



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 231 — Calculus II

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: October 26th, 4:00 - 5:20 pm Maya and Hriday

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/844>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Determine whether the integral converges or diverges. If so, what value does the integral converge to?

$$\int_5^{\infty} \frac{1}{n(\ln(n))^2}$$

2. We want to evaluate

$$\int_0^2 x^2 dx$$

using a Riemann sum of $n = 3$ terms. Let us define L_3 as the Riemann Sum if we choose the left endpoints and R_3 if we choose the right endpoints. Then:

- (a) $L_3 = \frac{9}{32}, R_3 = \frac{15}{32}$
- (b) $L_3 = \frac{40}{27}, R_3 = \frac{112}{27}$
- (c) $L_3 = \frac{15}{81}, R_3 = \frac{41}{81}$
- (d) $L_3 = \frac{7}{32}, R_3 = \frac{55}{36}$
- (e) $L_3 = \frac{53}{36}, R_3 = \frac{55}{36}$

3. Test the following improper integral for convergence. If it converges, find the value

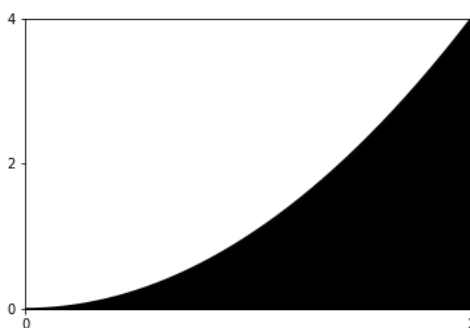
$$\int_0^{\infty} e^{-\frac{4}{3}y} dy$$

- (a) Diverges
- (b) Converges, value is $-\frac{3}{4}$
- (c) Converges, value is $\frac{4}{3}$
- (d) Converges, value is $\frac{3}{4}$
- (e) None of the above

4. Determine if the following integral is convergent or divergent:

$$\int_2^{\infty} \frac{\cos^2 x}{x^2} dx$$

5. Find the M_x , M_y , and the centroid of $y = x^2$ with density λ on $x \in [0, 2]$.



6. Consider the curve $y = 5\ln(x)$ between the points $(1,0)$ and $(e,5)$.
- A. SET UP, BUT DO NOT EVALUATE, a dx -integral which represents the arc length of the curve.
 - B. SET UP, BUT DO NOT EVALUATE, a dy -integral which represents the arc length of the curve.
 - C. SET UP, BUT DO NOT EVALUATE, a definite integral which represents the surface area of the surface obtained by rotating the curve around the line $y = 10$.
7. The profile $y = \sqrt{4 - x^2}$ on the interval $x \in [-1, 1]$ is revolved around the x -axis. Find the surface area of this surface.

8. Compute the arc length of the function $y = 1 + 2x^{\frac{3}{2}}$ between $x = 0$ and $x = 1$
- (a) $\frac{14}{9}$
 - (b) $\frac{10}{9}$
 - (c) $\frac{2}{9}\sqrt{10}$
 - (d) $\frac{2}{27}(10\sqrt{10} + 1)$
 - (e) None of the above
9. Which of the following is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
- (A) The series converges to $\pi^2/6$
 - (B) The series converges absolutely.
 - (C) Not enough information is given to make a determination.
 - (D) The series diverges.
 - (E) The series converges conditionally.
10. Which of the following is true about the series $\sum_{n=1}^{\infty} \left(\frac{6n+4}{15n+7}\right)^n$
- (A) The series diverges.
 - (B) Not enough information is given to make a determination.
 - (C) The series converges to $2/5$.
 - (D) The series converges conditionally.
 - (E) The series converges absolutely.
11. Consider the sequence $a_n = \frac{n^3 030^n}{30n!}$ and the series $\sum_{n=1}^{\infty} a_n$. Which of the following is true?
- (A) The sequence converges and series diverges.
 - (B) The sequence converges and the series converges conditionally.
 - (C) The sequence diverges and the series converges.
 - (D) The sequence converges and the series converges absolutely.
 - (E) Both the sequence converges and the series diverges.
12. Consider the sequence $a_n = \frac{2n+6}{3n+7}$ and the series $\sum_{n=1}^{\infty} a_n$. Which of the following is true?
- (A) None of the others are true.
 - (B) The sequence diverges and the series converges.

- (C) The sequence converges and series diverges.
(D) Both the sequence and the series diverge.
(E) Both the sequence and the series converge.
13. Consider the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$. Which of the following is true?
- (A) The series diverges.
(B) The series converges conditionally.
(C) The series converges absolutely.
(D) The series converges to $(e^2 - 1)/e$
(E) The series converges to 0.
14. First decide which test to use. After doing this for each question, determine if the series will diverge or converge.
- (i) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$
- (ii) $\sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n^3 + 4n + 1}$
- (iii) $\sum_{n=0}^{\infty} \frac{n^2}{n^3 + 1}$
- (iv) $\sum_{n=1}^{\infty} \frac{2n^3 + 7}{n^4 \sin^2(n)}$
- (v) $\sum_{n=3}^{\infty} \frac{e^{-n}}{n^2 + 2n}$
- (vi) $\sum_{n=2}^{\infty} \frac{1}{(2n+7)^3}$
- (vii) $\sum_{n=4}^{\infty} \frac{(-1)^{n+2}(1-n)}{3n - n^2}$
- (viii) $\sum_{n=1}^{\infty} \frac{\sqrt{2n^2 + 4n + 1}}{n^3 + 9}$
15. Come up with your own series problem! Have a partner answer it and explain what test you would use.