



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 241 – Calculus III

Midterm 2 CBTF Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 1, 5-7 pm Meredith and Lydia

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/845>
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Consider the vector function $\vec{r}(t) = \langle 2 \cos t, 2 \sin t, t \rangle$. Find the length from $(2, 0, 0)$ to $(2, 0, 4\pi)$ of the vector function.

2. Calculate the following derivatives:

- (a) Find $\frac{df}{dt}$ for $f(x, y) = xe^{xy}$, $x(t) = t^2$, $y(t) = \frac{1}{t}$
- (b) Find f_t for $f(x, y) = 2xy$, $x(s, t) = st$, $y(s, t) = s^2t^2$

3. What is the partial derivative of $f(x, y, z) = e^x \sin(yz)z^3 \ln(y)$ with respect to x .

4. Consider a function $f(t) = f(x(t), y(t), z(t)) = xyz - z^2$, where $x(t)$, $y(t)$, $z(t)$ are defined as followed:

$$x(t) = 2t^2 + 1$$

$$y(t) = 3 - \frac{1}{t}$$

$$z(t) = 3$$

Find the following values:

(a) $f_z(3, 1, 2)$

(b) $\left. \frac{dx}{dt} \right|_{(t=0)}$

(c) $\left. \frac{df}{dt} \right|_{(t=1)}$

5. Consider the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y^4 \cos^2 x}{x^4 + y^4}$$

Does this limit exist? If so, what is its value? Justify your answer.

6. If $f(x, y, z) = xye^z$, find the gradient of f and the directional derivative at $(2, 5, 0)$ in the direction of $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$.

7. Evaluate the following functions with $\lim_{(x,y) \rightarrow (0,0)}$:

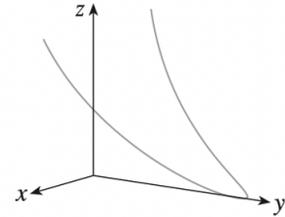
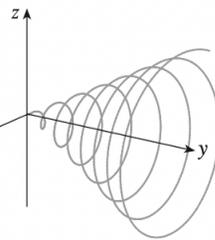
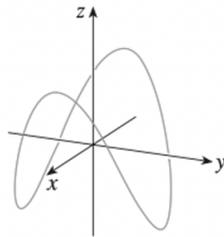
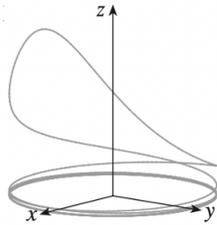
$$(a) f(x, y) = \frac{3xy - x^2y}{x^2 + y^2 + xy}$$

$$(b) f(x, y) = \frac{y \sin(x) + y^2 e^x}{y}$$

$$(c) f(x, y) = \frac{(x^2 + y^2)^5}{x^{10} + y^4}$$

Match the vector functions below to the corresponding graph.

8. (i) $x = t, y = 1/(1 + t^2), z = t^2$ (iii) $x = \cos t, y = \sin t, z = 1/(1 + t^2)$
 (ii) $x = \cos t, y = \sin t, z = \cos 2t$ (iv) $x = t \cos t, y = t, z = t \sin t, t \geq 0$



9. Find the limits for the following expressions:

(i)

$$\lim_{(x,y) \rightarrow (-1,0)} \frac{x^2 + xy + 3}{x^2y - 5xy + y^2 + 1}$$

(iii)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8}$$

(ii)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3}{x^2 + y^2}$$

(iv)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^6 + x^2y^4}{x^2 + y^2}$$

10. Find the derivatives of the function $f(x, y) = x^2 + y^2 + xy + y^3$.

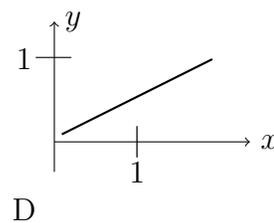
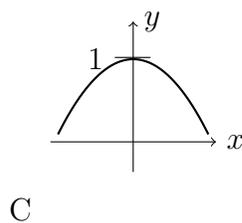
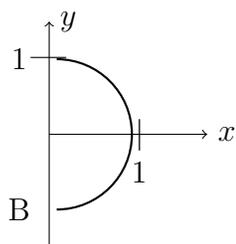
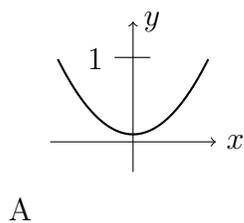
(i) f_x, f_y

(iii) f_{xx}, f_{yy}

(ii) Total differential

(iv) f_{xy}, f_{yx}

11. Let $r(t) = \langle \sin(t), \cos^2 t \rangle, 0 \leq t \leq 2\pi$. Which graph below represents this curve?



12. Assume you are walking around the surface of a spherical planet with a radius of 2. If you are walking clockwise on the xy-plane, what is the parametrization of the path after circling it twice?