Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Phys 214 – University Physics: Quantum Physics

Final Exam Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Monday, 3/9, 7 - 8:50 pm CIF 4031 (Zaahi, Aparna, Liz)

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. A wave propagating through the ocean is measured by a sensor and can be described by the equation $f(x,t) = \cos(0.4x - 2t)$.

   (i) What is the wavelength, frequency and amplitude of the wave?

   (ii) In which direction is the wave traveling?

   a) $-x$ b) $+x$ c) The direction is time dependent

(i) $\lambda = 15.71m$, $f = 0.318Hz$, $A = 1$
The general harmonic wave equation is

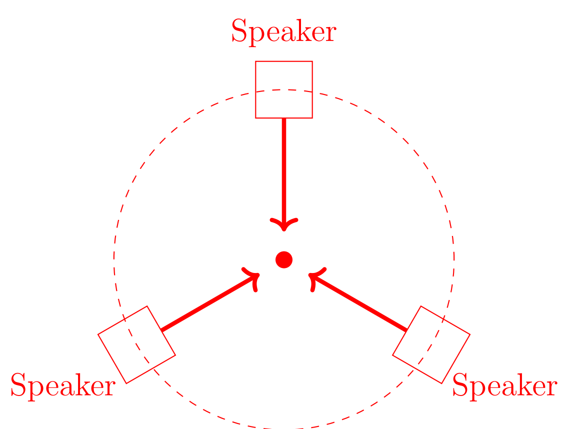
$$A\cos(kx - \omega t + \phi)$$

This can be used with $f = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ to get the frequency and wavelength while the amplitude is the coefficient A.

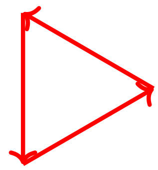
(ii) The answer is **(b)**. As we increase time, the value of x must also increase for the argument of the wave equation to stay constant.

2. Three speakers lie on the perimeter of a circle. The sound intensity at each source is I_0 while the total intensity at the center of the circle is observed to be zero. Use phasors to determine the relative phase shift of each speaker such that this is possible.
All three speakers are equidistant from the circle's center. This means that any phase angle between waves is a result of the sources being out of phase, not the path difference. The phasor diagram of this system must be an equilateral triangle so that the sum of the angles between the phasors adds up to 360° (the phase between each speaker can be deduced to be $360/3 = 120°$, 180 - 120 = 60 °).

Set-up:



Phasor Diagram:



3. An interferometer with equal arm lengths is sourced by a laser of wavelength 700 μm. If the length of one arm is increased by 0.12 mm, by what amount are the waves out of phase?

$$\phi = \frac{2\pi\delta}{\lambda} \rightarrow \delta = 0.24 \times 10^{-3} \text{ m}$$

$$\boxed{\phi = 2.15 \text{ radians}}$$

4. Continuing from the previous question, assuming that the intensity received at the detector was 4 W/m^2 when the arm lengths were equal, what is the new intensity?
Values are given in W/m^2

a) 2.95

b) 0

c) 0.898

d) 1.21

e) 4

The answer is **(c)**. Since the detector received 4 W/m^2 when the arm lengths were equal, we know that the source intensity is 4. We established the phase difference between these two waves in the previous question to be 2.15 radians. Using the equation

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

we can determine the new intensity in this interferometer. Remember that I_0 in this case is 1 W/m^2 because the beam is split twice. The source intensity is split on the first pass through and then each reflected beam is split, netting you two interfering beams with intensities that are $\frac{1}{4}$ of the intensity from the source.

5. The distance to the first minimum of a circular diffraction pattern is found to be 0.012 cm from the center. Assuming the distance to the screen is 10 mm and the diameter of the opening is 200 μm, what is the wavelength of the light used? Values are given in μm

a) 1.97

b) 2.28

c) 0.94

The answer is **(a)**. For a circular diffraction pattern, the equation

$$1.22\lambda = D\sin\left(\arctan\left(\frac{y}{L}\right)\right)$$

is used. However, a small angle approximation would be appropriate here since the angle is really small.

$$\lambda \approx \frac{Dy}{1.22L} = \frac{200\times10^{-6}(0.012\times10^{-2})}{1.22(10\times10^{-3})} = 1.97\times10^{-6}\text{ m}$$

Once converted to micrometers we get 1.97 μm

6. A single slit diffraction experiment is set up such that the central bright spot is 10 cm in width, and the screeen is 3 m away from the slit. Using light of 900 nanometers, calculate the slit spacing a.

The full width of the central bright spot is 10 cm in width, meaning the distance from y=0 to the first minima is 10/2 = 5 cm. Plugging this into our equation for slit theta:

$$\frac{y}{L} = \tan\theta \implies \theta \approx 0.05/3 = 1.67\times10^{-2}\text{ radians}$$

Solving for the slit width, we use a wavelenth of 900 nm

$$\frac{\lambda}{\sin\theta} = \frac{900\times10^{-9}\text{ m}}{0.0167} = a = 53.89\ \mu\text{m}$$

7. A laser beam with $\lambda = 200$ nm and power $P = 2.3\times10^{-4}$ W is incident on a material with work function $\Phi = 3.4$.

a) Calculate the energy E_{e^-}, the maximum energy of each ejected electron.

b) Calculate N_γ, the number of photons hitting the material per second.

c) Say we have a device that detects the power of the ejected electrons. Calculate the maximum power P this device could measure (assuming every photon ejects an electron, and each electron has maximum energy).

a) $\boldsymbol{E_{e^-} = 2.8}$ **eV**. The energy of each photon is equal to $hc/\lambda = (1240\text{ eV}\cdot\text{nm})/(200\text{ nm}) = 6.2$ eV. Subtracting the work function gives $E_{e^-} = 2.8$ eV.

b) $\boldsymbol{N_\gamma = 2.319\times10^{14}}$ **photons/s.** We know the power, and the energy per photon was found to be 6.2 eV $= 9.92\times10^{-19}$ J. Performing units analysis:

$$N_\gamma = \frac{2.3\times10^{-4}\text{J}}{\text{s}}\cdot\frac{\text{photon}}{9.92\times10^{-19}\text{ J}} = 2.319\times10^{14}\text{ photons/s}$$

c) **1.039×10^{-4} W.** If each photon is ejecting an electron of maximum energy, then 2.319×10^{14} electrons/s are being ejected, each with energy 2.8 eV. Multiplying these values yields $P = 6.492 \times 10^{14}$ eV/s $= 1.039 \times 10^{-4}$ W.

8. **True or False**: The light intensity at any location on a screen is proportional to the probability that a photon arrives at that location, and that probability density is given by the square of the absolute value of the wavefunction.
   True. The light intensity is correlated with the probability of detecting photons. That is, photons are more likely to be detected at those points where the wave intensity is high and less likely to be detected at those points where the wave intensity is low.

   By definition, the probability density function is found by square the magnitude of the wavefunction. The integral of this function, over all space, must equal one.

9. A wavefunction given by $\Psi(\mathrm{x}) = \mathrm{C}_1\Psi_1 + \mathrm{C}_2\Psi_2 + \mathrm{C}_3\Psi_3$ is a superposition of three eigenstates. Let $\mathrm{C}_1 = \mathrm{C}_2 = 0.5$. What must C_3 be for $\Psi(\mathrm{x})$ to be normalized?

$$1 = 0.5^2 + 0.5^2 + C_3^2 \text{ therefore } C_3 = \frac{\sqrt{2}}{2}$$

   Remember, it is the probabilities that must sum to 1, not the wavefunction coefficients.

10. Compute the magnitude of the normalization constant for $\Psi(x) = Ne^{ikx}$ over the interval $0 \leq x \leq 3$. Assume the wavefunction equals zero for all other regions of space.

a) 0.333

b) 0.577

c) 0.816

The answer is **(b)**. Taking the absolute square of this wavefunction will give N^2 as the probability density function

$$Ne^{ikx} * Ne^{-ikx} = N^2$$

If this is integrated from 0 to 3 and set equal to one (probabilities over all space must be equal to one), then we are left with $3N^2 = 1$, which when solved for N gives 0.577.

11. An electron ($m_{\mathrm{e}^-} = 9.109 \times 10^{-31}$ kg) in an infinite square well is found to have energy $E = 0.002407$ eV. We are told that this is the $n = 4$ energy eigenstate.

(a) Find the length of the well.

(b) Determine the ground state energy in eV.

(c) When the electron drops to a lower energy eigenstate, it releases its energy in the form of a photon. Determine all wavelengths of light this electron can produce from $n = 4$.

(a) $L = 50$ nm. We can rearrange the infinite square well energy equation to get

$$L = \sqrt{\frac{\hbar^2 n^2 \pi^2}{2mE}}$$

which will give us the length of the well.

(b) $E_1 = 1.505 \times 10^{-4}$ eV. Now that we know the length, we can find the ground state:

$$E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$$
$$= 1.505 \times 10^{-4} \text{ eV}$$

(c) Remember that we can write jumps in energy as

$$\Delta E = \frac{\hbar^2 \pi^2}{2mL^2}(n_i^2 - n_f^2)$$

We've already calculated the left term that multiplies the difference, and we know that the values for the difference must be $16 - 9,\ 16 - 4,$ and $16 - 1$. We get three energies:

$$\Delta E_{43} = 0.00105 \text{ eV}$$
$$\Delta E_{42} = 0.00181 \text{ eV}$$
$$\Delta E_{41} = 0.00226 \text{ eV}$$

Converting these to wavelengths, we get 1.117 mm, 0.687 mm, and 0.549 mm.

12. An electron is found to be in an infinite square well. From the ground state, we find two wavelengths of light that the electron is able to absorb: $\lambda_1 = 69.065$ nm and $\lambda_2 = 8.633$ nm. We are not given what these energy levels are, but we're told that they must be below $n = 7$.

(a) Determine the energy levels the electron is brought to by each light absorption.
Hint: Use the formula

$$E_n - E_1 = E_1(n^2 - 1)$$

coupled with the fact that n can only be natural numbers $(1, 2, 3 \ldots)$.

(b) Determine the ground state energy.

(c) Determine the length of the well, L.

(a) First, we know that an absorbing of energy translates to an energy difference between energy levels. We can solve for the energies given the wavelengths:

$$E_{\lambda_1} = \frac{1240 \text{ eV nm}}{69.065 \text{ nm}} = 17.954 \text{ eV} = 2.876 \times 10^{-18} \text{ J}$$
$$E_{\lambda_2} = \frac{1240 \text{ eV nm}}{8.633 \text{ nm}} = 143.635 \text{ eV} = 2.301 \times 10^{-17} \text{ J}$$

Then, we can relate them to the square well energies, E_n:

$$E_{\lambda_1} = E_{n_1} - E_1 = E_1(n_1^2 - 1)$$
$$E_{\lambda_2} = E_{n_2} - E_1 = E_1(n_2^2 - 1)$$

Even though we don't know the ground state energy E_1 yet, we can still solve for n_1 and n_2 by dividing the equations:

$$\frac{E_{\lambda_1}}{E_{\lambda_2}} = \frac{\cancel{E_1}(n_1^2 - 1)}{\cancel{E_1}(n_2^2 - 1)}$$

Plugging in the energies we calculated in the beginning, we get

$$0.125 = \frac{n_1^2 - 1}{n_2^2 - 1}$$

With this, we find values of n_1 and n_2 that coincide with this ratio (which are below $n = 7$). Then ones that work are $n_1 = 2$ and $n_2 = 5$.

(b) Now that we know each n value, we can go back and solve for E_1.

$$E_{\lambda_1} = E_1(n_1^2 - 1) \implies E_1 = \frac{E_{\lambda_1}}{n_1^2 - 1} = 5.985 \text{ eV}$$

(c) Recall the square well energy equation:

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

Let's use $n_1 = 2$. We can solve for E_2:

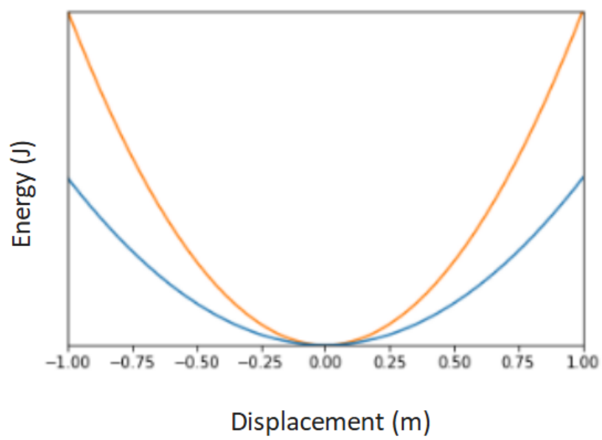
$$E_{\lambda_1} = E_2 - E_1 \implies E_2 = E_{\lambda_1} + E_1$$

Then, we solve for L:

$$L = \sqrt{\frac{\hbar^2 \pi^2 (2)^2}{2mE_2}}$$

Doing this, we get $L = 0.25$ nm.

13. Suppose two particles in a quantum harmonic oscillator have different values of k (spring constant), but identical mass. Which graph corresponds to the particle with lower k (spring constant)? Orange or Blue?



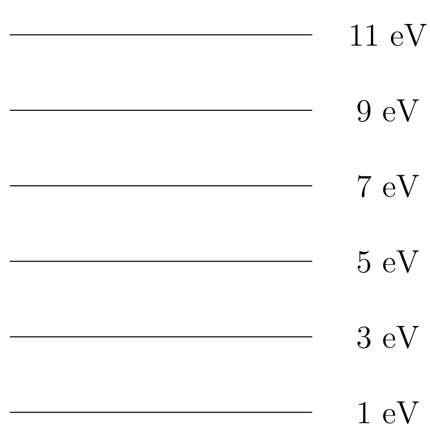
The blue graph since a lower value of k implies a wider parabola

$$U = \frac{1}{2}kx^2$$

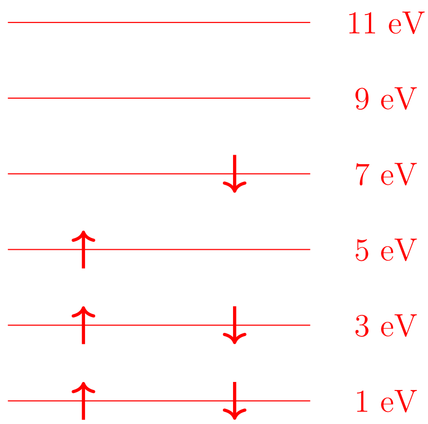
when these are approximated as small springs.

14. A particle is in the ground state of a quantum harmonic oscillator (ground state energy = $\frac{1}{2}\hbar\omega$), what would happen is some scientists shoot a photon with energy $\hbar\omega$ at it? What if they change the photon energy to $0.9\hbar\omega$? $1.1\hbar\omega$?
If the photon energy matches the energy gap between two allowed energy states, the particles will absorb the photon and jump over the energy gap to an excited state. If the photon energy does not match the gap, it will not be absorbed. Therefore, in the case of a photon with energy $\hbar\omega$, the particle will be in the first excited state and nothing happens for photons with energy 0.9 $\hbar\omega$ or 1.1 $\hbar\omega$.

15. Consider the oscillator below:



Six electrons are in a harmonic oscillator. Fill in the diagram to represent the oscillator's lowest excited state, using arrows to represent the spin of the electrons. What would the diagram look like if it is irradiated with light of energy 3 eV?



In the first excited state, the ground state is changed such that one electron is moved to a higher energy state corresponding to the lowest possible change in the energy of the system. Here, a single electron increased its energy by 2eV, moving to a higher band. Note that there can be two electrons in each band; however, they must have opposite spin per the Pauli Exclusion Principle. Also, the answer is correct even if the direction of the arrows in the 5eV and 7eV band are switched, since the spin in a band with a single electron is inconsequential for our purposes.

If the system is irradiated with 3eV light, it will not change. Electrons will only absorb certain quantized amounts of energy, corresponding to the amount of energy they need to reach a higher energy band. In the case of this diagram, the electrons can only absorb photons with energy that is a multiple of 2 eV, or photons with energy above 4 eV (which would free an electron from the oscillator).

16. We have a material that is transparent to yellow light ($\lambda = 750$ nm), but opaque to cyan light ($\lambda = 510$ nm).

(a) Given this information, give an energy range for the possible value of the electronic gap.

(b) Suppose we are now told that the material is opaque to green light ($\lambda = 550$ nm). Write a new, smaller range for the possible electronic gap.

(a) 1.65 eV $< E_g <$ 2.43 eV. We get these values by finding the energies that correspond with each wavelength. Since yellow light is not absorbed, we know E_g should be higher than yellow light's energy. The material absorbs cyan light, so E_g should be lower than cyan light's energy.

(b) 1.65 eV $< E_g <$ 2.25 eV. Knowing that the material is able to absorb green light, E_g should be lower than green light's energy. This lowers the upper bound of our inequality.

17. According to the band structure model, which of the following materials has the largest and smallest energy gap? Aluminum (good conductor), silicon (semiconductor), and silicon dioxide glass (good insulator).
According to the band structure model, no energy gap in the material leads to good conductance as it's easier for electrons to move. A large gap would make the material a good insulator because it is an energy barrier. And a medium gap usually indicates a semiconductor. For this question,

the energy gap for aluminum is less than that of silicon, which is less than that of silicon dioxide glass.

$$E_{aluminum} < E_{silicon} < E_{glass}$$

18. Suppose we have an unpolarized beam of photons with intensity $I_\circ$. We have a horizontal filter, i.e. a polarizer that only allows photons in the Ψ_H state to pass through.

(a) If the beam is incident on the horizontal filter, determine the resultant intensity in terms of $I_\circ$ as well as the wavefunction of photons exiting the filter.

(b) Now suppose we rotate the horizontal filter by 90°, effectively creating a vertical filter. Write the resultant wave function of the exiting photons in this scenario.

(c) Next, we rotate the filter such that it makes a 45° angle with the horizontal. Once again, write the resultant wave function.

(d) In (a)-(c), we didn't add any additional filters; we just rotated the prexisting one. Do these rotations influence the resultant intensity?

(e) Keeping the filter at 45°, we place another horizontal filter into the system, after the first filter. Determine the probability for a photon to pass through this second filter.

(a) $I = I_\circ/2$, $\Psi = \Psi_H$. When unpolarized light passes through a polarizer, the intensity is halve. Because a horizontal filter is used, the resultant wave function only has a horizontal component.

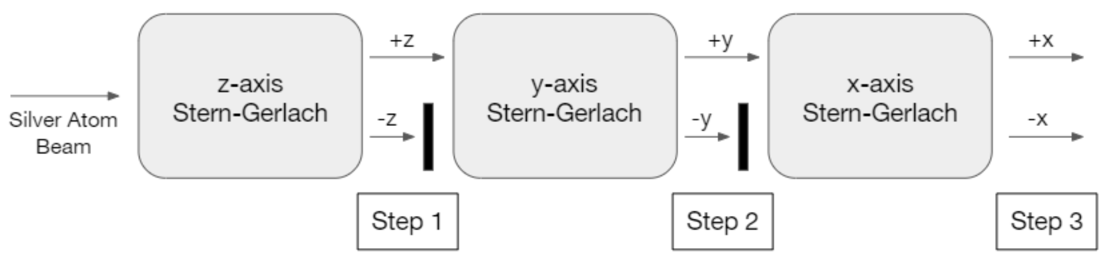
(b) $\Psi = \Psi_V$. Now that we're using a vertical filter, only the vertical component survives.

(c) $\Psi = \frac{1}{\sqrt{2}}(\Psi_H + \Psi_V)$. At 45°, portions of the horizontal and vertical components survive. The $1/\sqrt{2}$ factor normalizes the wavefunction.

(d) $I = I_\circ/2$ in all scenarios. Unpolarized light has no orientation; a linear filter rotated at any angle is still just a linear filter. The intensity is always halved when unpolarized light passes through.

(e) $P = 0.5$. The beam is diagonally polarized, meaning the wave function of each photon has an equal proportion of the horizontal and vertical components. Thus there is a 0.5 chance for each photon to be observed in the horizontal state.

19. We have the following Stern-Gerlach experiment setup:



(a) Assuming the silver atom beam is initially unpolarized in spin (i.e. it has equal components up and down), determine the probability of a silver atom to reach the $+x$ path in step three.

(b) Now let's assume the silver atoms are described by the wave function

$$\Psi = \frac{1}{5}\left(4\Psi_{\uparrow} + 3\Psi_{\downarrow}\right)$$

Given this information, determine the new probability of a silver atom reaching the $+x$ path.

(a) $P = 0.125$. Going step by step: first, the unpolarized silver atom beam is split 50/50 between the $\pm z$ paths (0.5). From Step 1 to Step 2, we can use the formula $P(S) = |S^* \cdot \Psi|^2$, with the initial state being $\Psi_{\uparrow}$:

$$\begin{aligned} P(+y|+z) &= \left|\frac{1}{\sqrt{2}}\left(\Psi_{\uparrow} - i\Psi_{\downarrow}\right) \cdot \Psi_{\uparrow}\right|^2 \\ &= 0.5 \end{aligned}$$

We do the same to find the probability from Step 2 to 3:

$$\begin{aligned} P(+x|+y) &= \left|\frac{1}{\sqrt{2}}\left(\Psi_{\uparrow} + \Psi_{\downarrow}\right) \cdot \frac{1}{\sqrt{2}}\left(\Psi_{\uparrow} + i\Psi_{\downarrow}\right)\right|^2 \\ &= \left|\frac{1}{2}(1+i)\right|^2 \\ &= \frac{1}{4}(1-i)(1+i) \\ &= 0.5 \end{aligned}$$

Thus the joint probability for all three steps is simply $(0.5)(0.5)(0.5) = 0.125$.

(b) $P = 0.64 * 0.5 * 0.5 = 0.16$. Given the new wave function, let's redo the first probability (from the initial silver atom beam to Step 1):

$$\begin{aligned} P(+x) &= \left|\Psi_{\uparrow} \cdot \frac{1}{5}\left(4\Psi_{\uparrow} + 3\Psi_{\downarrow}\right)\right|^2 \\ &= \left|\frac{4}{5}\right|^2 \\ &= 16/25 = 0.64 \end{aligned}$$

After this, the probabilities for the successive steps are 0.5, so the final probability is $(0.64) * (0.5) * (0.5) = 0.16$.

20. Consider the alpha decay of a radium-226 nucleus: ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

The decay releases a total energy of $Q = 4.87$ MeV, which goes into the kinetic energy of the α particle and the recoiling radon nucleus. Use the following constants:

- Mass of α particle (He): $m_\alpha = 6.64 \times 10^{-27}$ kg
- Mass of radon-222 nucleus: $M_{\text{Rn}} = 3.69 \times 10^{-25}$ kg
- 1 MeV $= 1.602 \times 10^{-13}$ J

*Assume the radium-226 nucleus is starting at rest.

(a) Using conservation of momentum, calculate the kinetic energy of the α particle and the radon nucleus.

(b) Determine the speed of the α particle and the recoiling radon nucleus (Assume p = mv).

(c) Estimate the mass of the radium-226 nucleus.

(a) By conservation of momentum:

$$\begin{aligned} p_\alpha &= p_{\text{Rn}} = p \\ K_\alpha + K_{\text{Rn}} &= Q \\ \frac{p^2}{2m_\alpha} + \frac{p^2}{2M_{\text{Rn}}} &= Q \\ p^2\left(\frac{1}{2m_\alpha} + \frac{1}{2M_{\text{Rn}}}\right) &= Q \\ p^2 &= \frac{2QM_{\text{Rn}}m_\alpha}{M_{\text{Rn}} + m_\alpha} \\ K_\alpha &= \frac{p^2}{2m_\alpha} = \frac{M_{\text{Rn}}}{M_{\text{Rn}} + m_\alpha}Q \\ K_{\text{Rn}} &= \frac{p^2}{2M_{\text{Rn}}} = \frac{m_\alpha}{M_{\text{Rn}} + m_\alpha}Q \\ K_\alpha &\approx 4.78 \text{ MeV}, \quad K_{\text{Rn}} \approx 0.09 \text{ MeV} \end{aligned}$$

(b) Speeds of the particles:

$$\begin{aligned} v_\alpha &= \frac{p}{m_\alpha} = \sqrt{\frac{2K_\alpha}{m_\alpha}} \approx 1.52 \times 10^7 \text{ m/s} \\ v_{\text{Rn}} &= \frac{p}{M_{\text{Rn}}} = \sqrt{\frac{2K_{\text{Rn}}}{M_{\text{Rn}}}} \approx 2.73 \times 10^5 \text{ m/s} \end{aligned}$$

(c) Estimate mass of radium-226 using mass-energy equivalence:

$$\begin{aligned} Q &= (M_{\text{Ra}} - M_{\text{Rn}} - m_\alpha)c^2 \\ M_{\text{Ra}} &= M_{\text{Rn}} + m_\alpha + \frac{Q}{c^2} \\ M_{\text{Ra}} &\approx 3.756 \times 10^{-25} \text{ kg} \end{aligned}$$

21. A simple entanglement example problem. Consider the following state describing particle one and particle two.

$$|\Psi\rangle = a(\uparrow\uparrow) + b(\downarrow\downarrow)$$

where the first arrow is the spin state of the first particle and likewise for the second particle. If the spin of particle one is measured to be up ($\uparrow$) what is the probability that particle two is:

(a) Spin down?

(b) Spin up?

(a) We know the first entry in the two particle state must be $\uparrow$ since we measured the first particle to be spin up. Notice, however, that this is only the case for the eigenstate $a(\uparrow\uparrow)$. Thus, by measuring the first particle to be spin up, we must measure the second particle to also be spin up, so the probability is 0.

(b) By our argument above, the probability to measure the second particle in the up state is 1.

What we have shown is that the state of particle two is completely determined by the state we measure particle one in. (And vice versa). This is what we mean by entanglement. In fact this state is known as a "maximally entangled state".

22. Consider a β^+ decay, where a proton inside a nucleus converts into a neutron, emitting a positron (e^+) and an electron neutrino (ν_e):

$$p^+ \to n^0 + e^+ + \nu_e$$

Suppose you measure the spins of the positron and proton and find that both are spin up (↑). Assume the system is initially in a spin-conserving state.

(a) Determine the two possible states that are consistent with total spin conservation.

(b) Determine the probability that the neutrino has spin down (↓).

(c) If you measure the neutrino spin and find it is up (↑), what is the probability that the neutron has spin down (↓)?

(a) Conservation of total spin requires that the sum of all spins equals the initial spin of the proton:

$$\Psi = a\,(\uparrow\uparrow\downarrow) + b(\uparrow\downarrow\uparrow) + c\,(\downarrow\uparrow\uparrow)$$

Since both proton and positron are measured spin up, the only two possibilities for the system are:

$$\Psi = a\,(\uparrow\uparrow\downarrow) + c\,(\downarrow\uparrow\uparrow)$$

(b) The the probability that the neutrino is spin down is

$$P(\nu_e \downarrow) = \frac{|a|^2}{|a|^2 + |c|^2}$$

(c) If you measure the neutrino spin and find it is up (↑), the system collapses to the state given by c. The probability that the neutron is spin down is then

$$\Psi = c\,(\downarrow\uparrow\uparrow)$$

$$P(n \downarrow) = \frac{|c|^2}{|c|^2} = 1$$

This means that the spin of the neutron has to be down. The solution tells us that if we measure 2 of the spins we know what the last particle's spin is due to conversation. In other words, conservation laws lead to a particle's state being entangled.

Lastly, keep in mind that coefficients for a, b, and c could be given as a complex numbers (similar to problems where you had to calculate the probability of a certain momentum)