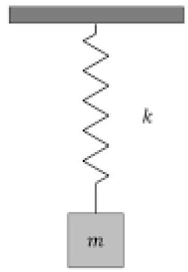
PHYS 211 PLT
Simple Harmonic Motion

1) A mass of M = 3 Kg is oscillating on a spring of stiffness K = 100 N/m.



a) Find the stretched position of the mass in equilibrium.

$$F = ma = kx$$

$$x = \frac{Mg}{k} = \frac{(3)(9.81)}{100} = .2943\,m$$

b) Now the mass is pushed downwards with a velocity of 5 m/s to begin oscillation. Write down the differential equation of the system, the initial conditions needed for solving, and the form of the solution for the solution.

$$m\frac{d^2x(t)}{dt^2} = -kx(t)$$

Also Written as:

$$\frac{d^2x(t)}{dt^2} = -\omega^2 x(t) \text{ since } \omega^2 = \frac{k}{m}$$

Initial Conditions:

$x(0) = 0$ since the mass starts at equilibrium

$v(0) = -5\,m/s$ since there is an initial downward velocity

Solution Form:

$$x(t) = A\cos(\omega t + \phi)$$

c) What is the maximum amplitude of this oscillation, A.

We know that $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{3}} = 5.77\, rad/s$

$$x(t) = Acos(\omega t + \phi)$$

$$v(t) = - A\omega sin(\omega t + \phi)$$

At x(0):

$$0 = Acos(\phi)$$

Therefore:

$$\phi = \frac{\pi}{2}$$

From Velocity:

$$- 5 = - 5.77Asin(5.77t + \frac{\pi}{2})$$

At t = 0

$$- 5 = - 5.77A$$

Therefore

$$A = 0.866\, m$$

d) What is the maximum acceleration of the block during its oscillation?

At Max acceleration, the cosine term can be neglected since it needs to be 1

Therefore

$$a(t) = A\omega^2$$

$$a(t) = 0.866 * 5.77^2 = 28.83\, m/s$$

e) At what time is the acceleration of the block zero?

$$a(t) = -A\omega^2 cos(\omega t + \phi)$$

For acceleration to be zero to cosine term must become zero:

$$0 = cos(\omega t + \phi)$$

Therefore

$$5.77t + \frac{\pi}{2} = \frac{3\pi}{2}$$

Therefore

$$t = 0.544\, s$$

f) What big assumptions was made regarding the spring for this problem

No Friction with the spring so no energy loss.

2) Write the resonance frequency ω for the following system. Explain how each is related.

a) Simple Pendulum (Mass connected to a massless string)

$$\omega = \sqrt{\frac{g}{L}}$$

b) Physical Pendulum (Rod with mass oscillating)

$$\omega = \sqrt{\frac{mgR}{I}}$$

c) Mass Connected to a Spring

$$\omega = \sqrt{\frac{k}{m}}$$

The similarities between these equations is that both relate some sort of component of stiffness to resistance.

The numerator of the frequency represents the ‘stiffness’ of the system. The denominator represents the ‘resistance’ to the system