1. For each of the functions below, determine the long-term behavior of the function.

   a. -(5 - x)(20x + 4)

f(x) = -(5 - x)(20x + 4) = $20x^2 - 96x - 20$. Leading term $20x^2$ -> $+\infty$ $as\ x \rightarrow \pm\infty$

   b. $8x^3 - 2x^2 + 3x + 15$

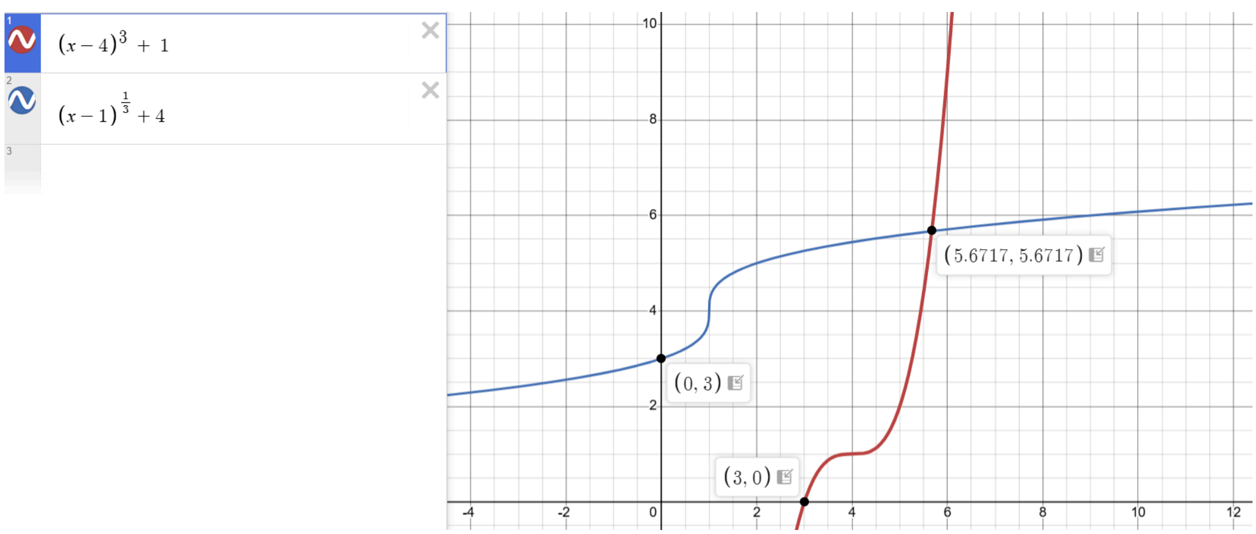
f(x) = $8x^3 - 2x^2 + 3x + 15$. Leading term $8x^3$ (odd,positive)
$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = -\infty$

   c. -$x^4 - 7x^3 - 7x^2 + 43x + 43$

Leading term: -x^4 (even, negative) $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

2. Use transformations to sketch the graph of g(x) = $(x-4)^3 + 1$. Then sketch the graph of its inverse.

Inverse: $g^{-1}(x) = \sqrt[3]{x-1} + 4$



3. Determine whether each statement below is true or false. If false, correct the statement.

   a. $log(100^x) = x$

False, $log_{10}(10^x) = x$

   b. $ln(e^x) = x$

True

4. Solve for x in the logarithmic equations.

   a. $log_8(4x + 1) = -1$

4x + 1 = 8^{−1} = 1/8 ⇒ 4x = −7/8 ⇒ x = −7/32.

   b. $4ln(2x - 1) + 3 = 11$

4 ln(2x − 1) = 8 ⇒ ln(2x − 1) = 2 ⇒ 2x − 1 = e^2 ⇒ x = (1 + e^2)/2

   c. $log_3(x^2 + 6x) = 3$

x^2 + 6x = 27 ⇒ x^2 + 6x − 27 = 0 ⇒ (x − 3)(x + 9) = 0 ⇒ x = 3 or x = −9. Check domain: both give positive arguments(27), so both are valid.

5. Consider the polynomial f(x) = $x^3 - 6x^2 + 3x + 10$.
   a. Write f(x) in factored form by using the Rational Roots Theorem and polynomial long division. Hint: the first step is listing possible rational roots.

Try x = 2 gives 8 − 24 + 6 + 10 = 0 ⇒ x = 2 is a root. Divide by (x − 2) ⇒ quotient x^2 − 4x − 5 ⇒ factor (x − 5)(x + 1). So f(x) = (x − 2)(x − 5)(x + 1).

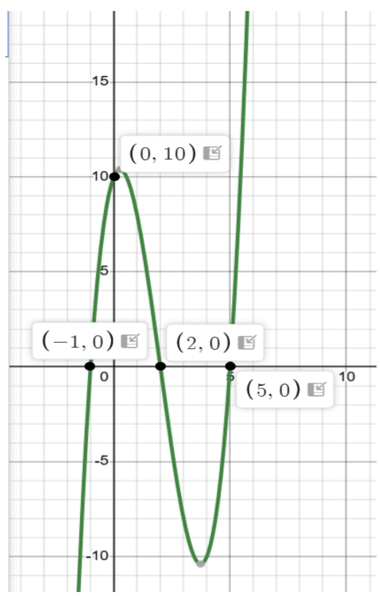
b. Determine the x and y intercepts of f(x).

x-intercepts: x = 2, 5, −1. y-intercept: f(0) = 10

c. What is the relationship between the zeros of a polynomial and its factored form?

The zeros of a polynomial are the solutions r where (x − r) is a factor; the factored form lists these factors.

d. Using your solutions above, draw a rough sketch of f(x). Label all intercepts.



6. Suppose $log_b a = 4$, $log_b c = 1$, $log_b d = 2$ Determine the exact value of the following expression.

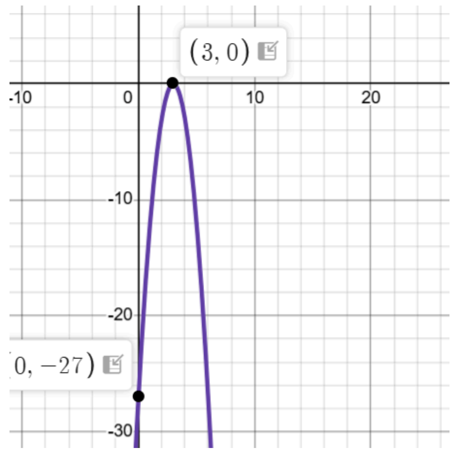
$$log_b\left(\frac{a^3 d^2}{c^5}\right)$$

$log_b(a^3 d^2 / c^5) = 3\ log_b(a) + 2\ log_b(d) - 5\ log_b(c) = 3(4) + 2(2) - 5(1) = 12 + 4 - 5 = 11.$

7. For each of the following polynomials, determine the long term behavior, intercepts, and where the polynomial is positive and negative. Use this to sketch the graph. Label all intercepts

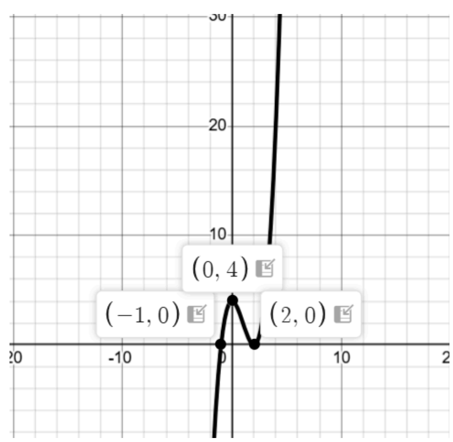
a. (3-x) (3x-9)

(3 − x)(3x − 9) = (3 − x)·3(x − 3) = 3(3 − x)(x − 3) = −3(x − 3)^2. So polynomials = −3(x − 3)^2 (a nonpositive parabola with double root at x=3). Long-term: leading term negative even ⇒ → −∞ b Intercept: x = 3 (double), y-intercept: plug x=0 ⇒ (3)(−9)= −27.



b. $(2 - x)^2(x + 1)$

Roots: x = 2, x = −1. Leading behavior: $-\ (x)^2 \cdot x \sim x^3$ with coefficient $(-\ 1)^2 \cdot 1 = +1$ ⇒ positive leading, so as x→∞ → ∞ and as x→−∞ → −∞. Sign chart: use multiplicities: at x=2 does not change sign; at x=−1 (odd) sign changes.



8. Suppose $f(x) = \frac{3}{2-x}$. What is the range f^{-1}?

Solve $y = \frac{3}{2-x} \Rightarrow$ swap: $x = \frac{3}{2-y} \Rightarrow (2-y) = \frac{3}{x} \Rightarrow y = 2 - \frac{3}{x}$. So f^{-1}(x) = $2 - \frac{3}{x}$. Domain of f is $x \neq 2$; range of f is $y \neq 0$ (since $3/(2-x)$ cannot be 0). Therefore the range of f^{-1} is all reals except 2.

9. Determine the following limits

a. $\lim_{x \to 2^+} \frac{5}{2-x}$

-9

b. $\lim_{x \to \infty} log_3(5 + x)$

∞

c. $\lim_{x \to -5} \frac{3}{x+5}$

D.N.E

d. $\lim_{x \to \infty} 250x$

∞

e. $\lim_{x \to -\infty} (-2 + 3x - 6x^2)$

-∞