CARE MATH 221
Exam 4 Review

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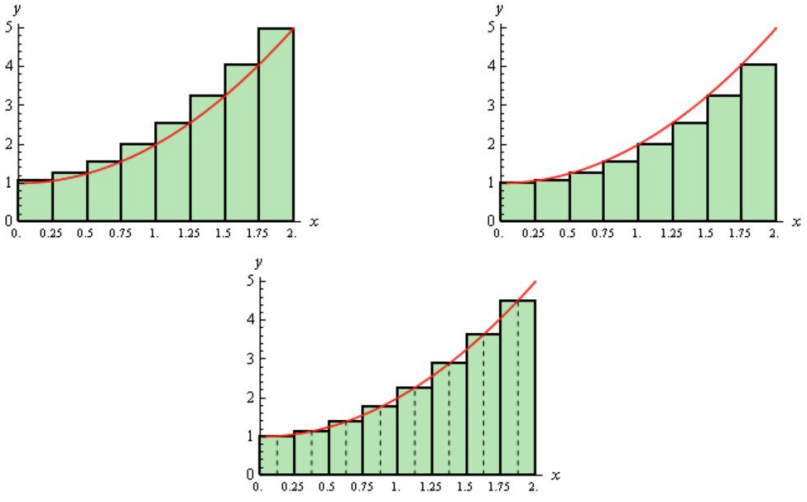
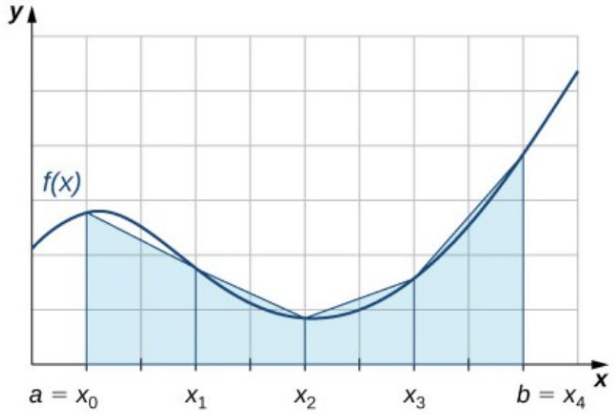
Riemann Sums

Base Rules for estimating area under a curve:

$$\int_a^b f(x)\,dx \qquad \Delta x = \frac{b-a}{n}$$





$$T_n = \tfrac{1}{2}\Delta x\,(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)).$$

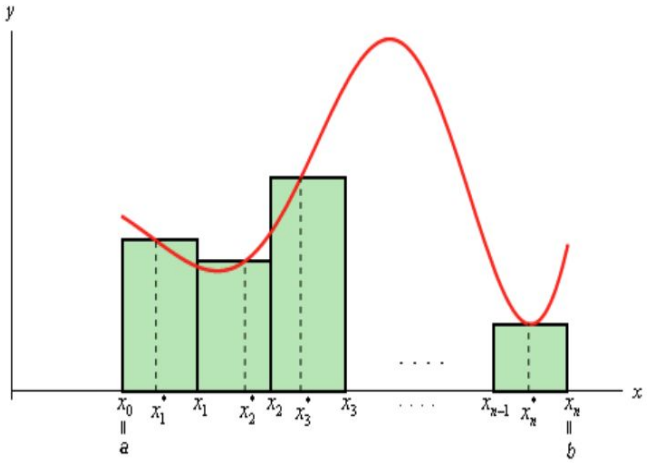
Four main types of Riemann Sums: Left endpoint, right endpoint, midpoint, and trapezoidal sums

Left and right endpoints can be over or underestimates depending on if the function is increasing or decreasing

Which sum is the most accurate?

Sigma Notation

Use summation notation to write an expression to estimate the area under the graph of $f(x) = \sin(x)$ with 4 rectangles of equal width and right endpoints on the interval $(-2, 6)$.



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$$

Δx = (b-a)/n

(x_i^*) = a +i Δx

$$R_n = \sum_{i=1}^{4} 2\sin\left(2i - 2\right)$$

Integral Properties

Properties of the integral

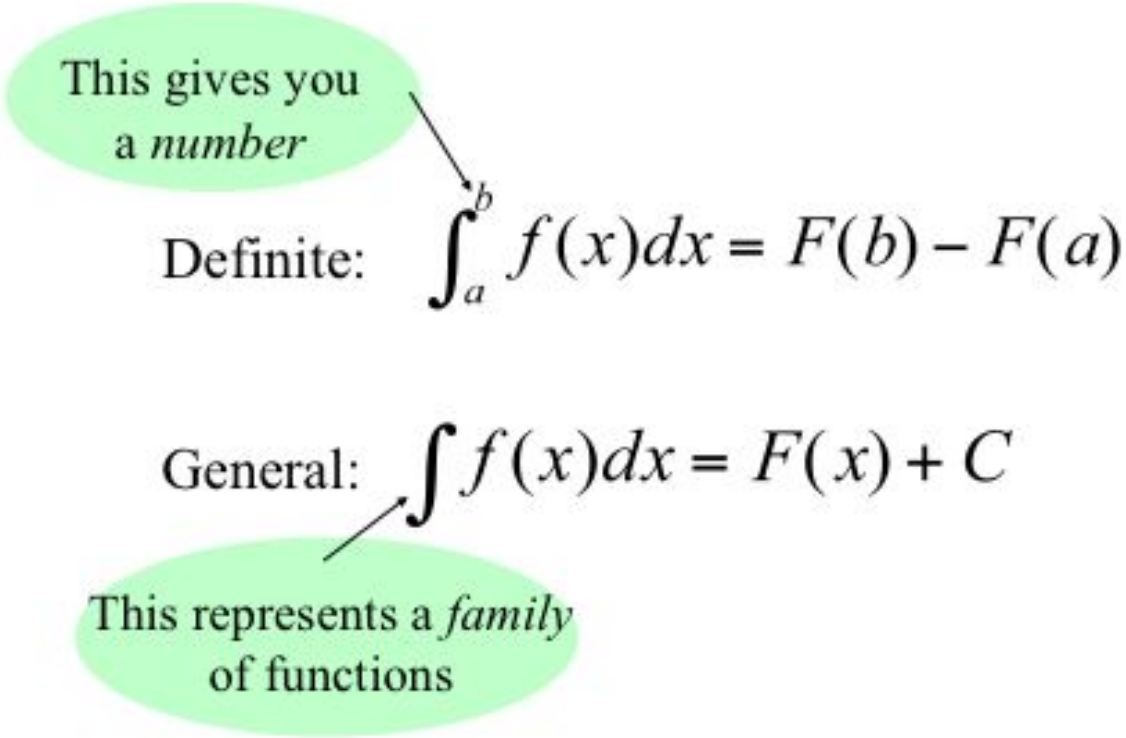
Theorem (Additive Properties of the Integral)

Let f and g be integrable functions on $[a, b]$ and c a constant. Then

1. $\int_a^b c\,dx = c(b-a)$
2. $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$
3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx.$
4. $\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx.$

Don’t forget to factor the constant back into your final answer!

Integrals: Definite vs. "General"



Note: Definite integrals require bounds!

FTC, Average Value of a Function

*Consider using the chain rule

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{1}{b-a}\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = F(b)-F(a)$$

$$\int_a^b f(x)dx = f(c)(b-a)$$

Common Antiderivatives

$$\int 0\,dx = C$$

$$\int 1\,dx = x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln(x) + C$$

$$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int -\csc(x)\cot(x)\,dx = \csc(x) + C$$

$$\int -\csc^2(x)\,dx = \cot(x) + C$$

Net Change Theorem

The definite integral of the rate of change of a quantity F'(x) gives the total, or net change, in that quantity on the interval [a, b].

$$\int_a^b F'(x)dx = F(b) - F(a)$$

- **Keywords to look for:**
  - "Net displacement"
  - "Total distance traveled"

Substitution Rule (U-sub)

$$\int f(g(x))g'(x)dx = \int f(u)\,du = F(u) + C$$

where $u = g(x)$ $du = g'(x)$

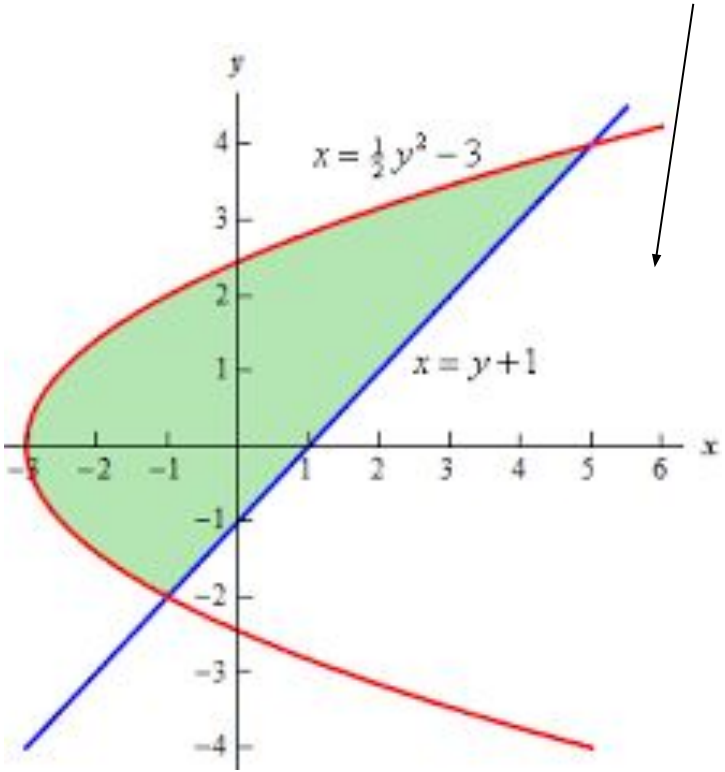
Areas Between Curves

$$A = \int_a^b (f(x) - g(x))dx$$

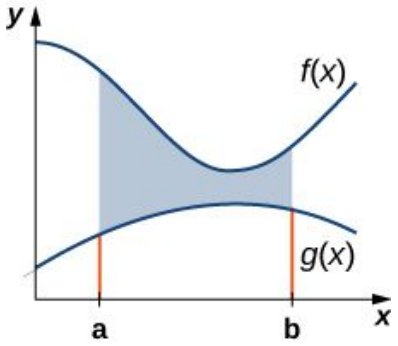
$$A = \int_a^b (top - bottom)dx$$

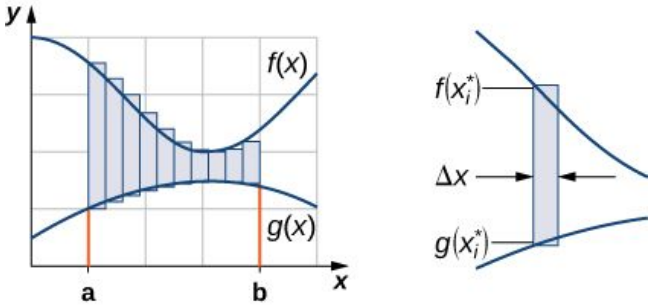
Take the integral of the least complex area

For dy integral, take the integral of right - left

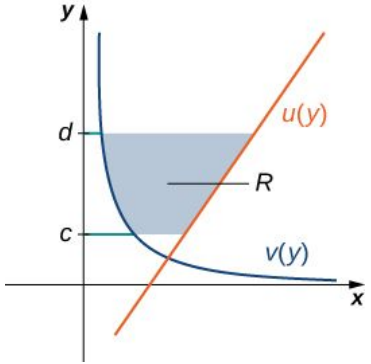


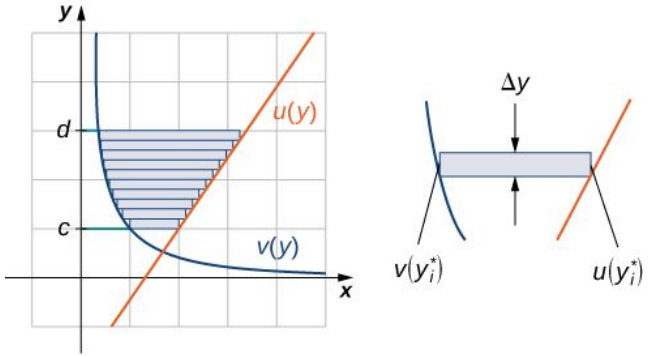
Area Between Curves: Vertical and Horizontal Rectangles





(a) (b)





(a) (b)

Work

$$W = \int_a^b F(x)\,dx$$

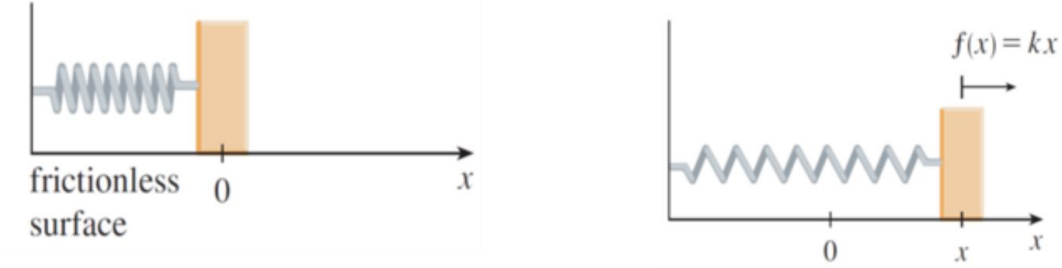
Remember: Work is only done if the object moves.

Hydrostatic Force:

$$F = mg = \rho V g$$

Hooke's Law:

$$F(x) = k\,x$$



(a) Natural position of spring

(b) Stretched position of spring

Hooke's Law
Figure 1

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