ILLINOIS

Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

MATH 257 – Linear Algebra with Computational Applications

Final Exam Extras Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Dec 10, 4:00 - 5:20 PM Rohan, Aman, Maheen

Session 2: Dec 12, 5:00 - 6:20 PM Rohan, Ryan, Nehan

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/955
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. For any $\boldsymbol{A_{nxn}}$ matrix, there exists an invertible matrix P and diagonal matrix D, such that $A = PDP^{-1}$ for all square matrices.

   $\boxed{False}$, A must have a valid eigenbasis and be square. A counterexample would be

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

2. True/False : Another way of representing matrix diagonalization is $L_{BB} = I_{BE}L_{EE}I_{EB}$, where the basis B represents the eigenbasis of a matrix A in the standard basis

   Let A be a collection of vectors $[\boldsymbol{a_1}, \boldsymbol{a_2}, .., \boldsymbol{a_k}] \in \mathbb{R}^n$. A is linearly independent iff Nul(A) $\neq 0$.

   $\boxed{False}$, the representation is actually $L_{EE} = I_{EB}L_{BB}I_{BE}$, because L_{EE} represents the transformation in the standard basis, which is given by A. L_{BB} is the transformation in the eigenbasis.

   True/False : Matrix A that has $\lambda_1 = \lambda_2 = 2$ and linearly independent eigenvectors V_1 and V_2 is diagonalizable.

   $\boxed{True}$, a matrix with an eigen value of mtiplicity ¿ 1 is still diagonalizable if the eigenvectors are independent

3. If A is a 3×3 matrix such that: $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\3\\0\end{bmatrix}$, $A\begin{bmatrix}-\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}-\frac{2}{\sqrt{2}}\\0\\\frac{2}{\sqrt{2}}\end{bmatrix}$, $A\begin{bmatrix}-\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}-\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix}$ What is A^2?

Observe that these are eigenvector equations, and the eigenvectors form an eigenbasis:

$$P = \begin{bmatrix}0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\1 & 0 & 0\\0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \; A = PDP^{-1} \text{ so, } A^2 = PD^2P^{-1} \; D = \begin{bmatrix}3 & 0 & 0\\0 & 2 & 0\\0 & 0 & -1\end{bmatrix}$$

$$A^2 = \begin{bmatrix}0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\1 & 0 & 0\\0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}9 & 0 & 0\\0 & 4 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\1 & 0 & 0\\0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^{-1}$$

4. What is the limit of $A^k v$ for all $v \in \mathbb{R}^n$ as $k \to \infty$ if $|\lambda_i| < 1$, where λ_i is an eigenvalue of matrix A?

$\lim_{k\to\infty} A^k v = 0$ because $A^k = PD^kP^{-1}$, and raising $|\lambda| < 1$ to infinitely large power results in a 0 matrix