



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 231 — Engineering Calculus

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Oct. 20, 7-8:30pm Grace and Sushrut

Session 2: Oct. 21, 7-8:30pm Sofi and Bella

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/844>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Evaluate the following integral:

$$\int e^x \cos(x) dx$$

- (a) $\frac{e^x}{4}(\sin(x) + \cos(x)) + C$
- (b) $\frac{e^x}{2}(\sin(x) + \cos(x)) + C$
- (c) $\frac{e^x}{2}(\sin(x)) + C$
- (d) $\frac{e^x}{6}(\sin(x) + \cos(x)) + C$

2. What is the best substitution to make in order to solve the following integral?

$$\int y^2 \sqrt{y^2 + 4} dy$$

- (a) $y = \ln(\theta)$
- (b) $y = 2 \sec(\theta)$
- (c) $y = \sin(\theta)$
- (d) $y = 2 \tan(\theta)$
- (e) $y = e^t + 1$

3. Evaluate the following integral:

$$\int_1^{e^3} 4x^3 \ln(x)$$

- (a) $\frac{1+e^8}{2}$
- (b) $\frac{11e^{12}+1}{4}$
- (c) $3 + e^4$
- (d) $\frac{1+e^{12}}{2}$
- (e) $\frac{7e^8+1}{4}$

4. Evaluate the following indefinite integral

$$\int x\sqrt{x^2 + 2}dx$$

- (a) $\frac{1}{2}x^2\sqrt{x^2 + 2} + C$
- (b) $\frac{2}{3}(x^2 + 2)^{\frac{3}{2}} + C$
- (c) $\frac{1}{3}(x^2 + 2)^{\frac{3}{2}} + C$
- (d) $(x^2 + 2)^{\frac{3}{2}} + C$
- (e) $\frac{2}{3}x(x^2 + 2)^{\frac{3}{2}} + C$

5. Let $y = f(x)$ be defined implicitly by $y^3 - xy = 10$, Find the $\frac{dy}{dx}$ in terms of y and x

- (a) $\frac{dy}{dx} = \frac{10y^2}{x^3}$
- (b) $\frac{dy}{dx} = \frac{10}{y^2 - x}$
- (c) $\frac{dy}{dx} = \frac{y}{3y^2 - x}$
- (d) $\frac{dy}{dx} = \frac{10}{3y^2} + \frac{x}{y}$

6. Suppose that $f(x)$ is continuous $[0, 1]$ and differentiable on $[0, 1]$ with $f(0) = 2$ and $f(1) = 5$. Exactly one of the following statements is true.

- (a) There exists a point c on $(\frac{1}{2}, 1)$ such that $f'(c) = 3$.
- (b) None of the other answers are true.
- (c) There exists a point c on $(0, 2)$ such that $f(c) = 0$.
- (d) There exists a point c on $(0, 1)$ such that $f'(c) = 3$
- (e) There exists a point c on $(2, 5)$ such that $f(\frac{1}{2}) = c$.

7. A 4m long ladder is propped up against a wall. The ladder begins to slip. At time $t = 3$ s, the base of the ladder is 2 m from the wall and moving away from the wall at 3 m/s. How fast is the top of the ladder moving vertically along the wall?

- (a) $-\sqrt{3}$ m/s
- (b) -2 m/s
- (c) $-3\sqrt{2}$ m/s
- (d) $-\frac{13}{4}$ m/s
- (e) $-3\sqrt{3}$ m/s

8. We want to evaluate

$$\int_0^2 x^2 dx$$

using a Riemann sum of $n = 3$ terms. Let us define L_3 as the Riemann Sum if we choose the left endpoints, and R_3 if we choose the right endpoints. Then:

- (a) $L_3 = \frac{9}{32}, R_3 = \frac{15}{32}$
- (b) $L_3 = \frac{40}{27}, R_3 = \frac{112}{27}$
- (c) $L_3 = \frac{15}{81}, R_3 = \frac{41}{81}$
- (d) $L_3 = \frac{7}{32}, R_3 = \frac{55}{36}$
- (e) $L_3 = \frac{53}{36}, R_3 = \frac{55}{36}$

9. A Farmer wants to build a rectangular field next to a river. The farmer will use the river as one side of the rectangle, but must build the other three sides of the rectangle with fence. There is a total amount of 120 meters of fence available. What is the largest possible area that can be enclosed?

- (a) 2400 m^2
- (b) 1600 m^2
- (c) 1800 m^2
- (d) 800 m^2

10. Let us define $f(x)$ by:

$$\int_{-3}^{x^2 \cos(x)} e^{-t^2} dt$$

Compute $f'(x)$

- (a) $2x \cos(x)e^{-x^4 \cos^2(x)} - x^2 \sin(x)e^{-x^4 \cos^2(x)}$
- (b) $x \cos(x)e^{-x^4 \cos^2(x)} + 2x^2 \sin(x)e^{-x^4 \cos^2(x)}$
- (c) $e^{-x^4 \cos^2(x)} - e^{-9}$
- (d) $2x \cos(x)e^{-x^2 \cos^2(x)} - x^2 \sin(x)e^{-x^2 \cos^2(x)}$
- (e) $e^{x^4 \cos^2(x)}$

11. Evaluate the following integral

$$\int \frac{5x - 1}{(x - 2)^2(x + 1)}$$

- (a) $\ln|x - 2| + \frac{4}{2-x} - 2 \ln|x + 1| + C$
- (b) $\frac{1}{3} \ln|x - 4| + \frac{3}{1-x} - \frac{2}{3} \ln|x + 3| + C$
- (c) $\ln|x - 2| + \frac{3}{2-x} - \ln|x + 1| + C$
- (d) $\frac{2}{3} \ln|x - 2| + \frac{3}{2-x} - \frac{2}{3} \ln|x + 1| + C$
- (e) $\frac{1}{4} \ln|x - 2| + \frac{3}{1-x} - \frac{2}{3} \ln|x + 1| + C$

12. Evaluate the following integral

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$

- (a) $\ln|x| + \ln(x^2 + 4) - \frac{1}{4} \arctan\left(\frac{x}{2}\right) + C$
- (b) $\ln|x| + \frac{1}{2} \ln(x^2 + 4) - \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$
- (c) $\ln|x| + \frac{1}{2} \ln(x^2 + 8) - \frac{1}{2} \arctan\left(\frac{x}{4}\right) + C$
- (d) $\frac{1}{2} \ln|x| + \frac{1}{2} \ln(x^2 + 4) - \frac{1}{3} \arctan\left(\frac{x}{2}\right) + C$

(e) $\frac{1}{2} \ln |x| + \frac{1}{2} \ln(x^2 + 2) - \frac{1}{3} \arctan\left(\frac{x}{4}\right) + C$

13. Evaluate the following integral

$$\int_0^4 \frac{dx}{(x^2 + 16)^{\frac{3}{2}}}$$

(a) $\frac{1}{8\sqrt{2}}$

(b) $\frac{1}{16\sqrt{3}}$

(c) $\frac{1}{16\sqrt{2}}$

(d) $\frac{1}{\sqrt{2}}$

(e) $\frac{1}{8\sqrt{3}}$