PHYS 214 Exam 1 Review

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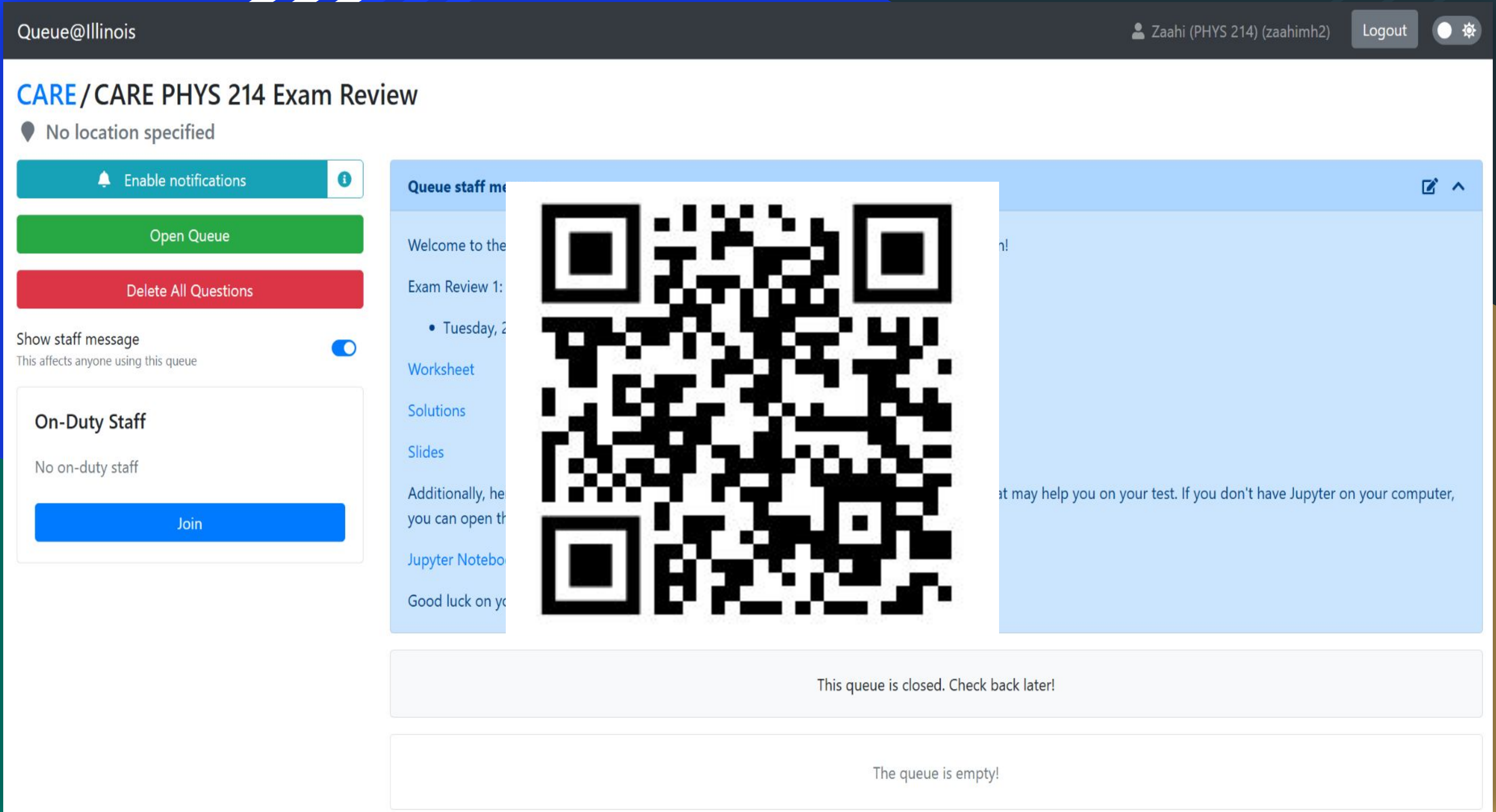
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Units for the Exam

- Waves
- Interference
- Diffraction

Wave Equation

General Wave Propagation: $y(x, t) = A\cos(kx - \omega t + \phi)$

k = wave number (how the wave repeats in SPACE) [m^{-1}]

ω = angular frequency (how the wave repeats in TIME) [rad/s]

ϕ = phase shift (the starting phase of the wave) [rad]

$kx - \omega t + \phi$ = phase

Properties of Waves

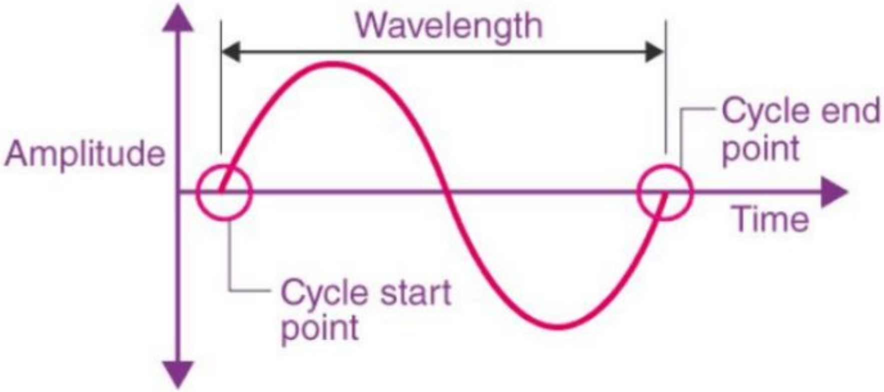
$\lambda = 2\pi/k$; $f = \omega/ 2\pi$

$v = \omega/k$ $v = \lambda f$

Intensity: $I(x,t) = |y(x,t)|^2$

$I_{average} = |A|^2/2$

$f = 1 / T$



Interference

Superposition (adding): A fancy way of saying that when two waves interact, the resulting wave is the **<u>SUM</u>** of the two individual waves

$y_1(x, t) = A_1\cos(k_1 x - \omega_1 t + \phi_1)$

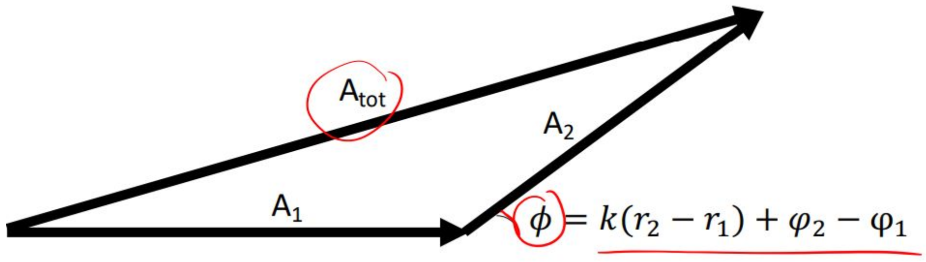
$y_2(x, t) = A_2\cos(k_2 x - \omega_2 t + \phi_2)$

$y_{tot}(x, t) = y_1(x, t) + y_2(x, t) = A_1\cos(k_1 x - \omega_1 t + \phi_1) + A_2\cos(k_2 x - \omega_2 t + \phi_2)$

If $\phi_1 = \phi_2$, the angular frequencies (ω) are the <u>SAME</u>, and the distance is the <u>SAME</u>, then the waves are <u>IN PHASE</u>

Phasors and Law of Cosines

$$A_{tot}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)$$

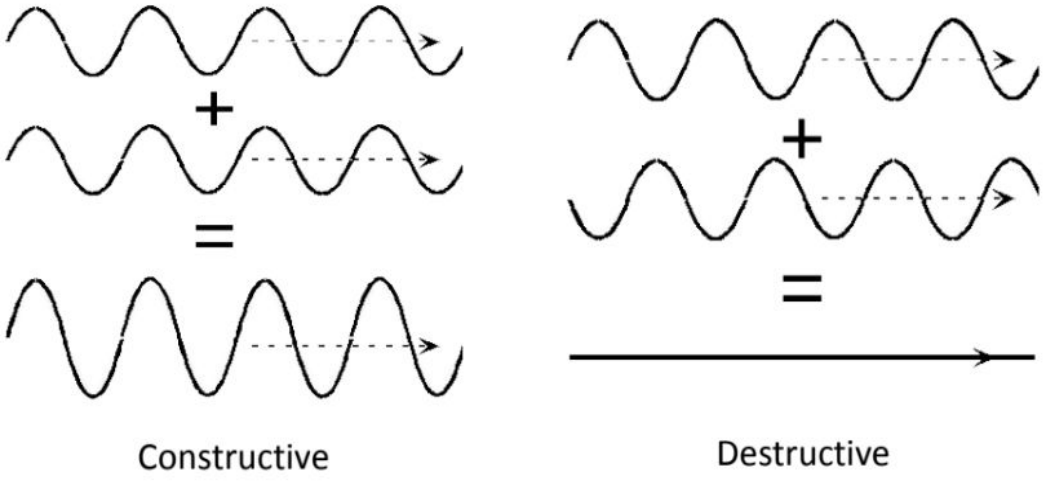


To note:

- The Law of Cosine usually has a -, but we are using the outside angle so it becomes +
- This is the **General Equation** and works for waves with different amplitudes

Interference (Cont.)

Phase difference = $k(r_2-r_1) = \phi$ for a two source system at different distances



In general, for two sources with the same amplitude/intensity:

$$I_{\text{tot}} = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

In your equation sheet, this is written as:

$$I_{\text{total}} = 2A^2\cos^2\left(\frac{kr_1+\phi_1-kr_2-\phi_2}{2}\right)$$

I_0 is the intensity from a single beam

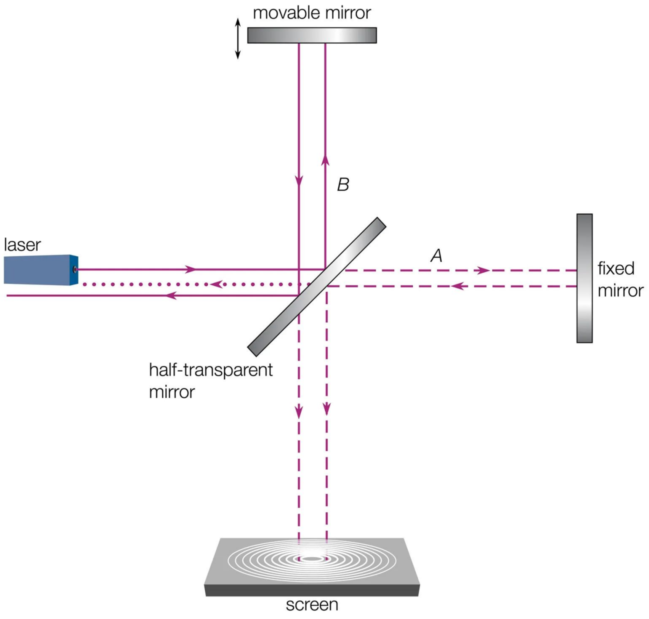
$I_0 = \frac{I_{source}}{4}$, this is due to the beam getting split twice

Example Problem - Interferometer

A Michelson interferometer is illuminated by a laser of power **P = 5 mW** and wavelength **λ = 632.8 nm**

You want to adjust one of the mirrors to get a **new power of 2 mW**.

How far do you have to move the mirror to achieve this new intensity?



Example Problem - Interferometer (Cont.)

$$I_0 = \frac{P}{4} = \frac{5}{4} = 1.25\ mW$$

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$$\Delta\phi = 2\cos^{-1}\left(\sqrt{\frac{I_{new}}{4I_0}}\right) = 2\cos^{-1}\left(\sqrt{\frac{2}{4(1.25)}}\right) = 1.77\ rad$$

Example Problem - Interferometer (Cont.)

$$I_0 = \frac{P}{4} = \frac{5}{4} = 1.25\ mW$$

$$\Delta\phi = \frac{2\pi}{\lambda}(L_2 - L_1) = \frac{2\pi}{\lambda}(2\Delta x)$$

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$$I_{new} = 4I_0\ cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta x = \frac{\Delta\phi\lambda}{4\pi} = \frac{1.77 * (600 * 10^{-9})}{4\pi} = 84.6\ nm$$

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Diffraction

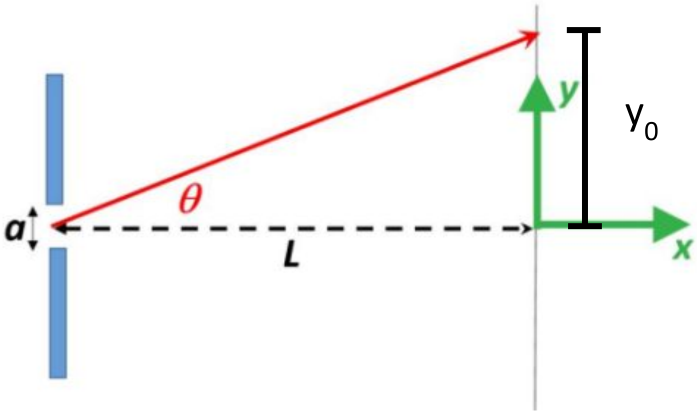
- Single slit diffraction:
  - a = slit width
  - θ_o = angle of first **minimum**
  - λ = wavelength

$$a \sin(\theta_\circ) = \lambda$$

- **Small a → Large θ_o**

- Small angle approximation:
  - $\theta \cong \sin(\theta) \cong \tan(\theta) \cong y_o/L$

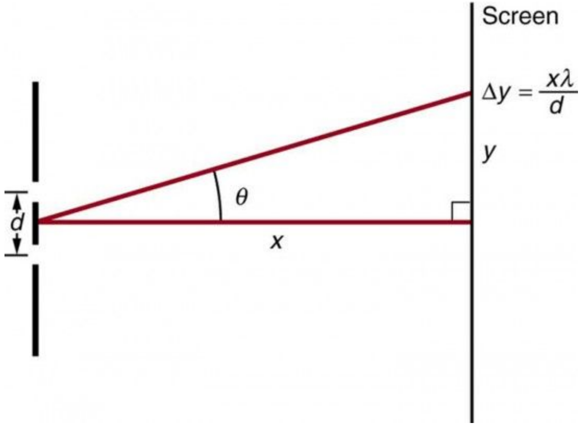
- Spot size:
  - Radius → $y_0 = L*\tan(\theta) \cong L*\theta$
  - Width → $2y_o = 2L*\tan(\theta) \cong 2*L*\theta$



Diffraction (cont.)

- Double slit diffraction:
  - *d* = distance between slits
  - θ_{max} = angle for mth **maximum**
  - θ_{min} = angle for mth **minimum**
  - m = integer (m = 0,1,2…)
    - Note: 0 denotes the central maximum

- **Small d → Large θ**
- Small angle approximation also applies these questions too



$$d\sin(\theta_{max}) = m\lambda$$

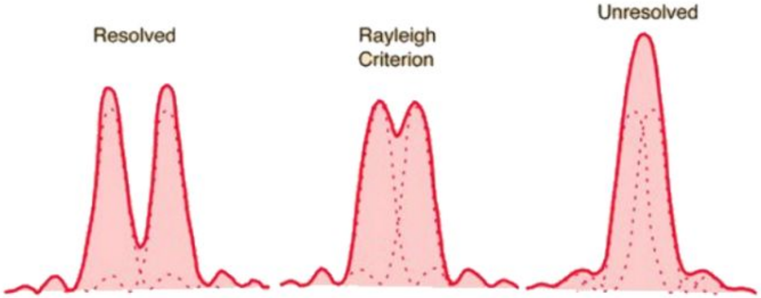
$$d\sin(\theta_{min}) = (m + \tfrac{1}{2})\lambda$$

Diffraction (cont.)

- Circular aperture diffraction
  - Similar to single slit; <u>1.22 factor</u>
- Rayleigh Criterion:
  - Center of the diffraction maximum from the first object falls onto the diffraction minimum from the second object
  - ie. $\theta_o \leq \theta_{objects}$
    - θ_o = angle of first minimum of central bright spot
    - For multiple objects θ_o is the minimum angle required to distinguish the two objects
    - $\theta_{objects}$ = angle between two bright spots

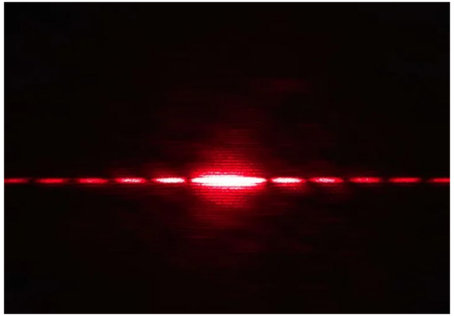
D - Diameter of lens

$$D \sin(\theta_\circ) = 1.22\lambda$$

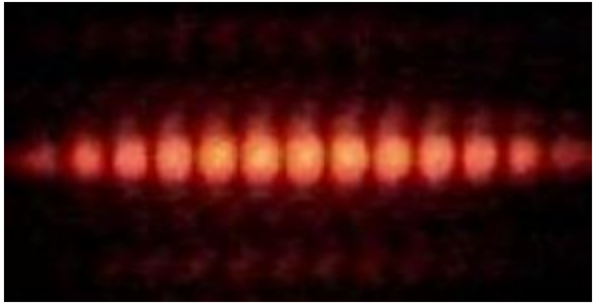


Diffractions In Real Life

Single Slit Diffraction



Double Slit Diffraction

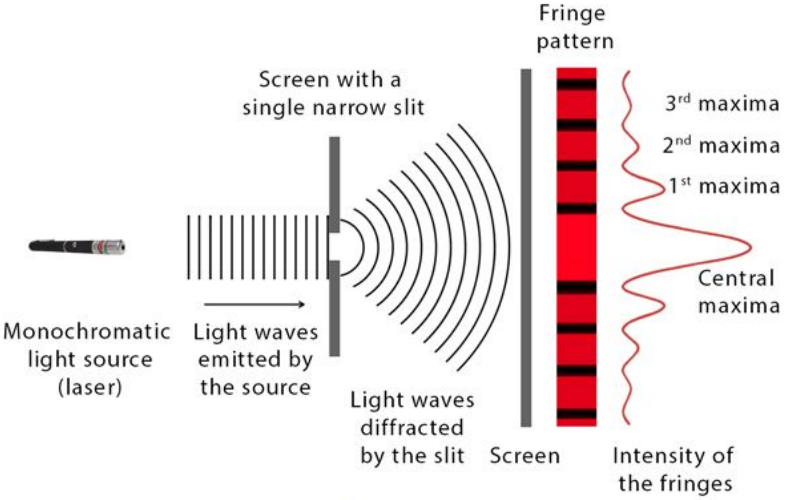


Circular Aperture Diffraction

Example Problem - Diffraction

A laser with wavelength **λ = 500 nm** illuminates a single slit width of ***a* = 0.1 mm**. A screen is placed ***L* = 2.00 m** from the slit.

Find the **vertical position y_4** on the screen of the fourth minimum measured from y = 0 (center of first maximum)



Example Problem - Diffraction (Cont.)

$$a\sin(\theta_0) = m\lambda$$

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$$a\theta_0 = m\lambda$$

$$\theta_0 = \frac{m\lambda}{a} = \frac{4(500 * 10^{-9})}{0.1 * 10^{-3}} = 0.02\,rad$$

Example Problem - Diffraction (Cont.)

$$a\sin(\theta_0) = m\lambda$$

$$y_4 = L\theta_0 = 2 * 0.02 = 0.04\,m$$

$$a\theta_0 = m\lambda$$

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Example Problem - Diffraction (Cont.)

$$a \sin(\theta_0) = m\lambda$$

$$a\theta_0 = m\lambda$$

$$\theta_0 = \frac{m\lambda}{a} = \frac{4(500 * 10^{-9})}{0.1 * 10^{-3}} = 0.02\, rad$$

$$y_4 = L\theta_0 = 2 * 0.02 = 0.04\, m$$

$$y_4 = 0.04\, m = 4\, cm$$

Good luck!

Feel free to ask any questions you may have.

You got this!