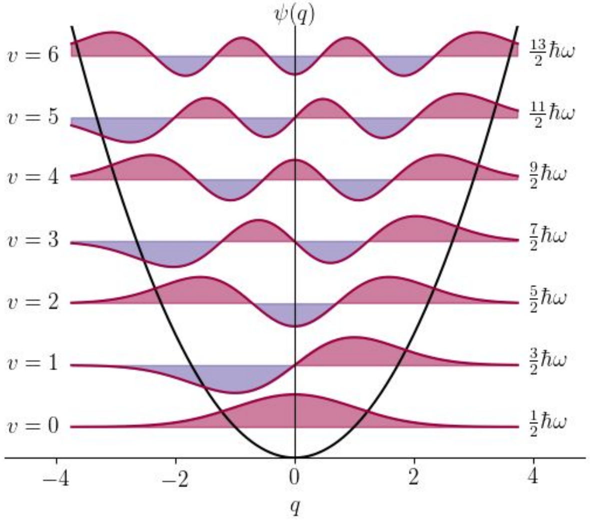
- n = number of “humps”
- Recognizing patterns and symmetries is easier than integrating
- Example: for the $n = 3$ case, what is the probability of finding the particle between 0 and $2L/3$?
    - Answer: ⅔ !
    - Two out of three “humps”

Harmonic Oscillator

- Potential in the form: $U(x) = \frac{1}{2}kx^2$
- Leads to the ground state wave function:

$$\Psi_0(x) = Ae^{-ax^2}$$

- Energy levels: $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$, n = 0, 1, 2 . . .
    - **Evenly spaced levels and n starts from 0**
- k - spring constant (N/m) → $\boxed{\omega = \sqrt{\frac{k}{m}}}$



Photon Emission

- When a multi-state system **drops to a lower energy level**, the system **emits a photon**

- Likewise, when a multi-state system **raises to a higher energy level**, the system **absorbs a photon**

- **Energy lost/gained by the system must be equal to the energy of the emitted/absorbed photon**

Infinite Square Well:

$$E_{n_1} - E_{n_2} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_1^2 - n_2^2\right) = E_1\left(n_1^2 - n_2^2\right)$$

Quantum Harmonic Oscillator:

$$E_{n_1} - E_{n_2} = \hbar\omega(n_1 - n_2)$$

Photon Energy:

$$E = hf = \frac{hc}{\lambda}$$

Example problem - Find the Smallest Frequency Emitted from a QHO and ISW

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- Given a harmonic oscillator with angular frequency **ω**
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- Given an ISW with length ***a*** and particles of mass ***m***
- Must use n = 1 and n = 2

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$$E_2 - E_1 = \frac{2^2\hbar^2\pi^2}{2mL^2} - \frac{1^2\hbar^2\pi^2}{2mL^2} = (2^2 - 1^2)E_1$$

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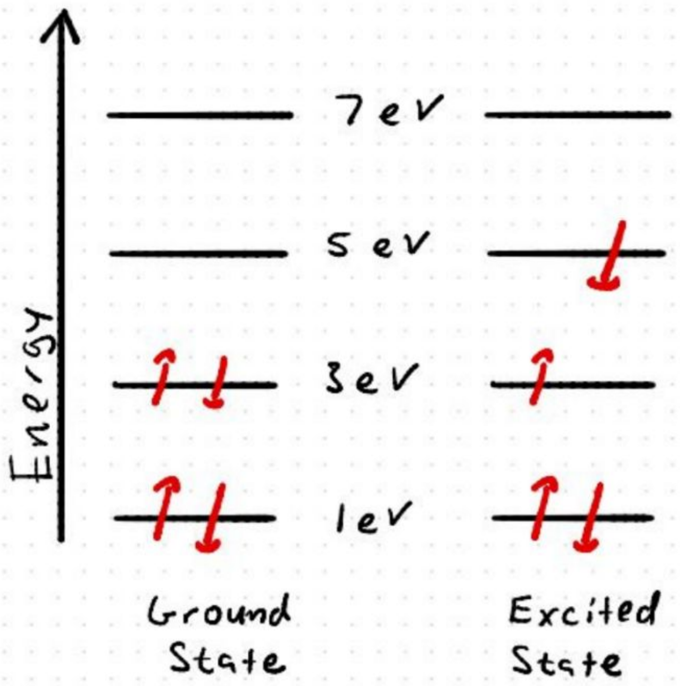
$$\Delta E_{min} = (2^2 - 1^2)E_1$$

Photon Energy:

$$\Delta E_{\min} = hf_{\min} \implies f_{\min} = \frac{\Delta E_{\min}}{h}$$

Multi-electron System

- **Aufbau Principle:** We fill energy states from bottom to top
- **Pauli-Exclusion Principle:** No two electrons can have the same spin in the same state
- Each excited state has the **next smallest possible energy**



Excited Energy States

Example: System with energy levels 3 eV, 8 eV, 11 eV, 12 eV; 3 electrons

Successive excited states are ALWAYS given by the SMALLEST CHANGE IN ENERGY!

If the system has strange spacing (like in the example above), then try different combinations to be sure

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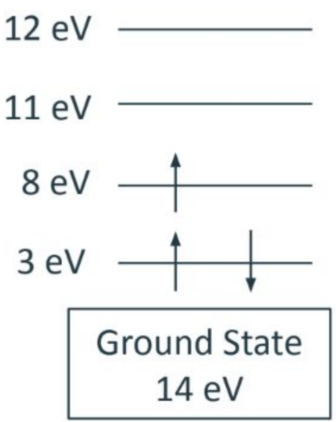

Ground State
14 eV

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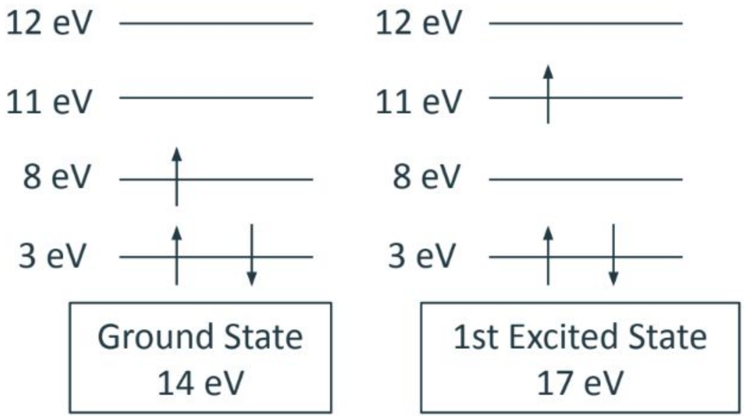

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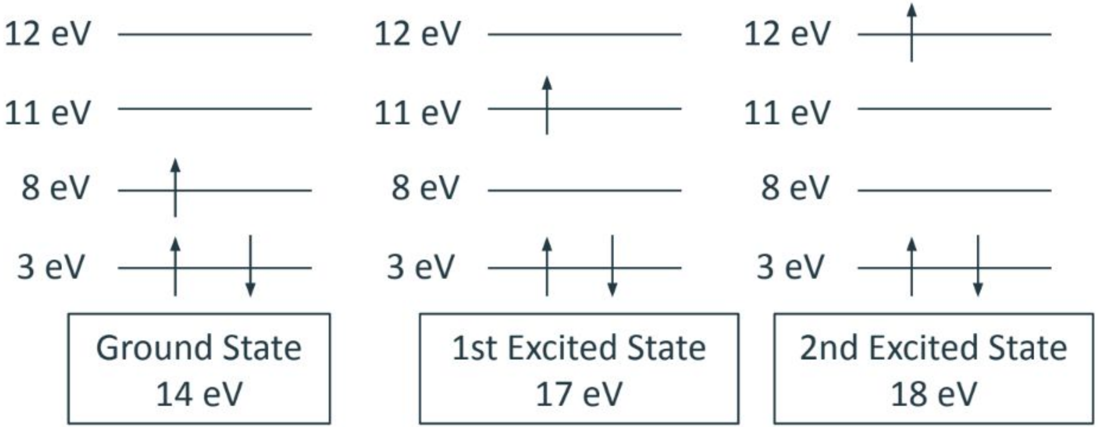

Ground State
14 eV

1st Excited State
17 eV

2nd Excited State
18 eV

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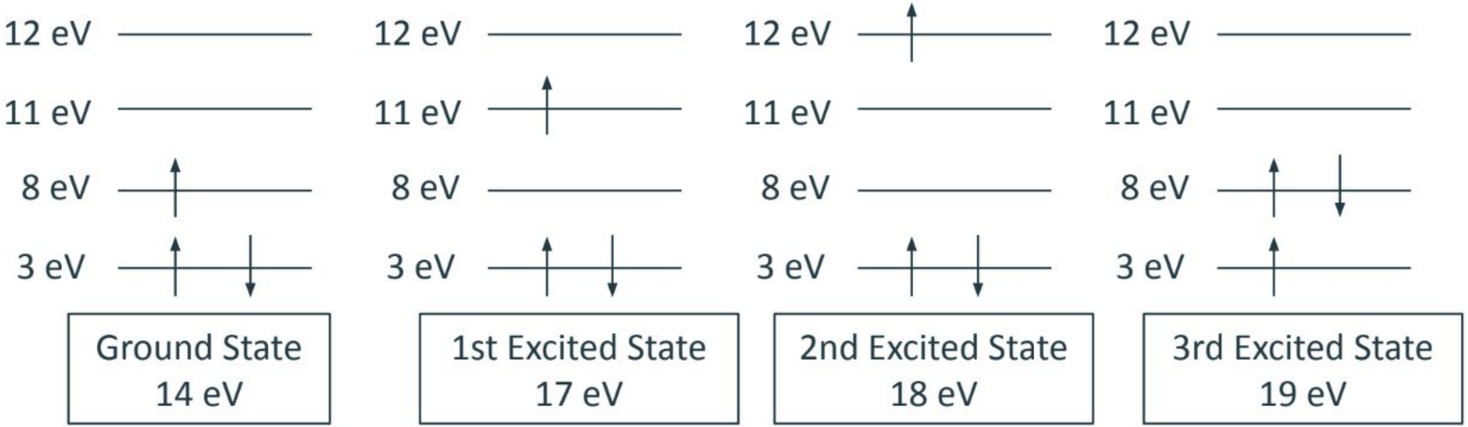

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If the system has strange spacing (like in the example above), then try different combinations to be sure

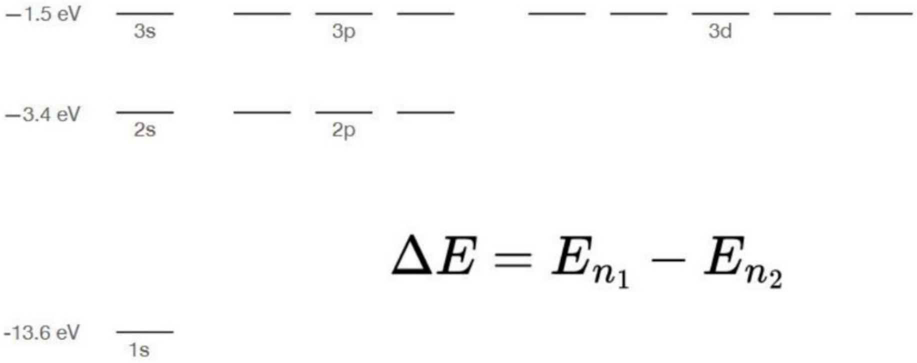
Hydrogen: Another Multi-Level System

- Energy levels are filled from **bottom to top**, and from **left to right**

- As usual, a **maximum of 2 electrons per state**

- There are many repeats of energy levels!
    - -13.4 eV has **1 level**
    - -3.4 eV has **4 levels**
    - -1.5 eV has **9 levels**

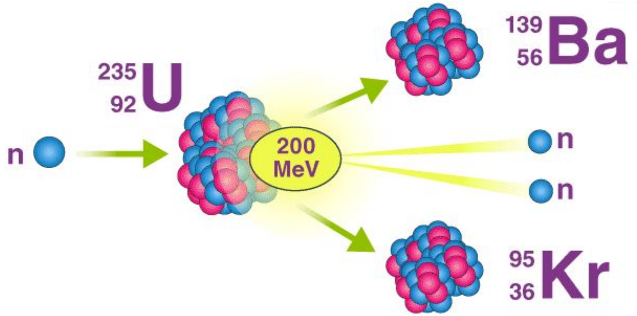
$$E_n = \frac{-13.6Z^2}{n^2}\ \text{eV}$$



$$\Delta E = E_{n_1} - E_{n_2}$$

Nuclear Physics

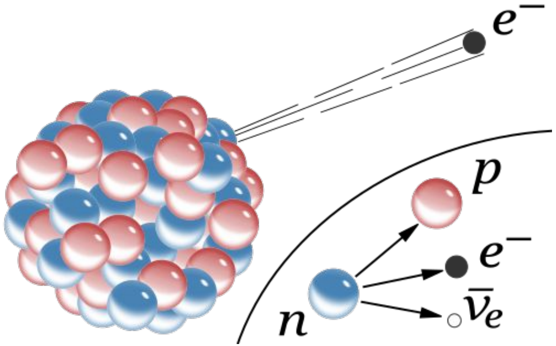
- **Atomic Number** = number of protons
- **Atomic Mass** = number of protons + number neutrons
  - Ex. $^{12}_{6}\mathrm{C}$ → 6 protons, 6 neutrons
- **Nuclei stability** is dependent on 2 forces:
  - **Coulomb force** → protons repel each other
  - **Strong Nuclear force** → binds protons and neutrons
  - Above forces compete leading to varying stability among atoms
- **Nuclear Reaction:**
  - If a nucleus has higher energy than another, it can transform into that lower energy state
  - Transformation gives off particles: gamma rays, electrons, positrons, neutrino, antineutrino, etc.



Types of Reactions

- **β− decay = neutron transforms into a proton**
  - Emits: electron, antineutrino, kinetic energy

$$\boxed{n^0 \rightarrow p^+ + e^- + \bar{\nu}_e}$$



- **γ decay = excited nuclei decays to ground state**
  - Emits: gamma photons
  - Nucleus in an excited state due to β− decay

- **β+ decay = proton transforms into a neutron**
  - Emits: electron, neutrino, kinetic energy

$$\boxed{p^+ \rightarrow n^0 + e^+ + \nu_e}$$

- **Energy released given by mass differences:** $Q = m_C c^2 - (m_N c^2 + m_e c^2)$

SAME Net Charge

Examples for Types of Reactions

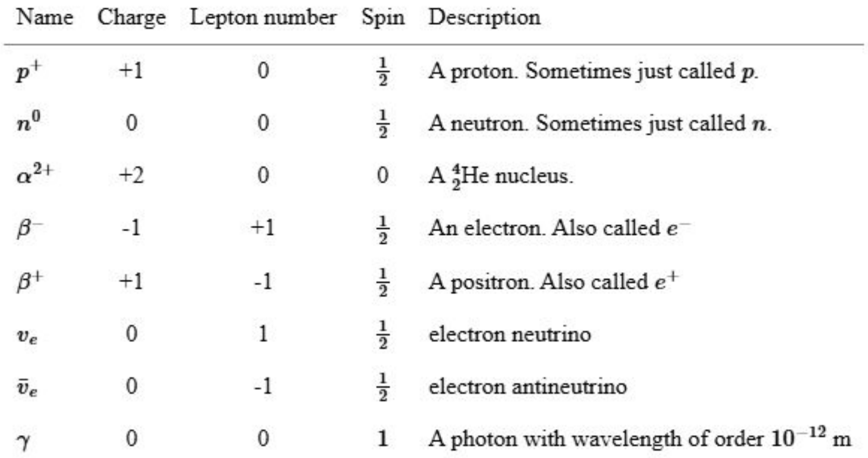
$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e$$

$${}^{60}_{27}\mathrm{Co}^{*} \rightarrow {}^{60}_{27}\mathrm{Co} + \gamma$$

$${}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + e^{+} + \nu_e$$

Conservation and Entanglement

- In nuclear reactions several things must be **CONSERVED:**
    - Energy
    - Lepton number = total count of electrons and electron neutrinos
    - Total charge
    - Total angular momentum (including spin)
- Conservation laws leads to entanglement of a particle's state



Name	Charge	Lepton number	Spin	Description
p^+	+1	0	$\frac{1}{2}$	A proton. Sometimes just called p.
n^0	0	0	$\frac{1}{2}$	A neutron. Sometimes just called n.
α^{2+}	+2	0	0	A ${}^4_2$He nucleus.
β^-	-1	+1	$\frac{1}{2}$	An electron. Also called e^-
β^+	+1	-1	$\frac{1}{2}$	A positron. Also called e^+
v_e	0	1	$\frac{1}{2}$	electron neutrino
$\bar{v}_e$	0	-1	$\frac{1}{2}$	electron antineutrino
γ	0	0	1	A photon with wavelength of order 10^{-12} m

Conservation and Entanglement (Cont.)

- Let’s say you have the following **β− decay**:

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

Conservation and Entanglement (Cont.)

- Let’s say you have the following **β− decay**:

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

- **You measure the spin of the neutron and find that it is spin up (↑)**
- Due to **conservation**, there are **3 possibilities** for the particle’s state:

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$$\Psi = a\left(\uparrow_{p^+}\uparrow_{e^-}\downarrow_{\bar{\nu}_e}\right) + b\left(\uparrow_{p^+}\downarrow_{e^-}\uparrow_{\bar{\nu}_e}\right) + c\left(\downarrow_{p^+}\uparrow_{e^-}\uparrow_{\bar{\nu}_e}\right)$$

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- **If you measure 2 of the spins, you know what the last particles spin has to be:**
  - If the proton’s spin and electron’s spin was found to be up (↑), the antineutrino spin has to be down (↓)
  - Essentially, the particle’s state is entangled due to conservation

Example Problem - Probability of specific spin

- Let’s continue with **β− decay** reaction from the previous slide:

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

- **You measure the spin of the neutron and electron and find that BOTH have spin up (↑)**

Determine the <u>2 possibilities</u> for the particle’s state:

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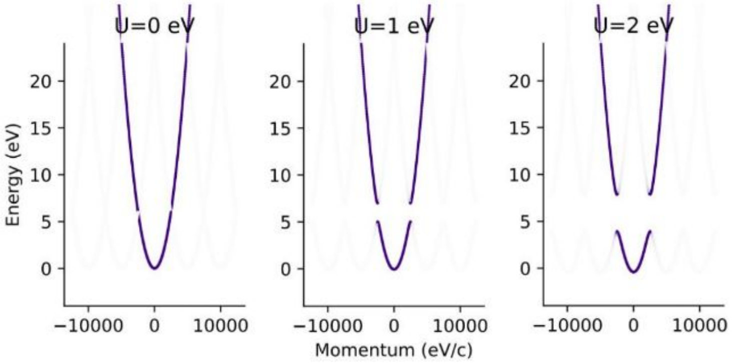
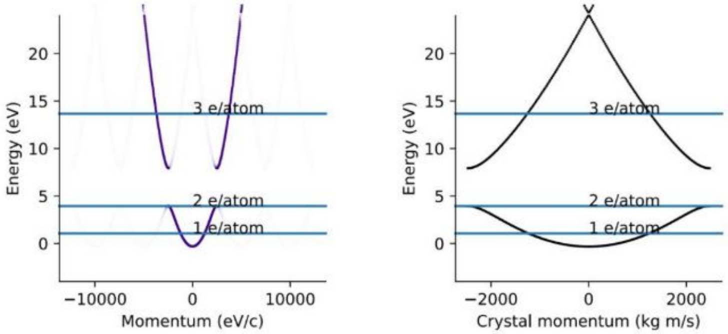
Determine the probability of spin up (↑) for the proton:

$$P(\text{proton spin } \uparrow) = \frac{|a|^2}{|a|^2 + |c|^2}$$

Band Structure

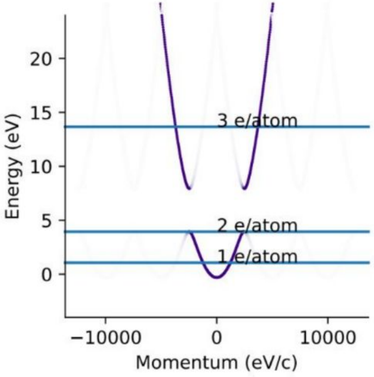
- Band structure describes the range of **energy levels** that electrons may have within it, as well as the ranges of **energy that it may not have**
- Gap: Energy it takes to get from one allowed state to a higher one
- **Metals/Conductors: <u>NO</u> gap**
- **Insulators: <u>LARGE</u> gap**
- **Semiconductors: <u>SMALL</u> gap**
    - Cooling down a semiconductor increases the size of the gap

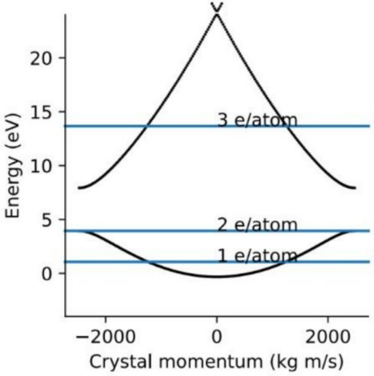




Band Structure (Cont.)

- In this diagram:
  - Blue line = highest electron occupancy
  - **1e/atom: conductor** (available states above highest electron occupancy)
  - **2e/atom: insulator** (no available states above highest electron occupancy)





- For a material with a band gap (E_g), if we send a photon with frequency f:
  - $hf < E_g$: photon passes through material
  - $hf > E_g$: photon is absorbed and electron is excited (overcomes gap)

Good luck!

Feel free to ask any questions you may have.

You got this!