Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 213 – University Physics: Thermal Physics

Final Review Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Wednesday, December 10, 6-8 pm Luke, Zaahi, Camille

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1 Chemical Potential and Phases

1. Label each statement as True or False.

(a) Temperature is to heat as chemical potential is to the number of particles.

(b) Suppose we are in a constant pressure and temperature scenario, and we have a substance such that $\mu_{\text{gas}} < \mu_{\text{liquid}}$. After reaching equilibrium, the substance will completely be gaseous.

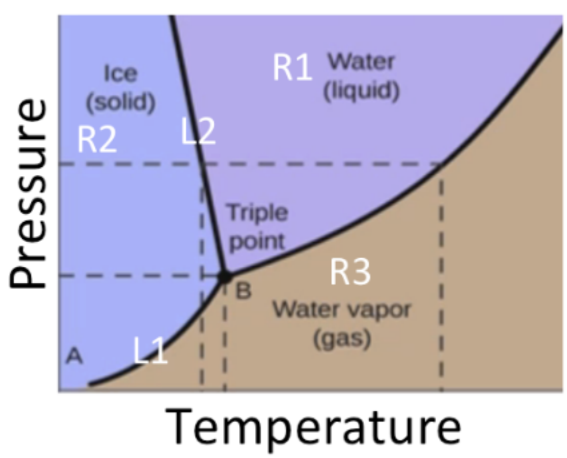
(c) In the situation in (b), μ_{gas} gets bigger when particles evaporate.

(d) By adding an ideal solute to water, we can lower both its melting point and boiling point.

(e) At a substance's melting point, $\mu_{\text{liquid}} = \mu_{\text{solid}}$.

(f) The temperature of a substance changes while it is undergoing a phase change.

2. Below is the phase diagram for water. Determine which chemical potential(s) is the lowest at the following points on the graph.



(a) region R1

(b) line L1

(c) extremely high temperatures, low pressures

3. Given a latent heat (vaporization or fusion), determine the heat energy required to perform the given phase change.

(a) heat of vaporization: 15 kJ/kg: boiling 20 kg of substance

(b) heat of fusion: 34 kJ/mol: melting 12 mol of substance

(c) heat of vaporization: 20 kJ/kg, boiling 15 mol of substance (molar mass: 40 g/mol)

4. Given a specific latent heat, a molar mass, and a phase transition temperature, determine the increase in entropy per molecule when the substance undergoes that phase change.

$$L = T\Delta S$$

(a) $L = 153$ kJ/kg, $M = 35$ g/mol, $T = 135$ K

(b) $L = 230$ kJ/kg, $M = 15$ g/mol, $T = 300$ K

(c) $L = 181$ kJ/kg, $M = 54$ g/mol, $T = 275$ K

5. We have a substance whose boiling point at 50 kPa is 230 K. Its latent heat per kilogram is 2500 J/kg, and its molar mass is 22 g/mol. Suppose we want to increase its boiling point by 0.5 K. This is small enough that we approximate $\mathrm{d}p \approx \Delta P$ and $\mathrm{d}T \approx \Delta T$. Assume the liquid phase of the substance has negligible volume.

(a) First, set up the differential relation between $\mathrm{d}\mu_L$ and $\mathrm{d}\mu_G$ at the boiling point (chemical potentials of the liquid and gas phases).

(b) Assume that the gas phase of this substance is ideal. Determine V_G/N_G, the volume per gas particle at the given pressure and temperature.

(c) Recall that for latent heat, $L = T\Delta S$. Use this to determine $\Delta S/N = \Delta S_G/N_G - \Delta S_L/N_L$, the change in entropy per particle when converting from liquid to gas.

(d) Use (a) - (c) to determine the change in pressure necessary to create the desired increase in boiling point temperature ($\Delta T = 0.5$ K).

6. Why does adding salt on the sidewalks in winter prevent ice from forming?

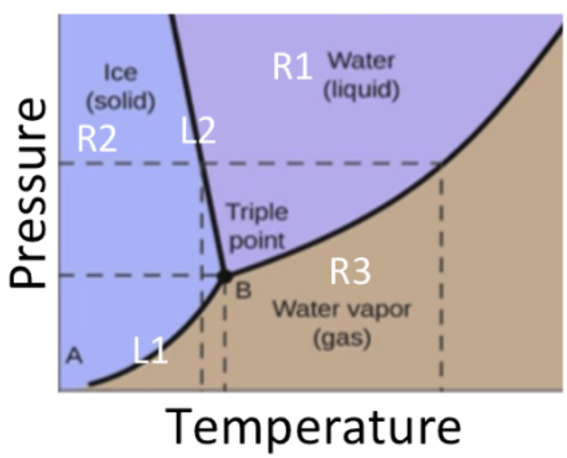
7. How does the phase diagram for water differ from other pure substances?

8. A substance has the following phase densities:

phase	ρ
solid	23 kg/m^3
liquid	25 kg/m^3
gas	1.3 kg/m^3

At extremely highly pressures, which phase will have the lowest chemical potential?

9. Here is the phase diagram for water.



(a) In which region or line is $\mu_{\text{liquid}} = \mu_{\text{solid}} < \mu_{\text{gas}}$?

(b) We have water exactly at its melting point (line L2). If we add an ideal solute to the water while keeping the temperature the same, which phase will the water become?

10. A sample of Substance A is at its melting point T = 313.3 K. Given that it takes 1566.5 J of energy to melt the sample at this temperature, what is the change in entropy per particle of the sample, given that the sample is made up of 1.5 moles of substance A?

(a) 5.5×10^{-24} J/K/particle

(b) 3.33 J/K/particle

(c) 5 J/K/particle

(d) 7.624×10^{-24} J/K/particle

2 The Boltzmann Factor

1. Label the following statements as true or false.

(a) A quantum harmonic oscillator can take on any value of energy.

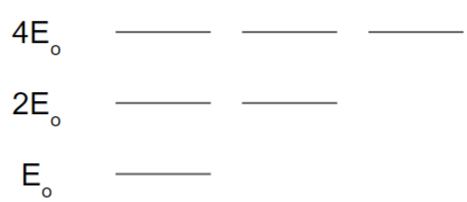
(b) The total energy of an array of quantum harmonic oscillators is constant, but the individual energies may change.

(c) The Boltzmann factors of each microstate of a system always sum to 1.

(d) As temperature goes to 0, the Boltzmann factor approaches 0 and each microstate of a system approaches an equal probability.

(e) As temperature goes to infinity, each microstate of a system approaches an equal probability.

2. Suppose we have a particle in contact with a reservoir at temperature T with the following microstates available to it:



Notice that certain energy levels have more than one valid microstate. At $T = E_\circ/k$:

(a) Determine the probability of the particle having energy $4E_\circ$.

(b) Determine the average energy of the particle in terms of $E_\circ$.

3. We have a system with a single quantum harmonic oscillator. In this system, one energy quantum is $\varepsilon = 1.5 \times 10^{-21}$ J, and there are a total of $q = 4\varepsilon$ energy quanta available. The oscillator is in a $T = 350$ K environment.

(a) Determine the ratio of probabilities for observing the $E = 4\varepsilon$ energy state and $E = 3\varepsilon$ energy state.

(b) As we increase the temperature T, what happens to the probabilities of each state?

4. We have a particle with two possible energy states $\pm\varepsilon$, where $\varepsilon = 6 \times 10^{-22}$ J. It is in contact with a thermal reservoir at temperature T.

(a) Determine the temperature T such that $P(-\varepsilon)/P(+\varepsilon) = 20$.

(b) What is the average energy at this temperature?

(c) What is the probability of measuring $+\varepsilon$ as T becomes extremely large? What is the average energy at extremely large temperatures?

5. An astrophysics major is trying to measure the temperature of a distant star. The star is primarily comprised of hydrogen. For simplicity, we'll describe the energy states of hydrogen as the following:

Here, $E_0 = 0$ J and $E_1 = 1.634 \times 10^{-18}$ J. Through science-y magic, the student determines that 99.9999993% of the hydrogen in the star is in the E_0 energy state.

(a) Calculate the ratio of hydrogen atoms in the E_0 state to those in the E_1 state.

(b) Determine the temperature. Remember that the energy E_1 has 4 available microstates.

6. We have a 10 km-tall cylinder filled with helium particles. The cylinder is heated to $T = 500$ K. The mass of a helium particle is $m_{\text{He}} = 6.643 \times 10^{-24}$ kg.

(a) Write the potential energy of a single Helium particle as a function of height h.

(b) Determine the ratio of probabilities between a particle at height h_1 and a particle at height h_2 as a function of h_1 and h_2.

(c) Locations of high probability in the cylinder correspond to higher-pressure areas, i.e.

$$p \propto P(h)$$

Determine the ratio of pressures between the top and bottom of the cylinder.

7. This question is pretty math/proof heavy and thus won't be asked on a test.
Let's create a formula for determining the heat capacity of a two-state system, $C(T)$.

(a) Suppose the two-state system has energies E_1 and E_2. Write the Boltzmann factor for each state.

(b) Write an expression for the average energy. This is the internal energy as a function of temperature T, $U(T)$.

(c) Find the heat capacity $C(T)$ by taking the derivative of $U(T)$ with respect to temperature (since $C = \mathrm{d}U/\mathrm{d}T$). In case you forgot your derivative rules:

$$x\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$$

You can also use an online derivative calculator ; this isn't a calculus class after all.

(d) Use this formula to find the heat capacity for a system with energy states $E_1 = 0$ J and $E_2 = 5.5 \times 10^{-20}$ J at temperature $T = 400$ K.

8. We are given a quantum system with 3 quantum harmonic oscillators and 7 Quanta of energy to distribute into them. How many different ways can we distribute the energy into the system?

9. Let's say we have an array of 8 quantum harmonic oscillators with $q = 20$ total energy quanta.

(a) How many microstates does this system have? What is the system's entropy?

(b) Now let's say we fix one of the oscillators to have 4 energy quanta (i.e. $E = 4\varepsilon$). How many microstates does *this* system have? What is its entropy?

(c) Without doing any calculations, if we were to fix one oscillator to 0 energy quanta instead of 4, would we expect the resulting entropy to be higher or lower?

3 Past Units

1. Consider 5 coins, each initially starting on heads.

(a) What is the entropy, S, of this system in its current configuration?

(b) List all the macrostates available to this system.

(c) Identify the most probable macrostates. Hint: there are two.

(d) How many microstates would lead to the macrostates identified above?

(e) Calculate the change in entropy, ΔS, if the system changed to either of its most probable macrostates.

2. Let's investigate the classic "cook a whole chicken by slapping it" experiment. The average whole raw chicken has a mass of 1.4 kg with a specific heat capacity of 3350 J/kg K. Your hand (along with a heat-insulating glove you're wearing) weighs 0.7 kg. Let's assume the chicken is in an insulated environment and is held in place, so the chicken cannot transfer heat to its surroundings and cannot move. The chicken starts out at room temperature, $T_\circ = 298.15$ K.

(a) Determine how much energy must be added to the chicken to fully cook it, i.e. bring it to $T_f = 350$ K.

(b) If you wanted to cook the chicken in one slap, determine how fast your hand must be moving during the slap.

(c) If you wanted to cook the chicken with multiple normal slaps (≈ 7 m/s), determine how many slaps you would need.

3. Two blocks, A and B, come in to contact. Block A starts out at $T_A = 150$ K, while block B starts at $T_B = 400$ K. The heat capacity of block A is 15 J/K, and that of block B is 5 J/K.

(a) Suppose Block A has a mass of 5 kg, for Block B, 1 kg. What would be the *specific* heat capacity for each?

(b) Determine the equilibrium temperature, T_f.

(c) Determine the net change in entropy. Which block lowered in entropy, and which block rose in entropy? Hint:

$$\frac{1}{T} = \frac{\partial S}{\partial U}, \quad C = \frac{\partial U}{\partial T}$$

4. Timmy buys an ice cream cone on a hot summer day, but he gets distracted and leaves it on a park bench. The specific latent heat of fusion of ice cream is 2.34×10^5 J/kg, and his scoop has a mass of 75 g.

(a) If the sun is adding energy to his ice cream at a rate of 5 W, estimate how long it will take for his ice cream to completely melt, assuming it's already at its melting point.

(b) Now, let's say the specific heat capacity of melted ice cream is 2400 J/kg K. Assuming that all of the ice cream has to melt before the liquid ice cream starts to increase in temperature, and that the melting point of ice cream is about 273.15 K (which is also its initial temperature), determine the total time for the ice cream to go from solid to a room temperature liquid (room temp. = 298.15 K).

5. Determine the specific heat capacity of solid aluminum via equipartition. Use the value of molar mass in the equation sheet.

6. We have helium gas at temperature of 400 K. The molar mass of helium is 4.003 g/mol.

(a) Determine the RMS velocity of the helium particles.

(b) How does the RMS velocity of these particles compare to the RMS velocity of neon gas at the same temperature? The molar mass of neon is 20.180 g/mol.

7. What is the relationship between volume and pressure during isothermal and adiabatic processes for an ideal gas, respectively?

8. The following two questions refer to the setup described below.

A piston of volume 0.05 m^3 contains 5 moles of a monatomic ideal gas at 300 K. If it undergoes an isothermal process and expands until the internal pressure matches the external pressure, $P_E = 1$ atm.

(i) How much work is done by the gas on the environment?

a) 7.42×10^3
b) 1.12×10^4
c) -1.12×10^4
d) 1.83×10^4
e) -1.83×10^4

(ii) Suppose that the piston undergoes an adiabatic expansion instead, what is the final volume of the piston, V_f? (Values have units of cubic meters)

a) 0.086
b) 0.095
c) 0.123

9. When a system is colder than the temperature of the environment (i.e. $T_{sys} < T_{eniv}$) its free energy is:

a) Smaller than its value when $T_{sys} = T_{eniv}$
b) Larger than its value when $T_{sys} = T_{eniv}$
c) The same as its value when $T_{sys} = T_{eniv}$

10. Using the second law of thermodynamics, show that it is impossible for a heat engine to operate at $\epsilon = 1$.