Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 213 – University Physics: Thermal Physics

Quiz 1 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Wednesday, November 5th, 6-7:50 pm Alex, Luke, and Sarah

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1 Here are some warm-ups to get you started!

1. Copper has a specific heat capacity of 376.812 J / kg K. Consider a copper ingot of mass 12 kg and at a temperature of 274.5 K. Given 3.67 kJ of energy delivered to the ingot:

(a) the heat capacity of the ingot

(b) the molar heat capacity of the ingot

(c) Calculate the ingot's final temperature

(a) Dimensional Analysis Again! Multiply the specific heat capacity by its mass to get the heat capacity of the copper ingot.
Answer: 4521.744 J/K

(b) To convert to molar heat capacity, we'll use dimensional analysis:

$$\frac{376.812 \text{ J}}{\text{kg K}} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{63.546 \text{ g}}{1 \text{ mol}} = \frac{23.94 \text{ J}}{\text{mol K}}$$

(c) Use $Q = mc\Delta T \implies T_f = \frac{Q}{mc} + T_i = 275.31\text{K}$

2. Below you are given an substance and its number of moles. Determine its heat capacity. Hint:

$$U = \frac{N_{\text{DOF}}}{2} NkT \implies C = \frac{N_{\text{DOF}}}{2} Nk$$

(a) N_2 gas, 4 moles

(b) solid aluminum, 6 moles

(c) argon gas, 8 moles

(a) 83.104 J/K

(b) 149.586 J/K

(c) 99.724 J/K

$N_{DOF} =$:

- N_2 Gas: 5(3 Translational,2 Rotational)
- Solid Aluminum: 6(3 Translational, 3 Elastic)
- Ar Gas: 3(3 Translational)

The number of atoms in a mole is Avogadro's Number$N_A = 6.022 \times 10^{23}$ Then the number of atoms: $N = nN_A$

3. Below you are given heat capacity functions as well as initial and final temperatures. Using the definition of temperature, determine the overall change in entropy. Hint:

$$\frac{1}{T} = \frac{\partial S}{\partial U} \implies \Delta S = \int \frac{\mathrm{d}U}{T} \implies \Delta S = \int_{T_i}^{T_f} \frac{C\mathrm{d}T}{T}$$

(a) $C(T) = 24$ J/K, $T_i = 300$ K, $T_f = 350$ K

(b) $C(T) = \alpha T^2$, $\alpha = 0.05$ J/K^2, $T_i = 300$ K, $T_f = 200$ K

(c) $C(T) = \beta T^4$ J/K, $\beta = 0.0001$ J/K^5, $T_i = 10$ K, $T_f = 130$ K

(a) $\Delta S = \int_{300}^{350} 24\frac{1}{T}dT = 24ln(\frac{350}{300}) = 3.7\frac{J}{K}$

(b) $\Delta S = \int_{300}^{200} \alpha\frac{T^2}{T}dT = \frac{\alpha}{2}(200^2 - 300^2) = -1250\frac{J}{K}$

(c) $\Delta S = \int_{10}^{130} \beta\frac{T^4}{T}dT = \frac{\beta}{3}(130^3 - 10^3) = 7140\frac{J}{K}$

4. Let's do some combinatorics! Find the number of possible combinations for N coin tosses resulting in t tails :

(a) N = 10, t = 10

(b) N = 20, t = 4

(c) N = 100, t = 74

(a) $\binom{10}{10} = \frac{N!}{t!(N-t)!} = 1$. This should hopefully make sense, there is only one way for 10 flips to give 10 tails!

(b) $\binom{20}{4} = 4845$

(c) $\binom{100}{74} \approx 7 \times 10^{23}$

2 Regular Problems

5. Label each statement as True or False.

(a) One mole of nitrogen gas has a different heat capacity than one mole of hydrogen gas.

(b) It is possible for the entropy of a single object to decrease.

(c) Equilibrium is defined as the point at which the entropies of each object in a system are all equal.

(d) As you keep adding internal energy to an object, it becomes harder and harder to increase its entropy.

(e) A copper block weighing 1 kg has a different heat capacity than a copper block weighing 2 kg.

(a) False. The two gases have the same number of degrees of freedom (5), so if they have the same number of moles, they have the same heat capacity.

(b) True. This is allowed if the *total* entropy of a system increases, such as in the heat transfer of two blocks at different temperatures.

(c) False. Equilibrium occurs when total entropy is *maximized*. Other quantities become equal at equilibrium, such as temperature.

(d) True. Look at the definition of temperature:

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

As you add energy, the temperature goes up, so $\partial S/\partial U$ decreases. This means that the rate at which the entropy increases as you add energy decreases.

(e) True. Specific heat capacity is unique to each material and has units J/ kg K. The two blocks would have different heat capacities because of their different masses, but their *specific* heat capacities would be the same since they're both copper.

6. Consider 5 coins, each initially starting on heads.

(a) What is the entropy, S, of this system in its current configuration?

(b) List all the macrostates available to this system.

(c) Identify the most probable macrostates. Hint: there are two.

(d) How many microstates would lead to the macrostates identified above?

(e) Calculate the change in entropy, ΔS, if the system changed to either of its most probable macrostates.

(a) The first thing to note here is that this system is in a 5H 0T *macrostate* (in this scenario, macrostates are the number of heads up (or tails) in a given configuration). There's only one way to attain 5 heads — all of them are heads! So there is one *microstate* corresponding to this macrostate. We can now use the statistical definition of entropy, $S = k_b \ln(\Omega)$, with the microstates in this configuration, Ω, equalling one. Thus,

$$\boxed{S = k_b \ln(1) = 0} \tag{1}$$

Which is a reasonable answer because this system is highly uniform.

(b) The macrostates of this system are all the combinations of heads (H) and tails (T) available to the 5 coins.

5H 0T
4H 1T
3H 2T
2H 3T
1H 4T
0H 5T

(c) The most probable macrostates have the highest number of associated microstates.

3H 2T
2H 3T

(d) To calculate the number of microstates in either of these macrostates, we must use the combination formula, ${}_nC_r$, where n is the total number of items in the set (5), and r is the number of items we're choosing (hence n *choose* r).

$$\boxed{{}_5C_3 ={}_5 C_2 = 10 \text{ microstates}}$$

(e) The change in entropy is found by taking the difference of the final state entropy, S_{final}, and initial state entropy, $S_{initial}$.

$$\Delta S = S_{final} - S_{initial} = k_b \ln(10) - k_b \ln(1) = \boxed{k_b \ln(10)}$$

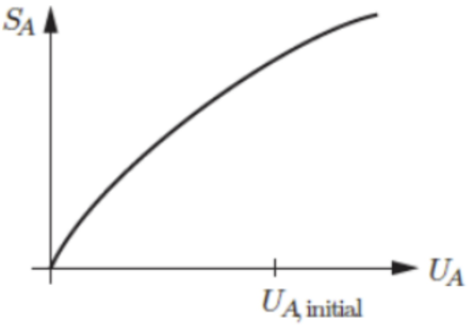
Note that the entropy increased, as we would expect by the second law of thermodynamics.

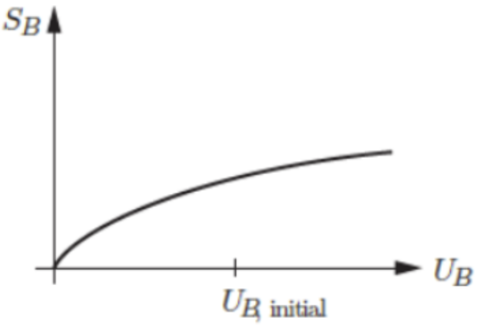
7. The heat capacity of a solid is linear with temperature. How does its entropy change with temperature?
The entropy will also increase linearly. To show this, let $C = \alpha T$

$$\Delta S = \int \frac{C}{T}\,\mathrm{d}T = \int \alpha\,\mathrm{d}T = \alpha \Delta T$$

Essentially, the T in the denominator reduces the heat capacity by a factor of T, but that factor is restored by the integral.

8. The figure below shows entropy S vs. internal energy U graphs for two objects, A and B. Both graphs are on the same scale. The internal energies of these two objects initially have the values indicated in the figure below.





(a) The objects are then brought into thermal contact with each other. Determine which of the following will happen.

i. Heat will transfer from object A to object B.
ii. Heat will not transfer between the objects.
iii. Heat will transfer from object B to object A.

(b) Which of the following is a correct statement?

i. $\Delta S_A = 0$, $\Delta S_B = 0$
ii. $\Delta S_A > 0$, $\Delta S_B > 0$
iii. $\Delta S_A < 0$, $\Delta S_B > 0$
iv. $\Delta S_A > 0$, $\Delta S_B < 0$

(a) Answer: iii. Heat will transfer from object B to object A. To answer this question we will need to use the following relationship:

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

The slope of graph A is larger than that of graph B. This means that object A is at a lower temperature than object B. This is because a larger slope means that the temperature has to be smaller (The slope of S vs U graphs is $1/T$, not T). Since object A is at a lower temperature than object B, heat must flow from object B to object A (heat flows from hot to cold).

(b) Answer: iv. We already established in part (a) that heat must flow from Object B to Object A (heat flows from hot to cold). If heat is flowing from object B to object A, then object B is losing energy while object A is gaining energy. This implies that $\Delta S_A > 0$ and $\Delta S_B < 0$. Another way of coming to this conclusion is to utilize the following two equations:

$$\frac{1}{T} = \frac{\partial S}{\partial T} \text{ and } C = \frac{\mathrm{d}U}{\mathrm{d}T}.$$

By performing some differential substitutions, we get

$$\Delta S = \int_{T_i}^{T_f} \frac{C}{T} \mathrm{d}T = C \ln(T_f/T_i).$$

Using the analysis above, we can see that ΔS will be negative if the equilibrium temperature is smaller than the initial temperature. And ΔS will be positive if the equilibrium temperature is greater than the initial temperature. Since heat is flowing from object B to object A, the equilibrium temperature will be smaller than the initial temperature of object B.

9. What is the difference between heat capacity, specific heat capacity, and molar heat capacity?

Physically, these three quantities are measuring the same thing: an object's tendency to increase in temperature as heat is added to it. However, the difference between these three is the units that they are expressed in.

Heat Capacity	Specific Heat	Molar Heat
$C \equiv \frac{\mathrm{d}Q}{\mathrm{d}T}$	$c_{sp} \equiv \frac{C}{m}$	$c_{mol} \equiv \frac{C}{n}$

Where m is the mass and n is the number of moles of the substance.

Therefore, the units on each of these quantities is

$$\text{Heat Capacity: } \frac{\text{J}}{\text{K}}$$
$$\text{Specific Heat: } \frac{\text{J}}{\text{kg} \cdot \text{K}}$$
$$\text{Molar Heat: } \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

10. Substance A has a heat capacity of 3 J/K while substance B has a heat capacity of 5 J/K. Starting from the same temperature, which one cools at a faster rate?

The units for heat capacity are J/K. Interpreting this, a heat capacity tells us the amount of energy required to raise or lower an object's temperature by 1 Kelvin. So you may be tricked into thinking at first that a *higher* heat capacity means a greater tendency to heat up, but that's not true. A *lower* heat capacity means less energy is required for temperature changes (heating and cooling), so objects with lower heat capacities will change temperature at faster rates.

$$\boxed{\text{Substance A}}$$

11. Explain why the heat capacity at constant volume of an ideal solid is twice that of the same amount of a monatomic ideal gas.
At constant volume, heat capacity is the derivative of internal energy

$$C_v = \frac{\mathrm{d}U}{\mathrm{d}T}$$

Where $U = \frac{D}{2}Nk_bT$ where D is the number of degrees of freedom.

For a ideal solid, there are six degrees of freedom; 3 translational and 3 vibrational ($D = 6$). But for a monatomic ideal gas, there are only 3 degrees of freedom; 3 translational ($D = 3$). When the derivative of internal energy is taken to obtain heat capacity, these constants remain.

12. A sealed container with a mass of 2.7 kg is filled with 4 moles of helium gas. Initially, the helium gas is at a temperature of 140°C and the container is at 38°C. The helium-container system is thermally isolated.

Note that the specific heat of the material making the container is 386 J/(kg·K) and the molar specific heat of helium is 12.5 J/(mol·K).

Find the equilibrium temperature of the system in Celsius and Kelvin.
The helium-container system is isolated from any heat conducting environment. So, by conservation of energy, we know that $Q_{He} = -Q_C$. One thing to see is that the heat capacities in the question have different units — one is specific heat, the other is molar heat. Using the fact that $Q = C\Delta T$ where C is regular heat capacity, we can do the following

$$Q_{He} = -Q_C$$
$$nc_{mol}(T_f - T_{He,i}) = -mc_{sp}(T_f - T_{C,i})$$

were n is the moles of Helium, m is the mass of the container, and T_f is the equilibrium temperature (which is the same for both the Helium and the container by the definition of thermal equilibrium). Solving for T_f

$$T_f = \frac{nc_{mol}T_{He,i} + mc_{sp}T_{C,i}}{nc_{mol} + mc_{sp}}$$

And when the values given above are put into this equation, we get

$$\boxed{T_f = 315.8 \text{ K} = 42.7°\text{C}}$$

Remember to convert the temperatures to Kelvin in the equations, then back to Celsius for the final answer.

13. (Note: this question uses fictional elements.) Consider a block of Wesleyium connected to a block of Vedhamite. A cylinder of Matthewide (thermal conductivity 10 $\frac{\text{W}}{\text{m K}}$ and length 2 m) connects the two. The Matthewide cylinder has a cross-sectional area of 1 m^3. The temperature difference between the two blocks is 20 degrees Celsius, and approximately 51 Joules of heat is transported between the two blocks through the cylinder. How long does this process take?

    We reference the equation:

    $$q'' = K\frac{T_2 - T_1}{L} = 10\frac{20}{2} = 100 \text{ W}/m^2 \tag{2}$$

    The heat flux q" can be written as $q'' = Q/(At)$ where Q is the total heat transferred (in joules) and t is the time taken to make the transfer. The area A is just 1 m^3, the cross-sectional area. Thus:

    $$t = Q/(q''A) = \frac{51}{100 * 1} \implies t = 0.51 \text{ seconds}$$

14. Let's say I have a gas of diatomic molecules such that at a high enough temperature, $T_{\text{crit}} =$ 500 K, the number of degrees of freedom for each molecules increases from 5 to 7. If I have 6 moles of this gas, determine the amount of energy I would need to add to it to go from $T_i = 200$ K to $T_f = 800$ K.

    $\Delta U = 89.752$ kJ. First, determine the amount of energy required to get to the transition temperature (500 K):

    $$\begin{aligned}\Delta U_1 &= \int_{T_i}^{T_{\text{crit}}} C \mathrm{d}T \\ &= C(T_{\text{crit}} - T_i) \\ &= \frac{5}{2}nR(T_{\text{crit}} - T_i) \\ &= 37.397 \text{ kJ}\end{aligned}$$

    Then, find the energy required to get to the final temperature with 7 DOFs:

    $$\begin{aligned}\Delta U_2 &= \int_{T_{\text{crit}}}^{T_f} C \mathrm{d}T \\ &= C(T_{\text{crit}} - T_i) \\ &= \frac{7}{2}nR(T_f - T_{\text{crit}}) \\ &= 52.355 \text{ kJ}\end{aligned}$$

    Adding these together, we get $\Delta U = 89.752$ kJ.

15. Suppose we have a box of two gases as the same temperature in an insulating chamber partitioned by an impermeable membrane. This membrane is free move back and forth, changing the left and right volumes, V_A and V_B. The entropy of gas A and B can be described with the following equations:

$$S_A = n_A R \ln(V_A) + f(U_A, N_A)$$
$$S_B = n_B R \ln(V_B - bn_B) + f(U_A, N_A)$$

Here, $n_A = 4$ mol $n_B = 3$ mol, and $b = 6 \times 10^{-5}$ m^3/mol. The total volume of the container is 6 m^3.

(a) Find the volumes V_A^f and V_B^f that result from the system achieving equilibrium.

(b) We are told the initial volumes were $V_A^i = 1$ m^3 and $V_B^i = 5$ m^3. Determine the change in total entropy ΔS, and confirm that this value is positive.

(c) Let's say that instead of a moving impermeable membrane, we had an *immovable permeable* membrane with the same initial volumes. How would the entropy maximization process change? What would stay constant, and what would change to maximize entropy? Do we have enough information to do this? (You don't have to do any math, just explain in words.)

(a) Remember that the system achieves equilibrium when entropy has been maximized. The only variable that can change in this system is the volume of each chamber, so at maximum entropy, the derivative with respect to volume should be zero. Let's differentiate with respect to V_A:

$$\frac{\partial S}{\partial V_A} = 0 = \frac{\partial S_A}{\partial V_A} + \frac{\partial S_B}{\partial V_A} = \frac{\partial S_A}{\partial V_A} - \frac{\partial S_B}{\partial V_B}$$

We know the total volume, which we can call V. Thus $V_B = V - V_A$:

$$\begin{aligned} 0 &= \frac{n_A R}{V_A} - \frac{n_B R}{V - V_A - bn_B} \\ \frac{n_A R}{V_A} &= \frac{n_B R}{V - V_A - bn_B} \\ V_A n_B + V_A n_A &= n_A(V - bn_B) \\ V_A &= \frac{n_A(V - bn_B)}{n_A + n_B} \\ &= 3.428 \text{ m}^3 \end{aligned}$$

Thus $V_A^f = 3.428$ m^3 and $V_B^f = 2.572$ m^3.

(b)

$$
\begin{aligned}
\Delta S_A &= n_A R \ln\left(V_A^f / V_A^i\right) \\
&= (4)(8.314)\ln(3.428/1) \\
&= 40.975 \text{ J/K} \\
\Delta S_B &= n_B R \ln\left(\frac{V_B^f - bn_B}{V_B^i - bn_B}\right) \\
&= -16.586 \text{ J/K} \\
\Delta S &= \Delta S_A + \Delta S_B \\
&= 24.389 \text{ J/K}
\end{aligned}
$$

As expected, this value is positive.

(c) Since the membrane is immovable, the volumes would stay the same. Instead, since the membrane is permeable, the gas particles of A and B would be able to traverse the membrane, thus N_A and N_B would change to maximize entropy. In order to do this, we would need to know the function f, as it depends on N_A and N_B, so we don't have enough given information.