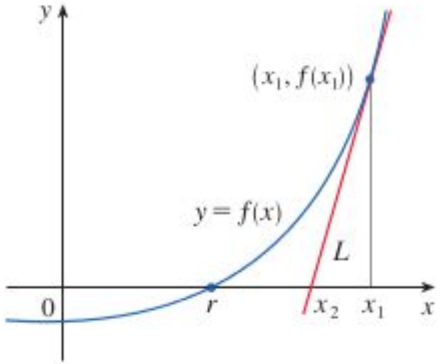
CARE
MATH 221
Exam 3 Review

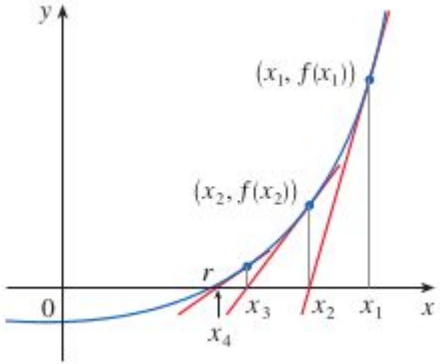
4.8 Newton’s Method

What: Iterative tangent line algorithm for finding root approximations

Why: Some root equations (eg. polynomials of degree 5 or higher) are impossible to solve algebraically

“The tangent line is close to the curve, so its intercept is near that of the curve.”





$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

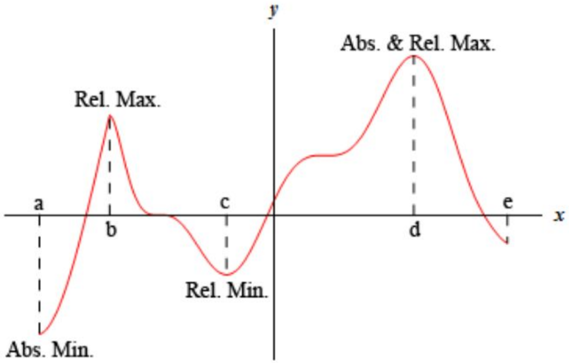
4.1 Maximum and Minimum Values

Terminology:

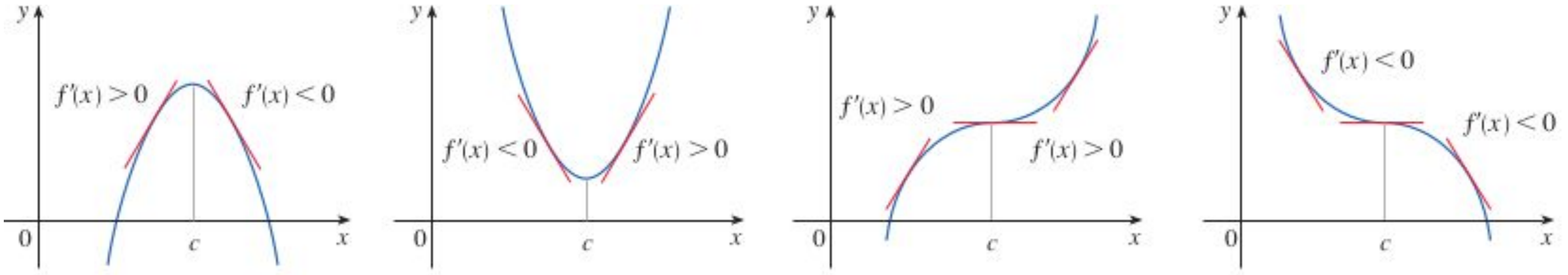
- **Extrema:** Max and min points, including both absolute and relative.
- **Absolute** (global) max/min: The greatest/least finite value of a function on a *specific interval of interest*, and/or possibly the whole function.
- **Relative** (local) max/min: The greatest/least finite value of a function on an **open** interval surrounding the point of interest.

Understand:

- Domain or interval of interest: there may be higher or lower values somewhere else, but we are not concerned with them
- **Relative extrema cannot occur at endpoints**



4.3 How Derivatives Affect the Shape of a Graph



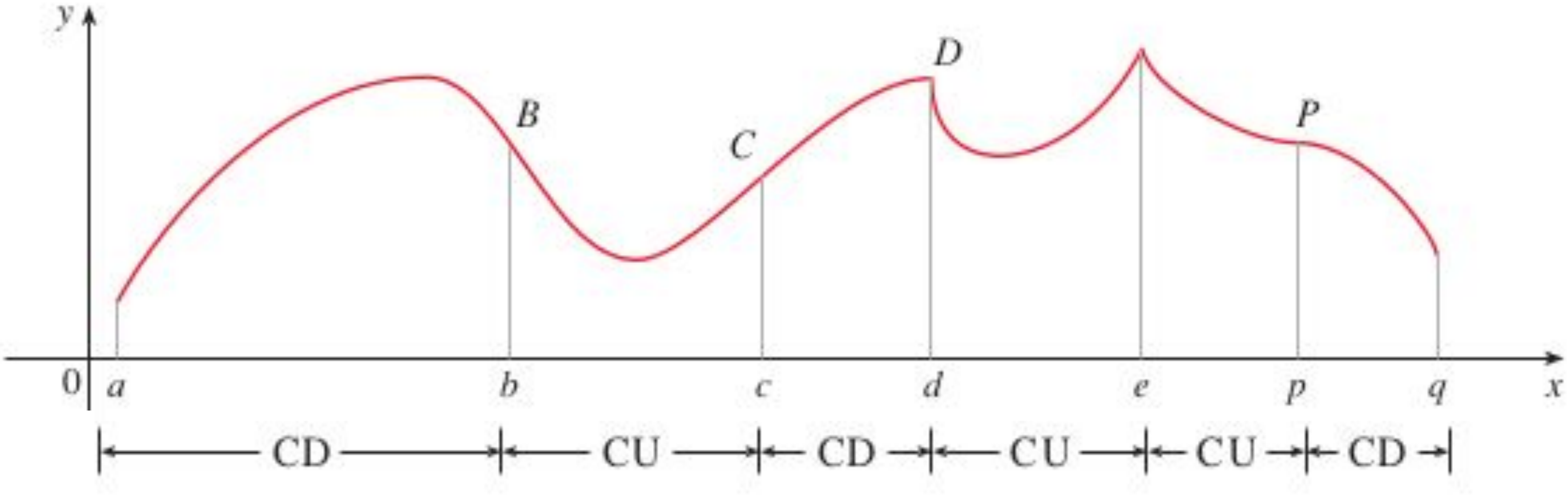
(a) Local maximum at c (b) Local minimum at c (c) No maximum or minimum at c (d) No maximum or minimum at c

Remark: The rate of change can equal zero (or be undefined) in a lot of different situations, therefore it is necessary to observe behavior on either side of the critical point to gain the full picture of base-function behavior

Ex: A woman can walk forward and turn the other way, therefore stopping at that instant to reorient, or stop and chill for a while and continue on in the same direction.

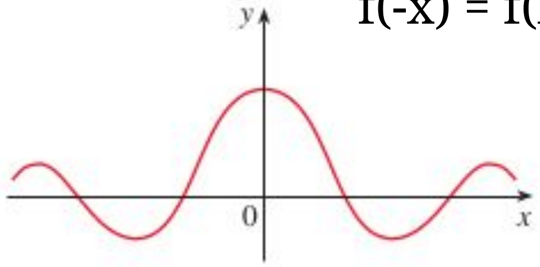
4.3 How Derivatives Affect the Shape of a Graph

Inflection Point - A point where the function is continuous and has a change in concavity



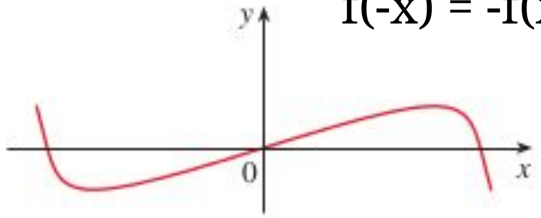
4.5 Curve Sketching

$f(-x) = f(x)$



(a) Even function: reflectional symmetry

$f(-x) = -f(x)$



(b) Odd function: rotational symmetry

- Domain/Range
- Intercepts
- Asymptotes (+ Slant asymptote)
- Intervals of increase/decrease
- Critical points and extrema
- Points of inflection
- Intervals of concavity

Related Rates

1. Draw a picture with arrows to indicate known motion
2. Write out all relevant equations
3. Determine the differential form of the rate of interest
4. Rewrite equations to be in terms of shared variables
5. Differentiate

Antiderivatives

$$\int 0dx = C$$

$$\int 1dx = x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln(x) + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C$$

$$\int \sin(x)dx = -\cos(x) + C$$

$$\int \cos(x)dx = \sin(x) + C$$

$$\int \sec^2(x)dx = \tan(x) + C$$

$$\int \sec(x)\tan(x)dx = \sec(x) + C$$

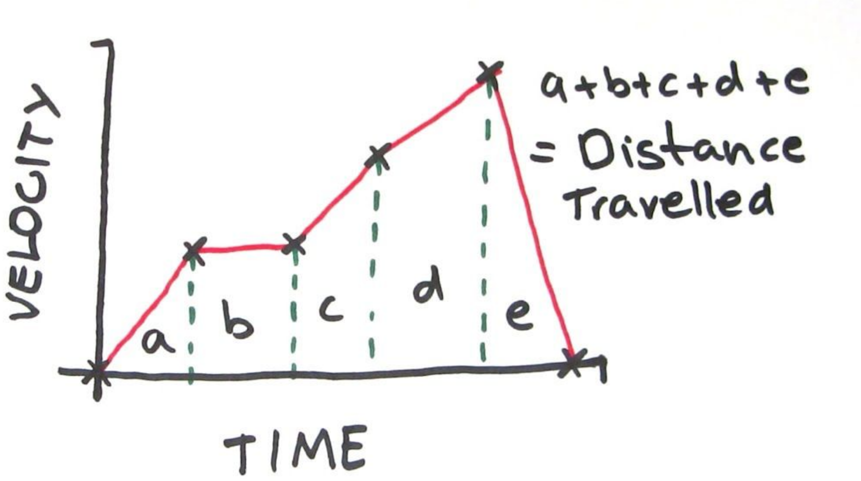
$$\int -\csc(x)\cot(x)dx = \csc(x) + C$$

$$\int -\csc^2(x)dx = \cot(x) + C$$

Distances

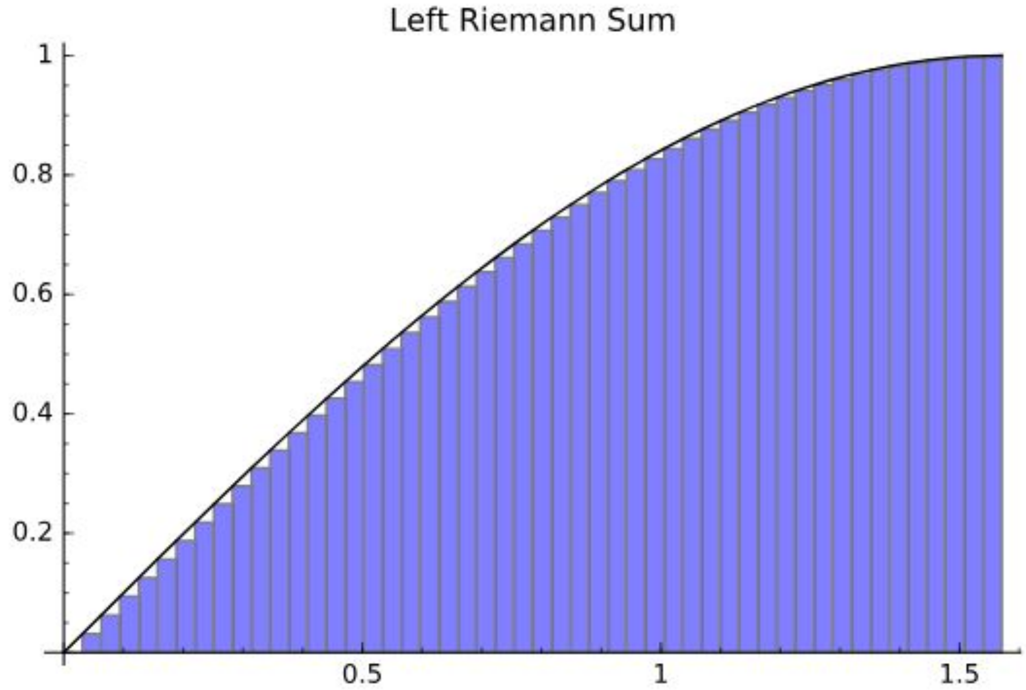
- **Distance = Velocity X Time**
  - *Hint:* think about the units → (meter) = (meters/seconds) X (seconds)

Given a plot of velocity vs time, distance traveled would be the area under the curve:



Area Under A Curve

Integrals can be thought of as the concept of dividing our area into sections of **rectangles** and then taking the **sum** of them all.

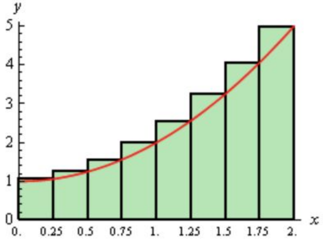


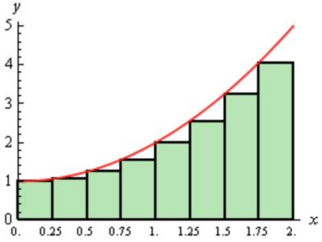
Riemann Sums

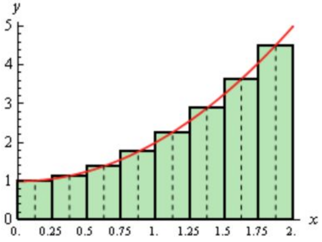
Base Rules for estimating area under a curve:

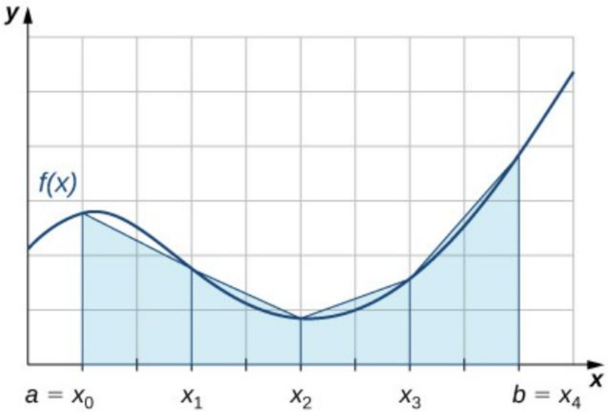
$$\int_a^b f(x)\,dx.$$

$$\Delta x = \frac{b-a}{n}$$









$$T_n = \frac{1}{2}\Delta x\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right).$$

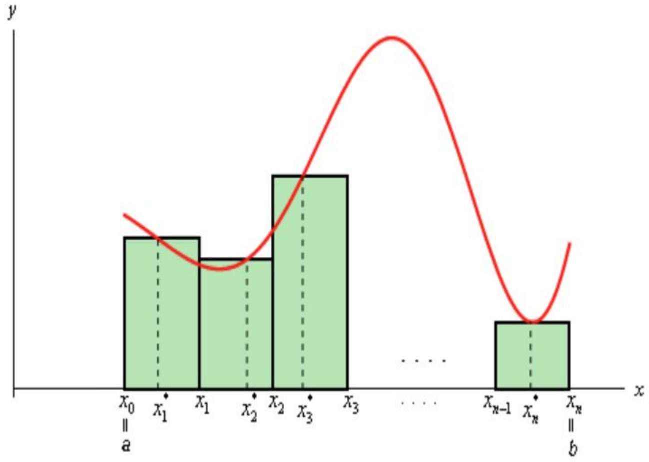
Four main types of Riemann Sums: Left endpoint, right endpoint, midpoint, and trapezoidal sums

Left and right endpoints can be over or underestimates depending on if the function is increasing or decreasing

Which sum is the most accurate?

Sigma Notation

Use summation notation to write an expression to estimate the area under the graph of $f(x) = \sin(x)$ with 4 rectangles of equal width and right endpoints on the interval $(-2, 6)$.



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$$

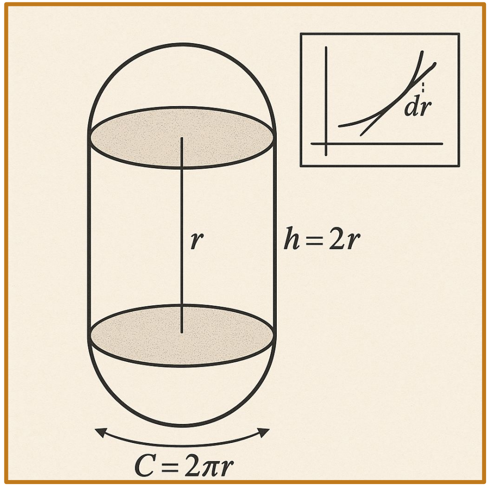
Δx = (b-a)/n

(x_i^*) = a +i Δx

$$R_n = \sum_{i=1}^{4} 2\sin\left(2i - 2\right)$$

Error Estimation Using Differentials

1. Determine what quantity you're finding the error in (volume, area, surface area)
2. Write its formula in terms of the measured variable(s)
    a. Example: $V = \pi r^2 h$
3. Relate all variables to the measured one
    a. Example: given radius is twice the height, $r = 2h$
        i. Now, $V = \pi(2h)^2 h$
4. Take the total differential of your function
    a. $dV = [\pi(2h)^2 h]' = 12\pi h^2\, dh$
5. Substitute in known quantities to solve



Constant acceleration problems

1. Identify knowns and unknowns

- Write down what’s given
- Choose a direction convention

2. Write fundamental relationships

- Integral or Derivative Form

3. Use separation of variables and integrate

- $\int v\, dv = \int a\, dy$ $\int v dv = \int a dy$
- Apply initial conditions to remove the constant (+C)

5. Substitute known values

- Plug in v, v_0, y, y_0, to solve for a or y

6. Check results

- Confirm sign direction (up or down)

	Derivative Form	Integral Form
Position	$r(t)$	$r(t) = r_0 + \int_0^t v\,dt'$
Velocity	$v(t) = \frac{dr}{dt}$	$v(t) = v_0 + \int_0^t a\,dt'$
Acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2}$	$a(t)$

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