



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 221 – Calculus I

Midterm 3 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: October 27, 6:00pm-7:20pm Jiya and Patrick

Session 2: October 29, 7:00pm-8:20pm Lucy and Emma

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/1056>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Match each of the following expressions with the correct description.

#	Expression	Answers
1	$F(x) = \int_a^x f(t) dt \implies F'(x) = f(x)$	
2	$a + (i - 1)\Delta x$	
3	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	
4	$a + i\Delta x$	
5	$\int_a^b F'(x) dx = F(b) - F(a)$	
6	$\ln x + C$	

- *A*: Newton's Method
- *B*: Antiderivative of $\frac{1}{x}$
- *C*: Height of the i -th rectangle in a left Riemann sum
- *D*: Net Change Theorem
- *E*: Height of the i -th rectangle in a right Riemann sum
- *F*: Fundamental Theorem of Calculus, Part 1

2. Consider the region R bounded by $y = \frac{1}{x}$, $y = -\frac{1}{x} + 3$, and $x = 4$.

- (a) Sketch a rough graph of this region R .
- (b) Set up, but do not evaluate, a definite integral that will give the area of R .

3. Evaluate the following indefinite integrals:

a. $\int \frac{\sin^2(x)}{\sec(x) \csc^4(x)} dx$ b. $\int \frac{e^{9x}}{e^{18x} + 1} dx$

4. Find the average value of the function below on the interval $[1, 9]$. Simplify.

$$f(x) = \frac{8x}{x^2 + 9}$$

5. Consider the function $f(x) = \sqrt{x+2}$ over the interval $[0, 3]$.

- (a) Use a right Riemann sum with $n = 6$ rectangles to estimate the area under the curve.
- (b) Is the right Riemann sum an underestimate or an overestimate? Explain.
- (c) Write the right Riemann sum over the same interval for n rectangles in summation form.

6. At t hours, a population of bacteria is growing at a rate of

$$r(t) = \frac{21e^{\sqrt{t}}}{\sqrt{t}} \text{ bacteria per hour}$$

Compute the change in population size between times $t = 169$ s and $t = 225$ s. Simplify your answer.

7. The function $f(x) = 10x^3 - 20x + 1$ has one root in the interval $[1, 2]$. In order to approximate this root, begin with an initial estimate of $x_1 = 2$ and use Newton's Method to obtain a second estimate x_2 , then write it in decimal form.

8. Express the definite integral as the limit of Riemann Sums. Do not evaluate the limit.

$$\int_{-3}^5 x^2 e^{\sin(x)} dx$$

9. Evaluate the definite integral:

$$\int_1^3 \frac{12x + 18}{x^2 + 3x + 10} dx$$

10. Check if the Fundamental Theorem of Calculus applies, then evaluate the given integral if it does.

(a)

$$\frac{d}{dx} \int_{-\pi}^x \sin(t) dt, \quad x \in [-\pi, \pi]$$

(b)

$$\frac{d}{dx} \int_{-5}^x \frac{1}{t} dt, \quad x \in [-5, 3]$$