Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

MATH 257 – Linear Algebra with Computational Applications

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb. 7, 5:00 - 6:20 PM Aman, Nehan, Aidan

Session 2: Feb. 8, 5:00 - 6:20 PM Rohan, Siddh, Serge

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/955
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Consider the system of linear equations:

$$\begin{cases} x_1 + 4x_2 + 2x_3 & = 2 \\ x_1 + 4x_2 + \quad +2x_4 & = 2 \end{cases}$$

a) Create an augmented matrix for this system and put it into reduced row echelon form.

b) Which variables are basic? Which variables are free?

c) State the general solution in **parametric** form. (Express the solution vector as the sum of a fixed vector and vectors that are multiplied by the free variables)

2. Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be defined as follows:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 4 & 1 \\ 6 & 0 \end{bmatrix} \text{ and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$$

Compute the following:

a) $\boldsymbol{A} + \boldsymbol{C}$

b) $\boldsymbol{2B} + \boldsymbol{A}$

c) $\boldsymbol{4C^T} - \boldsymbol{B}$

d) $\boldsymbol{C} + \boldsymbol{\tfrac{1}{2}B^T} - \boldsymbol{A}$

3. Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be defined as follows:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \; \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix} \text{ and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 8 & 7 \end{bmatrix}$$

a) Compute the following or state it is not defined:

i) $\boldsymbol{BC}$

ii) $\boldsymbol{ACB}$

b) The matrix $\boldsymbol{B}$ is an elementary matrix. What row operation would $\boldsymbol{B}$ do if it was multiplied on the left to a matrix $\boldsymbol{M}$ of an appropriate size?

4. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be three nonzero vectors in $\mathbb{R}^3$ and let $\mathbf{c}$ be in span$\{\mathbf{a}, \mathbf{b}\}$.

(a) Describe span$\{\mathbf{a}, \mathbf{b}\}$

(b) Suppose these three vectors are arranged in a matrix $\boldsymbol{A}$ column-wise. How many pivots can $\boldsymbol{A}$ have at most?

5. True or False: A square matrix $\boldsymbol{A}$ is invertible if and only if $\det \boldsymbol{A} = 0$.

6. Let $\boldsymbol{A}$ be an $m \times n$ matrix. What are the dimensions of $\boldsymbol{A}^T\boldsymbol{A}$?

7. Consider a linear system $\boldsymbol{Ax} = \mathbf{b}$. Given that the row reduced echelon form of $\boldsymbol{A}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which of the following are true?

I) The system has exactly one solution
II) The system has no solutions
III) There are three pivots associated with $\boldsymbol{A}$
IV) $\boldsymbol{A}$ is invertible

8. Which of the following matrices is the inverse to $\boldsymbol{A}$?

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 0 & 1 & -2 & -2 \\ -1 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 & -2 & 0 \\ -1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$