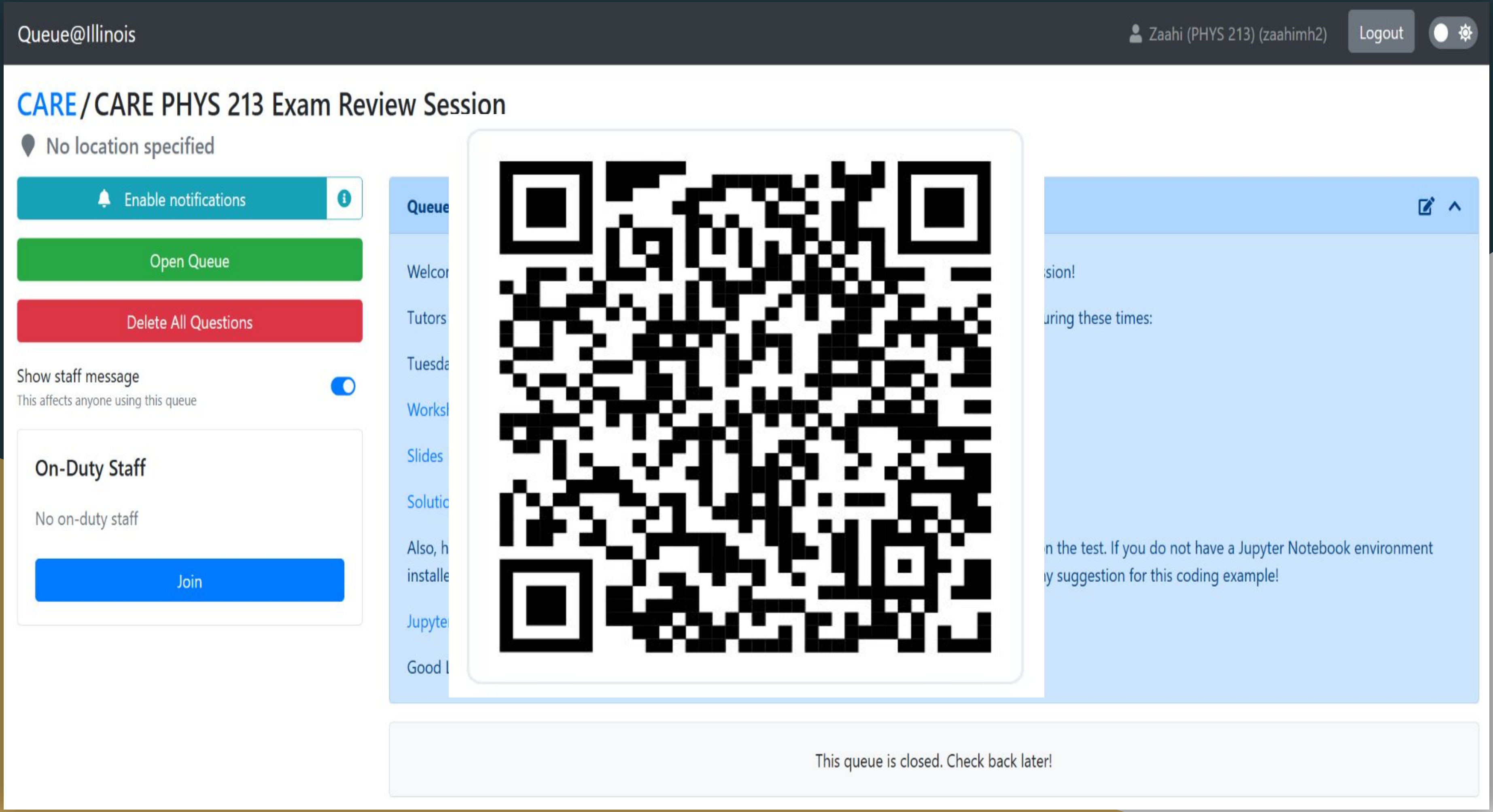
# C.A.R.E. PHYS 213 Quiz 1 Review Session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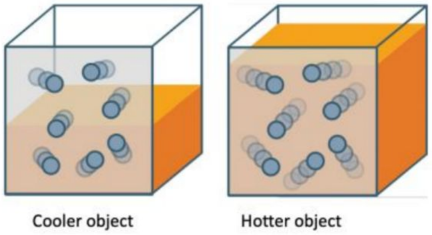
# Units for the Exam

- Internal Energy
- Temperature
- Heat Capacity
- Entropy

# Internal Energy




- Total energy is <u>**ALWAYS**</u> conserved

- **Positive work on a system increases the system's internal energy**

- **Higher temperature → More Internal Energy**

- First law of thermodynamics:

$$\Delta U = W_{on} + Q$$

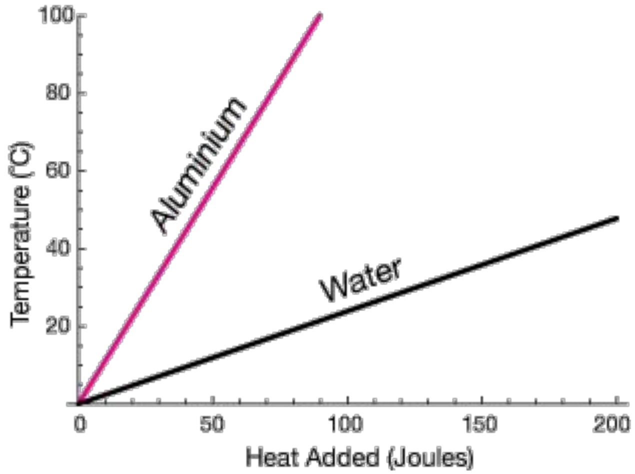
Change in internal energy | Work done on the system | Heat added to the system

# Temperature & Heat Capacity

- **Heat Capacity** (*C*) - how much energy it takes to **increase** the temperature of a substance by 1 K/°C
  - Units of J/K

- **Larger *C*** $\rightarrow$ **More energy** is required to **increase the temperature** of the object

$$C = dQ/dT$$



# Temperature & Heat Capacity

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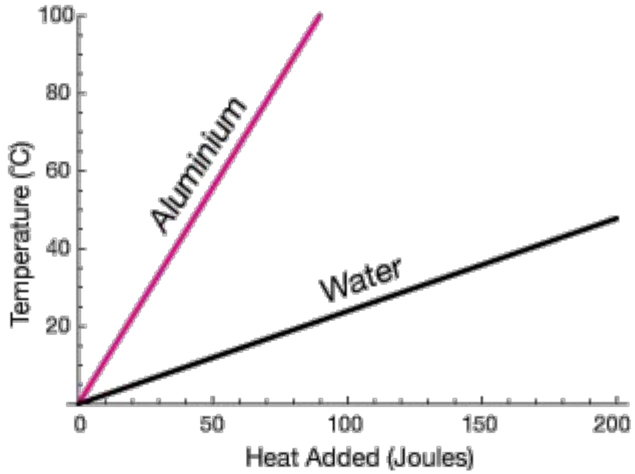
- Water has a larger heat capacity than Aluminum



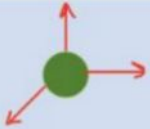
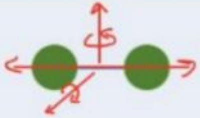
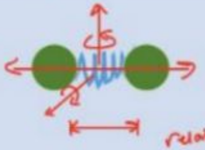
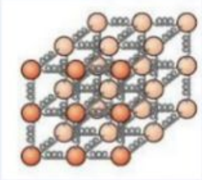
# Types of Heat Capacity

- **Molar Heat Capacity [J/mol K]:** The amount of heat required to raise the temperature of **1 mole** of a substance by 1 K/°C
  - $c_M = C/n$, where n is the number of moles

- **Specific Heat Capacity [J/kg K]:** The amount of heat required to raise the temperature of **1 kg** of a substance by 1 K/°C
  - $c = C/m$, where m is the mass [kg]

- **Heat Capacity at a Constant Volume and Constant Pressure:**
  - $C_v = dU/dT$
  - $C_p = dU/dT + p\, dV/dT$

# Equipartition

| | Molecule | Degrees of freedom | U | $C_V$ |
|---|---|---|---|---|
| Monatomic: DOF = 3 |  | $N_{DOF} = 3$<br>x, y, z momentum | $U = \frac{3}{2} NkT$ | $C_V = \frac{3}{2} Nk$ |
| Diatomic: DOF = 5 |  | $N_{DOF} = 5$<br>x, y, z momentum<br>2 rotation axes | $U = \frac{(3+2)}{2} NkT$ | $C_V = \frac{5}{2} Nk$ |
| Vibrational: DOF = 7 |  relative | x, y, z momentum<br>2 rotation axes<br>vibration mode<br>(momentum+potential)<br>$N_{DOF} = 7$ | $U = \frac{(3+2+2)}{2} NkT$ | $C_V = \frac{7}{2} Nk$ |
| Solid: DOF = 6 |  | x, y, z momentum<br>x, y, z spring modes | $U = \frac{(3+3)}{2} NkT$ | $C_V = 3Nk$ |

# Equipartition & Heat Capacity

- Only need to memorize DOFs for monatomic gas, diatomic gas, and solids (3, 5, and 6, respectively)

- For substances under the **equipartition assumption:**

$$U = \frac{N_{\mathrm{DOF}}}{2} NkT \implies C = \frac{\mathrm{d}U}{\mathrm{d}T} = \frac{N_{\mathrm{DOF}}}{2} Nk$$

****Nk = nR***

$R$ = 8.314 J/(mol K)

$R$ = 0.08206 L atm/(mol K)

# Entropy

- Microstate vs Macrostate:
  - Microstate: individual, **specific** arrangement
  - Macrostate: property that arises from the microstates
  - **Many microstates can lead to the same macrostate**
    - Two people have the same weight (macroststate), but the distribution of the weight can be different (microstate)

# Entropy (Cont.)

- Entropy (*S*) is a measure of the degree of 'diversity' associated with a macrostate
    - $S = k\ ln(\Omega)$, where $\Omega$ is the number of microstates
    - **Second Law of Thermodynamics:** $\Delta S \geq 0$

- Equilibrium
    - Occurs when the macrostate of the system ceases to change
    - The **most probable macrostate** is the one with the **highest entropy (most microstates)**
    - **Equilibrium** is achieved when **S, entropy, is maximized**

# Binomial Coefficient

If I have ***N* coins** and I am looking for the macrostate with ***q* heads**, the number of microstates:

$$\binom{N}{q} = \frac{N!}{q!(N-q)!}$$

Example: If I have **20 coins**, how many microstates are associated with the macrostate of getting **7 heads**?

**Answer:**

# Binomial Coefficient

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Example: If I have **20 coins**, how many microstates are associated with the macrostate of getting **7 heads**?

**Answer:**

$$\binom{20}{7} = \frac{20!}{7!\,(20-7)!} = 77520$$

# Entropy (Cont.) - Dice Example

- **Microstate**:
- **Macrostate**:
- What is the most likely macrostate?
    -
    -
- What is the macrostate with the highest entropy?
    -

| | ⚀ | ⚁ | ⚂ | ⚃ | ⚄ | ⚅ |
|---|---|---|---|---|---|---|
| ⚀ | 2 | 3 | 4 | 5 | 6 | 7 |
| ⚁ | 3 | 4 | 5 | 6 | 7 | 8 |
| ⚂ | 4 | 5 | 6 | 7 | 8 | 9 |
| ⚃ | 5 | 6 | 7 | 8 | 9 | 10 |
| ⚄ | 6 | 7 | 8 | 9 | 10 | 11 |
| ⚅ | 7 | 8 | 9 | 10 | 11 | 12 |

- **Entropy of a macrostate is simply a measure of the number of microstates associated with it**
- **More microstates → Higher Entropy, Higher Probability**

# Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**:
- What is the most likely macrostate?
    - 
    - 
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| | | | | | | |
|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 |
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# Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
    - 
    - 
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# Entropy (Cont.) - Dice Example

- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
  - 7: has the largest number of microstates associated with it
  - Probability = 6/36 = 0.167
- What is the macrostate with the highest entropy?
  - 

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- **Microstate**: set of individual die values
- **Macrostate**: sum of die values
- What is the most likely macrostate?
  - 7: has the largest number of microstates associated with it
  - Probability = 6/36 = 0.167

- What is the macrostate with the highest entropy?
  - Again, 7: has the largest number of microstates associated with it

- **Entropy of a macrostate is simply a measure of the number of microstates associated with it**
- **More microstates ⟶ Higher Entropy, Higher Probability**

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# Important Equations

Start with the definition of temperature

$$\boxed{\frac{1}{T} \equiv \left(\frac{\partial S}{\partial U}\right)_{V,N}}$$

multiply by dU

$$\frac{1}{T} dU = dS$$

integrate

$$\boxed{\Delta S = \int_{U_i}^{U_f} \frac{1}{T} dU}$$

at constant volume
$dQ = dU$

$$C_V = \left(\frac{dU}{dT}\right)_{V,N}$$

multiply by dT

$$C_V dT = dU$$

plug dU into $\Delta S$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C_V(T)}{T} dT}$$

Or the definition of heat capacity

$$\boxed{C_V = \left(\frac{dQ}{dT}\right)_{V,N}}$$

multiply by dT

$$C_V dT = dQ$$

integrate

$$\boxed{\Delta U = \int_{T_i}^{T_f} C_V(T) dT}$$

# Differential Manipulation

- Know these tricks!

- Heat Capacity is the link between *dU* and *dT*
  - **If you know the heat capacity and the temperature change, you can find the change in internal energy and the change in entropy**

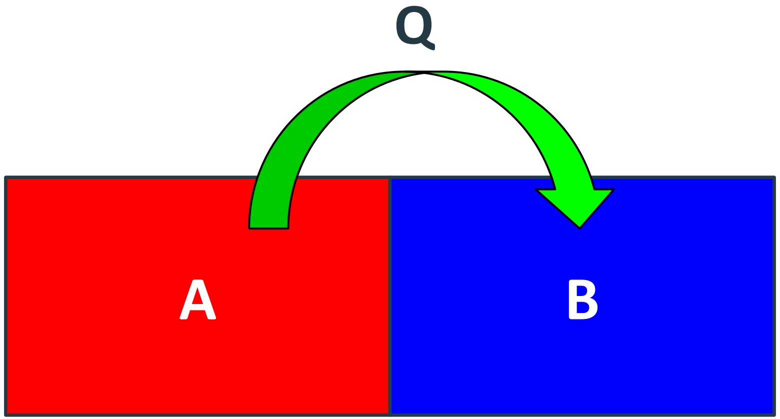
$$C = \frac{\partial U}{\partial T} \implies \Delta U = \int_{T_i}^{T_f} C \mathrm{d}T$$

$$\frac{\partial S}{\partial U} = \frac{1}{T} \implies \Delta S = \int \frac{\mathrm{d}U}{T} = \int_{T_i}^{T_f} \frac{C \mathrm{d}T}{T}$$

# Entropy of Two Blocks in Thermal Contact - Example

Two blocks A and B, are in thermal contact and insulated from surroundings. Initially, block A is at a higher temperature than block B. **Each block has a temperature-dependent heat capacity given by $C = pT^2$.**

**Determine the entropy change for each block**



# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

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$$= \int_{T_i}^{T_f} pT\,dT$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

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$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

$$= \int_{T_i}^{T_f} pT\,dT$$

$$\boxed{= p\frac{T_f{}^2}{2} - p\frac{T_i{}^2}{2} = \frac{p}{2}\left[T_f{}^2 - T_i{}^2\right]}$$

# Entropy of Two Blocks in Thermal Contact - Example (Cont.)

$$dS = \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$C = \frac{dQ}{dT}$$

$$\boxed{\Delta S = \int_{T_i}^{T_f} \frac{C\,dT}{T}}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{pT^2}{T}\,dT$$

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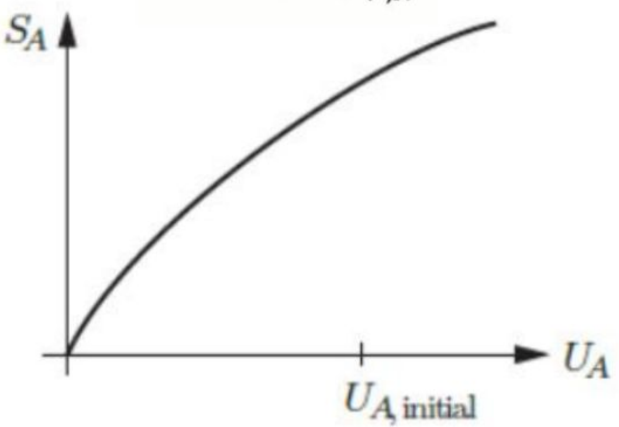
$$\boxed{= p\frac{T_f{}^2}{2} - p\frac{T_i{}^2}{2} = \frac{p}{2}\left[T_f{}^2 - T_i{}^2\right]}$$

$$\boxed{\begin{aligned} \Delta S_A &= \frac{p}{2}\left[T_{A,f}{}^2 - T_{A,i}{}^2\right] \\ \Delta S_B &= \frac{p}{2}\left[T_{B,f}{}^2 - T_{B,i}{}^2\right] \end{aligned}}$$

# Entropy (S) vs. Internal Energy (U)

- Since slope is always positive, temperature is always positive
- **More energy = greater entropy**
- **Diminishing returns:** it gets harder and harder to increase the entropy as internal energy increases
- **Decreasing slope = increasing temperature**
  - **More energy means greater temperature**

$$\frac{1}{T} \equiv \left(\frac{dS}{dU}\right)_{V,N}$$



# Good luck!

Feel free to ask any questions you may have.

**You got this!**