Exam 2 Review Session
Math 231E

Please join the queue
for attendance!

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Outline

1. Please join the queue
2. Mini review of some topics covered
3. Practice! → CARE Worksheet, Practice Exams
    a. Please raise hands for questions rather than put them in the queue

Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	4pm-10pm	1pm-5pm 8pm-10pm		1pm-5pm 8pm-10pm	6pm-8pm		2pm-4pm

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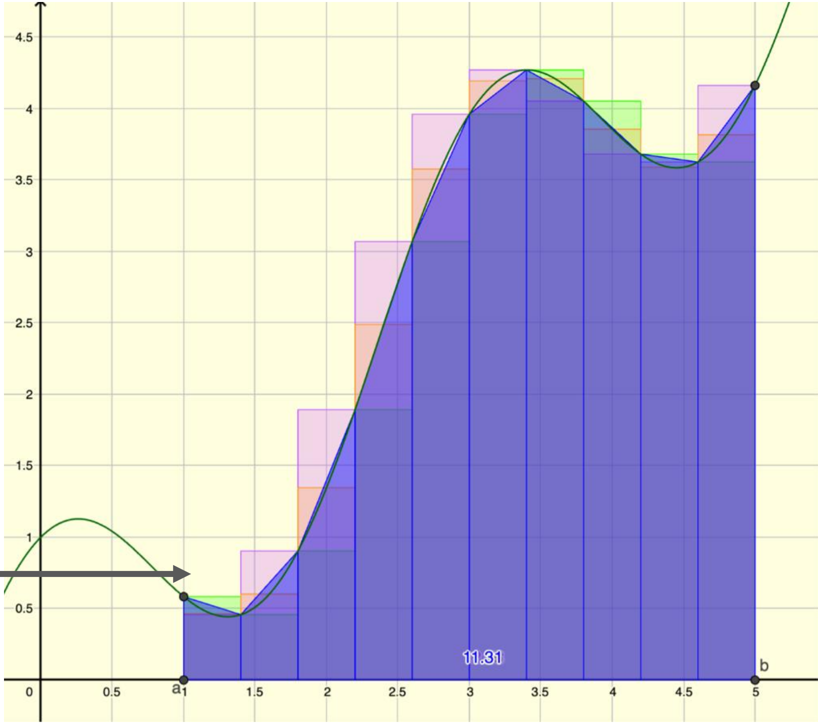
Content Review

Riemann Sums

- Approximation for the area under a curve
- General form: $\sum_{x=a}^{b} f(x_k{}^*)\Delta x$
  - $\Delta x = \frac{(b-a)}{n}$ → difference of the two values divided by the number partitions
  - $x_k^* = a + n\Delta x$ → the leftmost value plus the difference times the number partition you are on
- Types
  - Left endpoint
  - Right endpoint
  - Midpoint
  - Trapezoidal



https://www.geogebra.org/m/WTgyER9x

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Integrals and the Fundamental Theorem of Calculus

- What exactly is an integral? → Riemann sum where each "rectangle" has an infinitely small width

$$\lim_{n\to\infty} \sum_{x=a}^{b} f({x_k}^*)\Delta x = \int_a^b f(x)dx$$

- Representations: area under a curve, accumulation of change

- Fundamental Theorem of Calculus: $\int_a^b f'(x)dx = f(b) - f(a)$
  - An integral "undoes" differentiation → the resulting function from integrating is an antiderivative

Known Antiderivatives

- For some common functions the antiderivative is straightforward:

Constant: $\int A dx = Ax + C$

Exponential: $\int e^x dx = e^x + C$

Power rule: $\int x^n dx = \frac{x^{n+1}}{(n+1)} + C$

Sin(x): $\int sin(x) dx = -cos(x) + C$

1/x: $\int \frac{1}{x} dx = ln(x) + C$

Cos(x): $\int cos(x) dx = sin(x) + C$

Is the function I'm integrating a known derivative of another function?

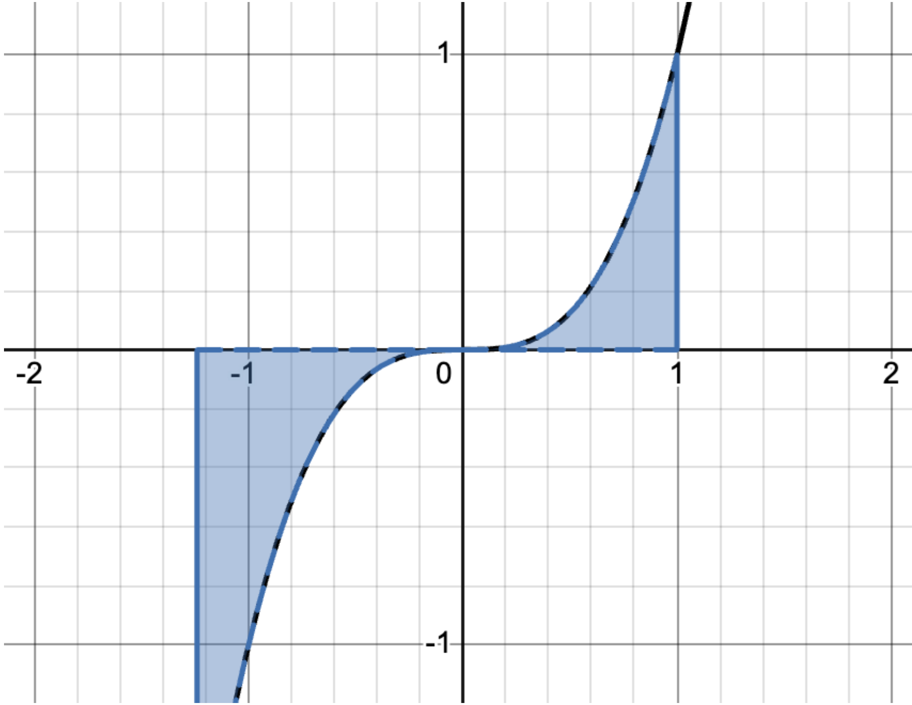
Integral Properties

1. $$\int Cf(x)dx = C\int f(x)$$

1. $$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$



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Methods for Evaluating More Complicated Integrals

U-Substitution

- If you notice one part of the integrand is the derivative of another part …

$$\int f(g(x)) \cdot g'(x)dx = \int f(u)du$$

$$u = g(x)$$

$$du = g'(x)dx$$

…you can substitute a new variable "u" and take its derivative to plug back into the integral

Example: $\displaystyle\int \frac{x}{\sqrt{1-x^2}}dx$ $\quad u = 1 - x^2$, $du = -2x$ $\longrightarrow$ $\displaystyle\int \frac{-1}{2\sqrt{u}}du$

Remember to put your answer back in terms of x

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Integration by Parts

- If you notice that two functions that are not derivatives of each other are multiplied together …

$$\int u dv = uv - \int v du$$

…you can decompose the integrand such where …

$$u = f(x) \qquad v = \int g(x)dx$$

$$du = f'(x) \qquad dv = g(x)$$

You may need to do multiple integration by parts to fully work through the integral!

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Integration by Parts → LIATE Method

	u	Examples
L - logarithmic	$\downarrow$	L - $ln(x)$
I - inverse trigonometric		I - $sin^{-1}(x)$
A - algebraic		A - $x^2 + 3x$
T- trigonometric		T- $cos(x)$
E - exponential	dv	E - e^x

You may need to do multiple integration by parts to fully work through the integral!

Partial Fractions

- If you have a rational function with a polynomial in the denominator …

Rational Function	Partial Fraction
$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2-bx-c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{(x^2-bx-c)}$

… from the original numerator you can solve for the A,B,C, etc.

Trigonometric Integrals

- If you have an integrand with trig functions, you can use known trig identities to simplify the integral and use another method if needed

$$cos^2(x) + sin^2(x) = 1$$

$$tan^2(x) + 1 = sec^2(x)$$

$$tan^2(x) = sec^2(x) - 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$cos^2 x = \frac{1}{2}\,(1 + \cos(2\mathrm{x}))$$

$$\cos(2x) = cos^2(\mathrm{x}) - sin^2(\mathrm{x})$$

$$= 1 - 2sin^2(\mathrm{x})$$

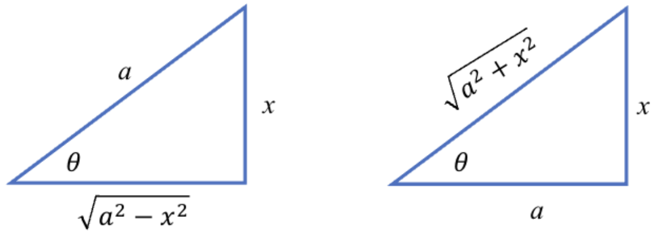
$$= 2cos^2(\mathrm{x}) - 1$$

Trigonometric Substitution

- If you notice a complicated function inside of a square root …

Format	Substitution	Derivative Substitution	Trig Identity
$\sqrt{a^2 - x^2}$	$x = a * \sin(\theta)$	$dx = a * \cos(\theta)\, d\theta$	$cos^2(\theta) + sin^2(\theta) = 1$
$\sqrt{a^2 + x^2}$	$x = a * tan(\theta)$	$dx = a * sec^2(\theta)\, d\theta$	$tan^2(\theta) + 1 = sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a * sec(\theta)$	$dx = a * \sec(\theta)\tan(\theta)\, d\theta$	$tan^2(\theta) = sec^2(\theta) - 1$

… you can substitute a known trig identity and solve the integral…



… and can convert back using a triangle

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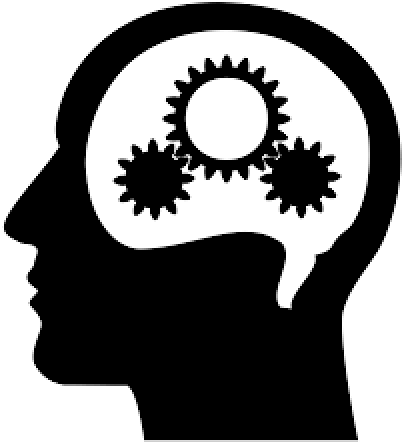
What method would you use?



$$\int \frac{1}{\sqrt{16+x^2}}dx$$

$$\int sin(x)e^x dx$$

1. $$\int cos^3(x)sin^2(x)dx$$

$$\int \frac{3x+11}{(x-3)(x+2)}dx$$

$$\int ln(x)dx$$

$$\int (2x+2)e^{x^2+2x+3}dx$$

5. $$\int sec(x)dx$$

$$\int cos(2x)dx$$

1. Trig substitution

1. Integration by parts

1. Trigonometric integrals

1. Partial fractions

5. Integration by parts

5. U-substitution

5. Integration by parts with u-sub (or known antiderivative)

5. U-substitution (or known antiderivative)

Good luck on your exam!

You can use the rest of the time to practice

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