MATH 231 REVIEW

Topics Covered

- Integration by Parts
- Trigonometric Integrals
- Trigonometric Substitution
- Integration of Rational Functions by Partial Fractions
- Approximate Integration
- Improper Integrals

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Where

$u = f(x)$	$du = f'(x)$
$dv = g(x)dx$	$v = + g(x)dx$

Note: May have to repeat process more than once to completely solve

How to Choose “u”?

Use

LIATE!

L-ogarithmic	$\ln(x)$
I-nverse Trig	$sin^{-1}(x)$
A-lgebraic	$x^2 + 3x$
T-rigonometric functions	$\sin(x)$
E-xponential functions	e^x

Trigonometric Integrals

- $cos^2(x) + sin^2(x) = 1$
- $tan^2(x) + 1 = sec^2(x)$
- $tan^2(x) = sec^2(x) - 1$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $sin^2 x = \frac{1}{2}(1 - \cos(2x))$
- $cos^2 x = \frac{1}{2}(1 + \cos(2x))$
- $\cos(2x) = cos^2(x) - sin^2(x)$

  $= 1 - 2sin^2(x)$

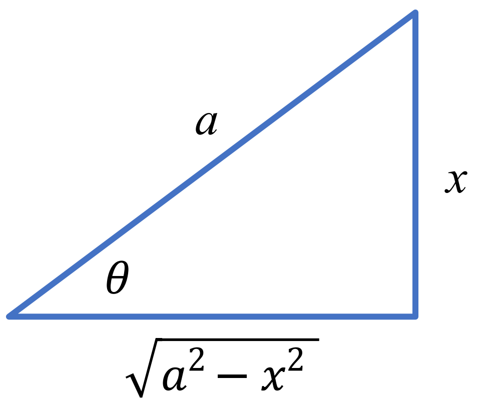
  $= 2cos^2(x) - 1$

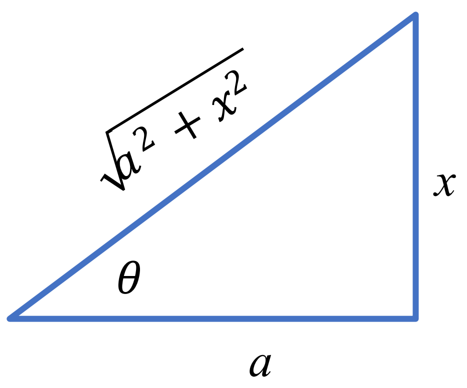
- Used the following trig identities and others to rewrite and simplify trig equations under an integral

Ex: $\int \sin(x)^3 \cos(x)^5 dx$

Trigonometric Substitution

Format	Substitution	Derivative Substitution	Trig Identity
$\sqrt{a^2 - x^2}$	$x = a * \sin(\theta)$	$dx = a * \cos(\theta)\, d\theta$	$cos^2(\theta) + sin^2(\theta) = 1$
$\sqrt{a^2 + x^2}$	$x = a * tan(\theta)$	$dx = a * sec^2(\theta)\, d\theta$	$tan^2(\theta) + 1 = sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a * sec(\theta)$	$dx = a * \sec(\theta)\tan(\theta)\, d\theta$	$tan^2(\theta) = sec^2(\theta) - 1$





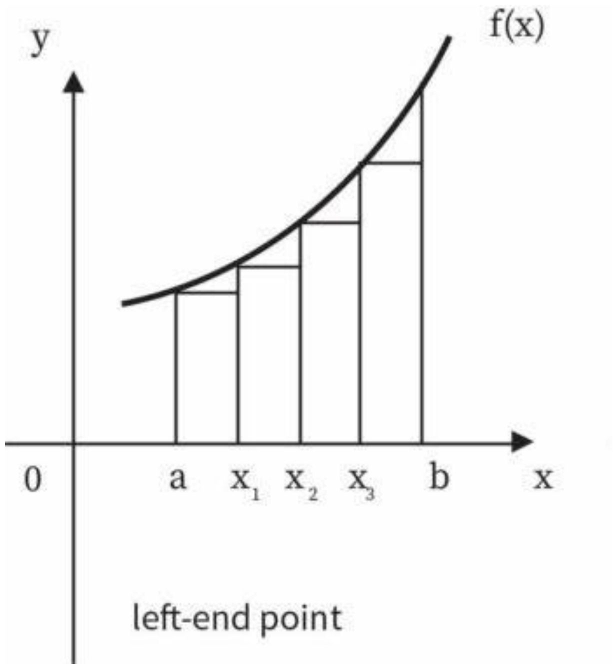
Steps to Solve:

1. Identify format
2. Replace x and dx
3. Simply and/or use trig identity
4. Convert back to numerical using triangle

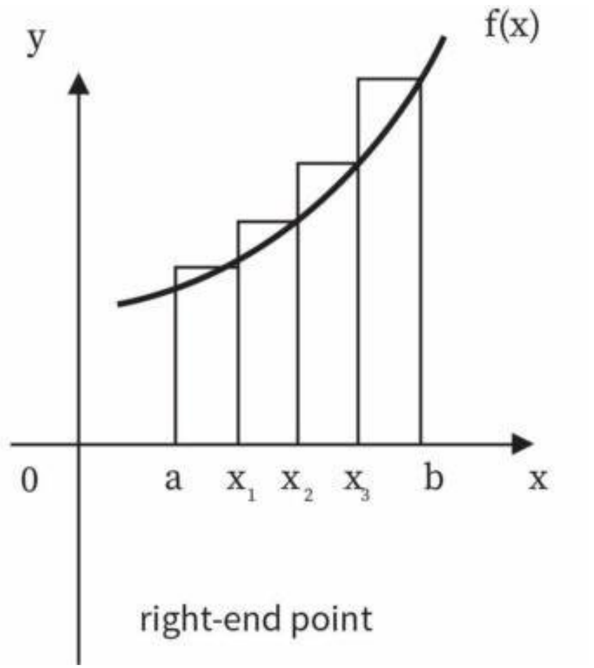
Integration of Rational Functions by Partial Fractions

Rational Function	Partial Fraction
$\dfrac{px+q}{(x-a)(x-b)}, \mathrm{a} \neq b$	$\dfrac{A}{(x-a)} + \dfrac{B}{(x-b)}$
$\dfrac{px+q}{(x-a)^2}$	$\dfrac{A}{(x-a)} + \dfrac{B}{(x-a)^2}$
$\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\dfrac{A}{(x-a)} + \dfrac{B}{(x-b)} + \dfrac{C}{(x-c)}$
$\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$	$\dfrac{A}{(x-a)} + \dfrac{B}{(x-a)^2} + \dfrac{C}{(x-b)}$
$\dfrac{px^2+qx+r}{(x-a)(x^2-bx-c)}$	$\dfrac{A}{(x-a)} + \dfrac{Bx+C}{(x^2-bx-c)}$

Approximate Integration



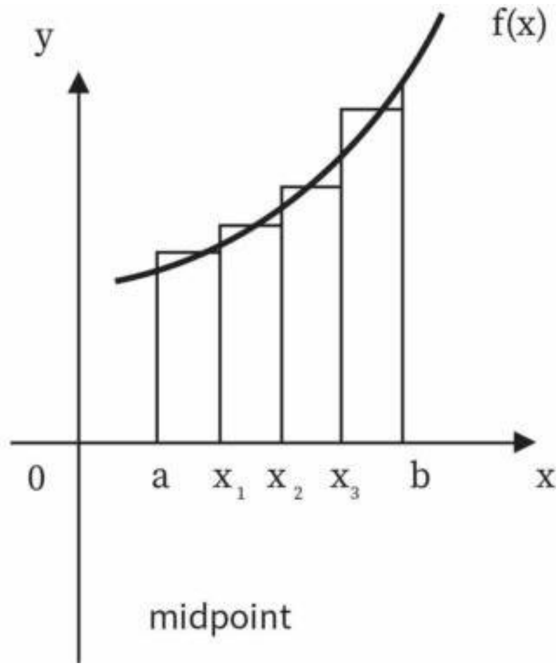
Where $\Delta x = \frac{b"a}{n}$



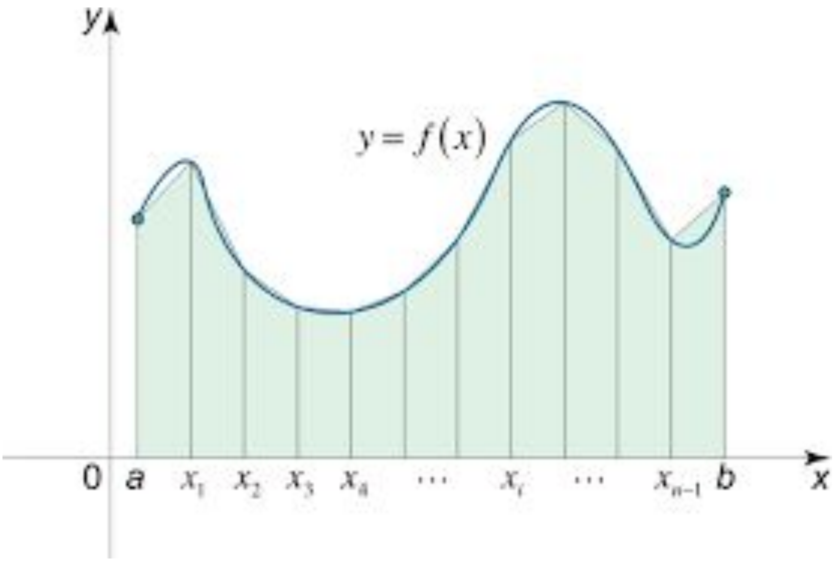
$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \cdots f(x_{n-1})\Delta x = \sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \cdots f(x_n)\Delta x = \sum_{i=0}^{n-1} f(x_i)\Delta x$$

Approximate Integration



Trapezoidal Rule



$$M_n = f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + \cdots f(\bar{x}_n)\Delta x = \sum_{i=0}^{n-1} f(\bar{x}_i)\Delta x$$

Where $\bar{x} = \frac{x_{i-1} - x_i}{2}$

$$T_n = \frac{\Delta x}{2}(f(x_0)\Delta x + \ 2f(x_1)\Delta x + \cdots 2f(x_{n-1}) + f(x_n)) = \sum_{i=0}^{n-1}(f(x_{n-1}) + f(x_n))\,\frac{\Delta x_n}{2}$$

Approximate Integration: Simpson's Rule

$$S_n = \frac{\Delta x}{3}(f(x_0)\Delta x + \; 4f(x_1)\Delta x + 2f(x_2)\Delta x + \cdots f(x_n)) \approx \int_0^n f(x)$$

- Has the lowest error, therefore the most accurate
- Closest of the methods to finding the actual integration or area under the curve

Improper Integrals

- Improper Integrals: FTC does not hold since functions are **not continuous along the interval of integration.**
- Type I: Infinite Interval

$$\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

- Type II: Discontinuous Interval

$$\int_a^{b} f(x)dx = \lim_{t\to b-} \int_a^t f(x)dx$$

$$\int_a^{b} f(x)dx = \lim_{t\to a+} \int_t^b f(x)dx$$

$$\int_a^{b} f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Questions?