Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 285 – Intro Differential Equations

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Febuary 18 from 4:00-5:50 pm in 433 Grainger Library - Regina, Sean

Session 2: Febuary 19 from 6:00-7:50 pm in 433 Grainger Library - Clive, Eric, Kimaya

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/846
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Consider the following differential equation and initial condition:

$$(16 - t^2)y' + t^3 y = \cos\left(\frac{t}{2}\right) \qquad y(1) = -5$$

On what interval is the unique solution certain to exist?

A) $(0, 2\pi)$

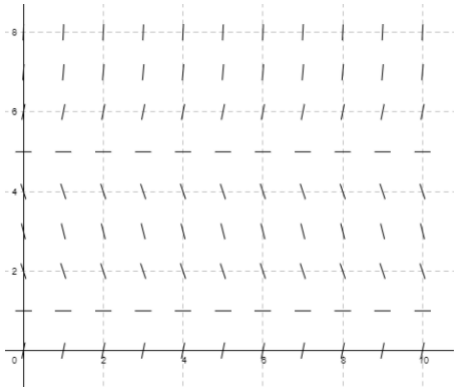
B) $(-2\pi, 0)$

C) $(-4, 0)$

D) $(-4, 4)$

E) $(-\infty, -4)$

2. Which equation produces the direction field below?



A) $\frac{dy}{dx} = (x-1)(x-5)$

B) $\frac{dy}{dx} = xy$

C) $\frac{dy}{dx} = (y-1)(y-5)$

D) $\frac{dy}{dx} = (y+1)(y+5)$

E) $\frac{dy}{dx} = y^2$

3. Which of the following equations are linear? Note: u_* denotes a first order partial derivative and u_{**} is a second order partial derivative.

$$\text{(I)}\ \ \frac{\mathrm{d}^2y}{\mathrm{d}t^2} + e^y = 6t + 5$$

$$\text{(II)}\ \ (2t^3 + 6)\frac{\mathrm{d}^5y}{\mathrm{d}t^5} - \frac{\mathrm{d}^3y}{\mathrm{d}t^3} + 4y = t\cos(t-1)$$

$$\text{(III)}\ \ u_y = uu_{xx} - u_{xy}$$

$$\text{(IV)}\ \ u_{xx} + xu_{xt} + t^2u_{tt} = \sin(x + 2t)$$

A) (I), (II), (III)

B) (II) and (IV)

C) (I) and (IV)

D) (II)

E) (IV)

4. What is the order of the following differential equation?

$$\cot(y)y''' + (t^2 + t + 9)y' - \ln\big(xy^2\big)y + 6y^9 = \sin\big(3t^5 + 1\big)$$

A) 9

B) 1

C) 3

D) 5

E) None

5. Consider the following autonomous equation: $y' = (y^2 - 9)(5 - y)$
   List the equilibrium solutions and classify them as stable, semi-stable, or unstable.

6. Prove that the following ODE is exact and find the general solution using the exact equations method with ψ: $(3x^2 - 3y^2)\frac{dy}{dx} + (3x^2 + 6xy) = 0$

7. Solve $x(\frac{dy}{dx}) = 2y + x$

8. Find the solution to the following initial value problem

$$(x^4 + 1)\left(\frac{dy}{dx}\right) = 2x^3y^2 \qquad y(0) = \left(\frac{3}{2}\right)$$