

Math 231 Exam 3 Review Session

MATH 231

Power Series

- ▶ Can be defined by the form:

$$\sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots$$

- ▶ C_n are the coefficients
- ▶ Series is a function of x

- ▶ Can be centered at any number:

$$\sum_{n=0}^{\infty} C_n (x - a)^n = C_0 + C_1 (x - a) + C_2 (x - a)^2 + C_3 (x - a)^3 + \dots$$

Power Series

- ▶ Domain of Convergence: For what values will the series converge?
 - ▶ Use tests to find out what values of x satisfies convergence criteria.

Theorem 3.1. For any power series

$$\sum_{n=0}^{\infty} c_n(x-a)^n,$$

there are exactly three possibilities for the domain of convergence (DOC) and radius of convergence (ROC).

1. Converges only at $x = a$, or
DOC = $\{a\}$, ROC = 0;
2. Converges for all x , or
DOC = $(-\infty, \infty)$, ROC = ∞ ;
3. There is an R such that the power series converges for $|x - a| < R$ and diverges for $|x - a| > R$,
ROC = R .

Remark 3.2. In the case with a radius of convergence R with $0 < R < \infty$, we have to check the endpoints “by hand”.

Taylor Series

- ▶ Essentially a way to estimate a function about a point!

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \dots + \frac{f^n(a)}{n!}(x - a)^n$$

- ▶ Big O Notation

- ▶ “Everything else”

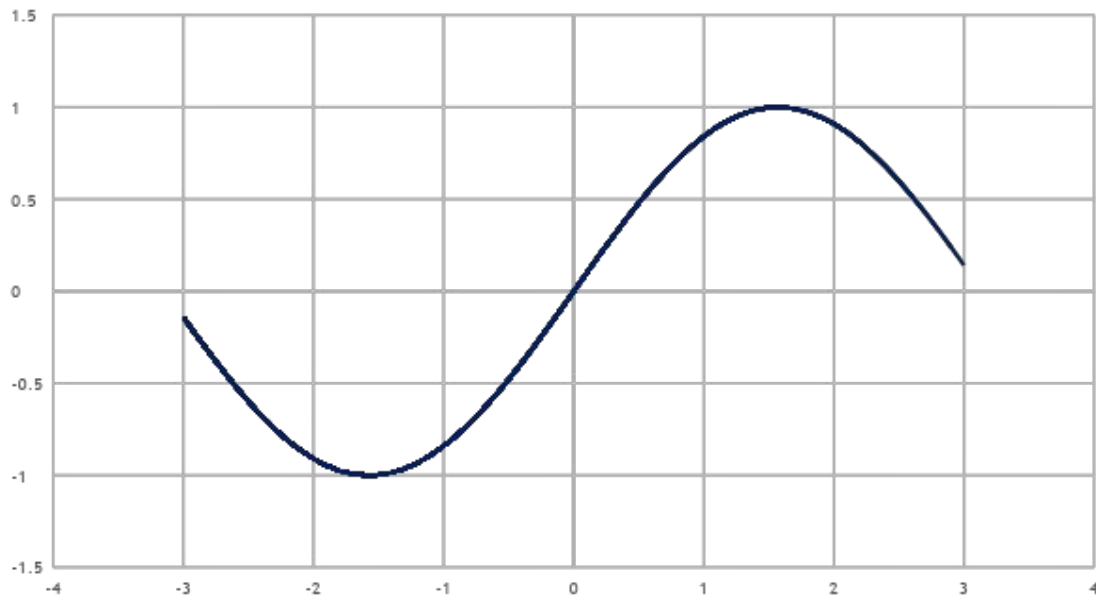
- ▶ Anything past a point **will not affect** the function much.

- ▶ $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + O(x^4)$

- ▶ If $a = 0$, it's called a Maclaurin Series for $f(x)$

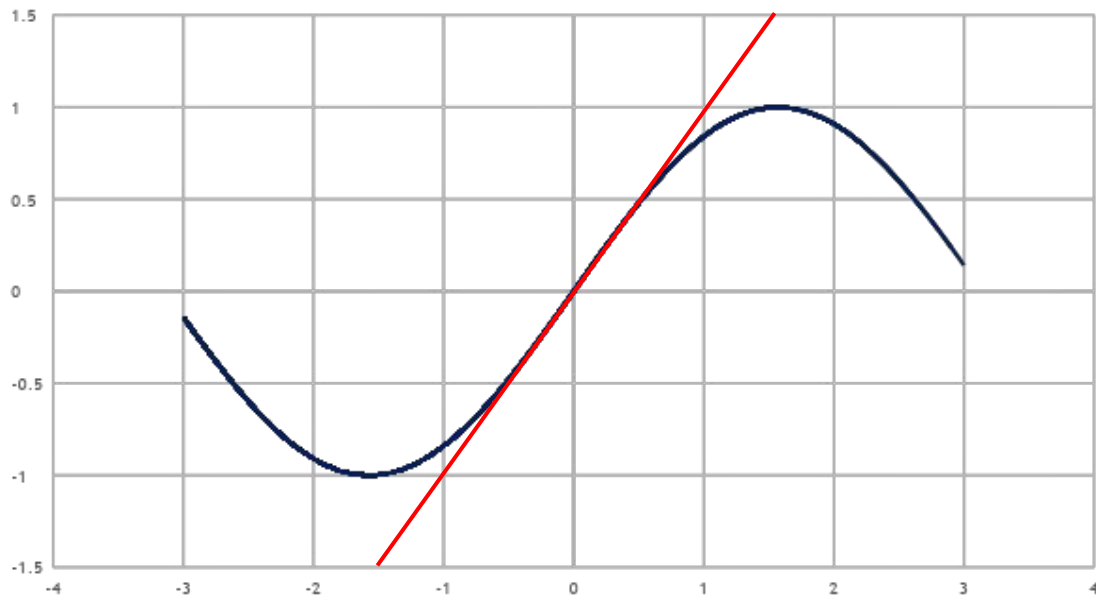
$$f(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \dots + \frac{f^n(0)}{n!}(x)^n$$

Taylor Series



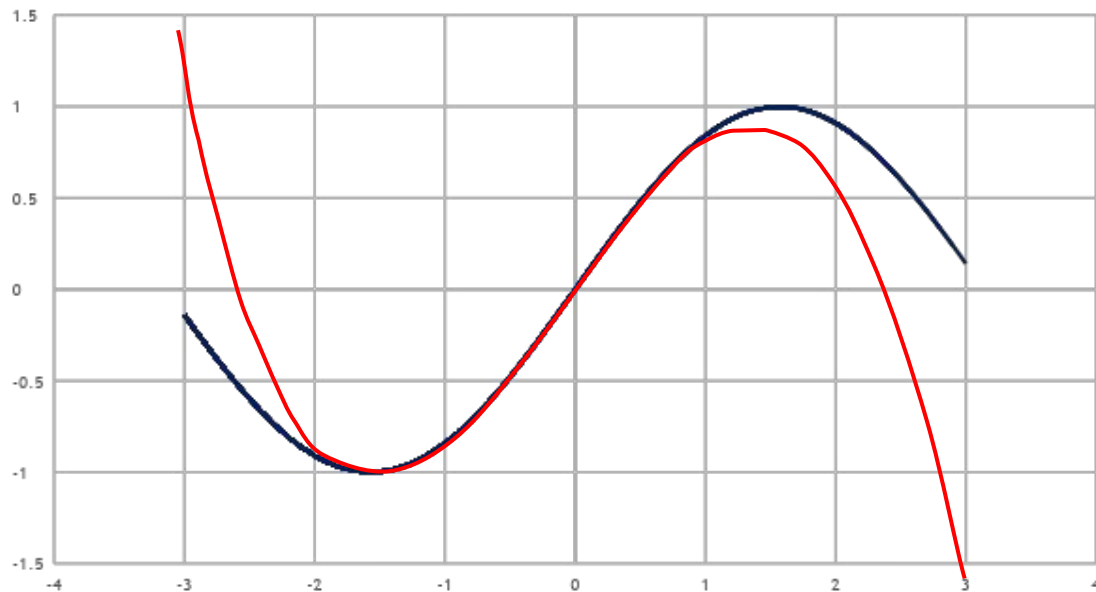
$\sin(x)$

Taylor Series



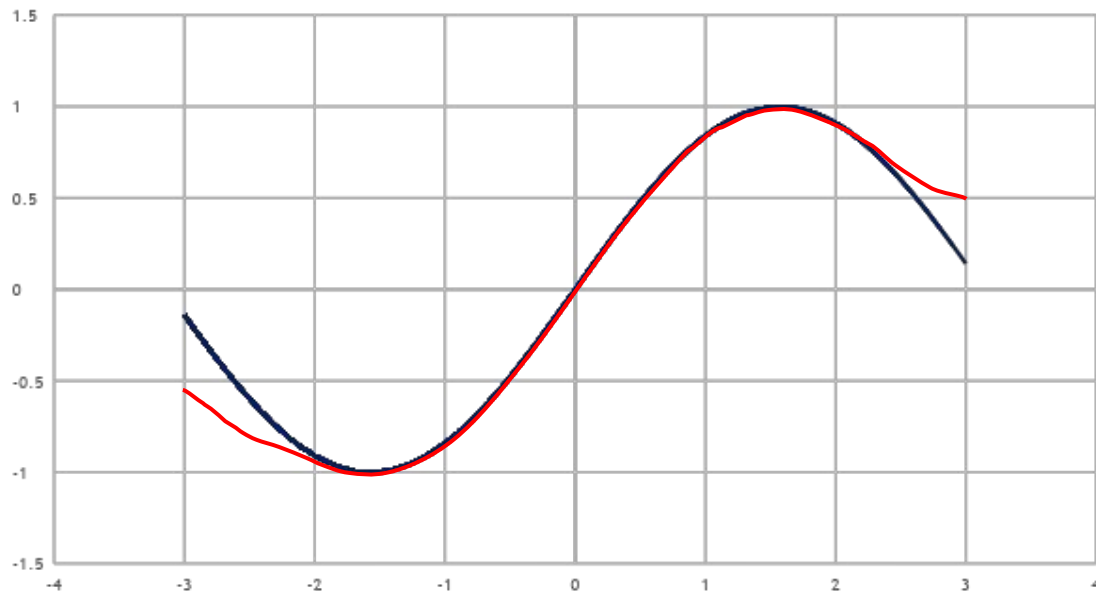
$$\sin(x) = x$$

Taylor Series



$$\sin(x) = x - \frac{x^3}{3!}$$

Taylor Series



$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Taylor Series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{1}{x} = \frac{1}{x-1+1} = \frac{1}{x-1} \cdot \frac{1}{1-\frac{1}{x-1}} = \frac{1}{x-1} \left(1 + \frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{(x-1)^3} + \frac{1}{(x-1)^4} + \dots \right)$$

Parametric Curves

- ▶ In most of your math classes, you have only dealt with curves that have functions in terms of one other variable such as $f(x) = y$.
- ▶ What if a graph doesn't pass the vertical line test?
 - ▶ Can we still graph it?
- ▶ Yes!
- ▶ Let's introduce a new variable: t
- ▶ The variable x and y can now be put into terms of t :
 - ▶ $x = g(t)$
 - ▶ $y = h(t)$

Parametric Curves, but with Calculus

- ▶ Derivatives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d(dy/dx)}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{dy/dt}{dx/dt} \right)}{\frac{dx}{dt}}$$

- ▶ Area:

$$\int_a^b y \, dx = \int_c^d g(t) f'(t) \, dt. \quad x = f(t), \quad y = g(t), \quad t \in [c, d],$$

- ▶ Arc Length:

$$\int_c^d \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt$$

Polar Coordinates

- ▶ Define coordinates relative to the origin:
 - ▶ “ r ” - the distance from the origin
 - ▶ “ θ ” - the angle in between the line and the x axis
- ▶ Transformation from regular Cartesian coordinates (x & y).
 - ▶ $x = r \cos \theta$
 - ▶ $y = r \sin \theta$
 - ▶ $r^2 = x^2 + y^2$
- ▶ Not all polar coordinates are unique:
 - ▶ Ex. Repeating after 2π

Polar Coordinates, but with Calculus

► Area

$$A = \frac{1}{2} \int f(\theta)^2 d\theta = \frac{1}{2} \int r^2 d\theta$$

► Arc Length

$$AL = \int \sqrt{(f(\theta))^2 + (f'(\theta))^2} d\theta = \frac{1}{2} \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$