CARE MATH 221
Midterm 2 Review

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Rates of Change in the Natural and Social Sciences

Velocity: rate of change of distance with respect to time

Acceleration: rate of change of velocity with respect to time

1. To find a velocity at t, take the derivative of the position function.
2. Particle is at rest when the velocity function = 0
3. Find what direction particle is moving in by using critical points.
4. Find total distance traveled during a specific time interval

- Remember what direction the particle is moving in when adding the distances together

Exponential Growth and Decay

$$y(t) = a \times e^{kt}$$

Where **y(t)** = value at time "t"

a = value at the start

k = rate of growth (when >0) or decay (when <0)

t = time

Implicit Differentiation

$$y^5 + 2y = x^2$$

Take the derivative of each term

Related Rates

Steps to solving related rates problems:

1. Draw a picture with arrows to indicate known motion
2. Write out all relevant equations
3. Determine the differential form of the rate of interest
4. Rewrite equations to be in terms of shared variables
5. Differentiate

Example Problem

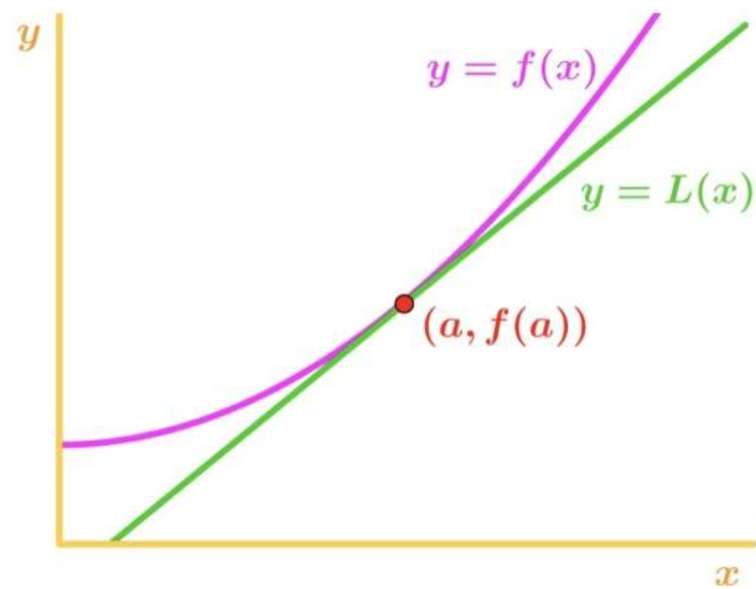
A cone-shaped tank of water is being filled at a rate of 12 m^3/s. The base radius is 26 m and its height is 8 m. At what rate is the depth of water changing when the top of the water surface has $r = 10$ m?

Linearization and Approximation of f(x)

Linear Approximation (output evaluated at a certain point)

$$L(x) = f(a) + f'(a)(x - a)$$

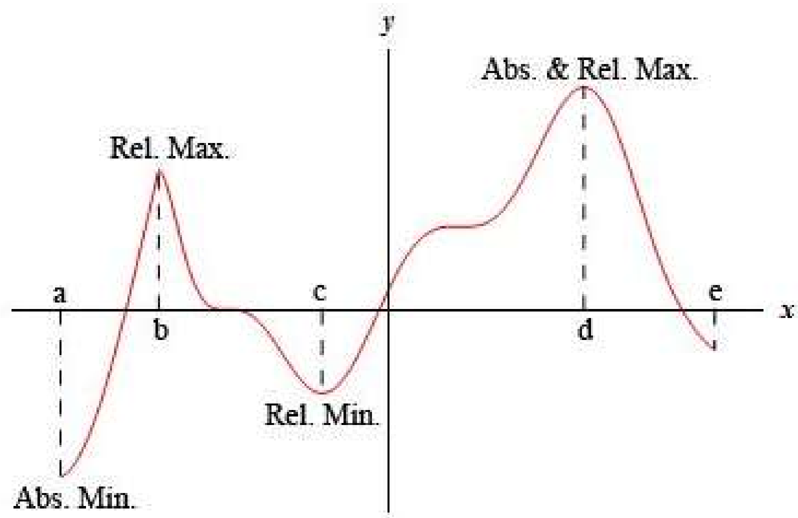
Linearization (input function)



Maximums and Minimums

Local maximum and minimum can only occur at critical values

Critical numbers are x values that will make the first derivative of your function undefined or equal to zero



Solving Max/Min Problems

Steps to solving problems:

1. Evaluate if boundary restrictions are included
2. Take derivative of function
3. Identify values that will make your derivative function equal to 0 (critical values)
4. Plug values back into original function
5. Evaluate magnitude of values compared to values for the **boundary restrictions**

Mean Value Theorem

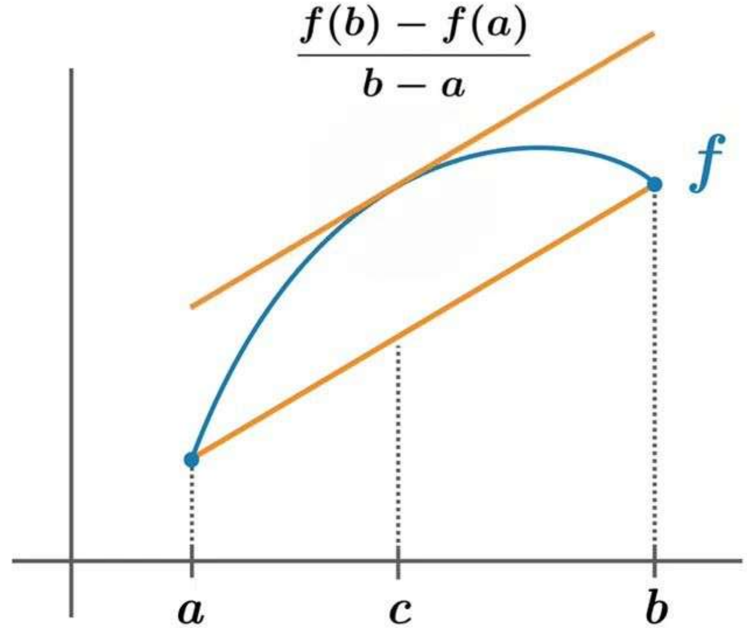
The Mean Value Theorem

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). Then there exists some argument c such that

$$a < c < b$$

and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Rolle's Theorem

Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

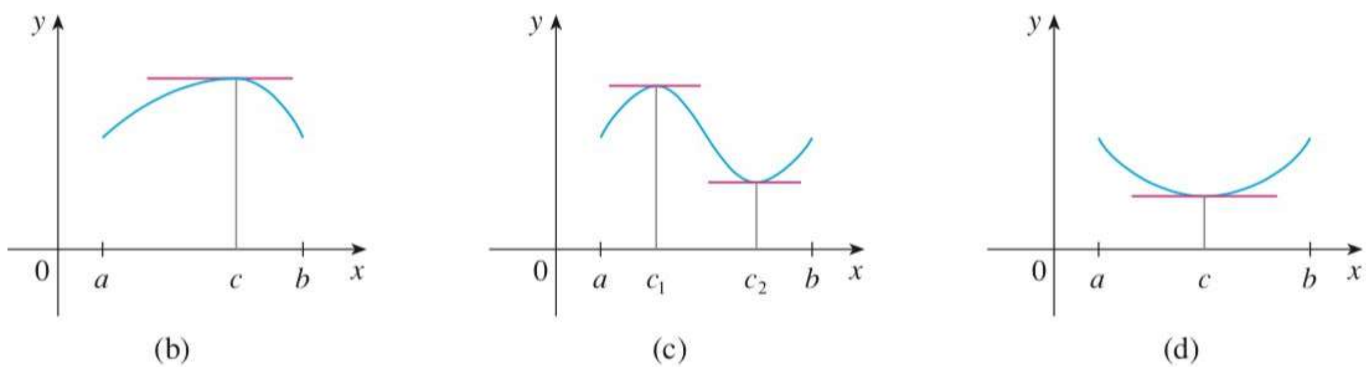
1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

Before giving the proof let's take a look at the graphs of some typical functions that satisfy the three hypotheses. Figure 1 shows the graphs of four such functions. In each case it appears that there is at least one point $(c, f(c))$ on the graph where the tangent is horizontal and therefore $f'(c) = 0$. Thus Rolle's Theorem is plausible.



1st & 2nd Derivative Test

First derivative:

y' is positive $\Rightarrow$ Curve is rising.

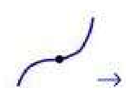
y' is negative $\Rightarrow$ Curve is falling.

y' is zero $\Rightarrow$ Possible local maximum or minimum.

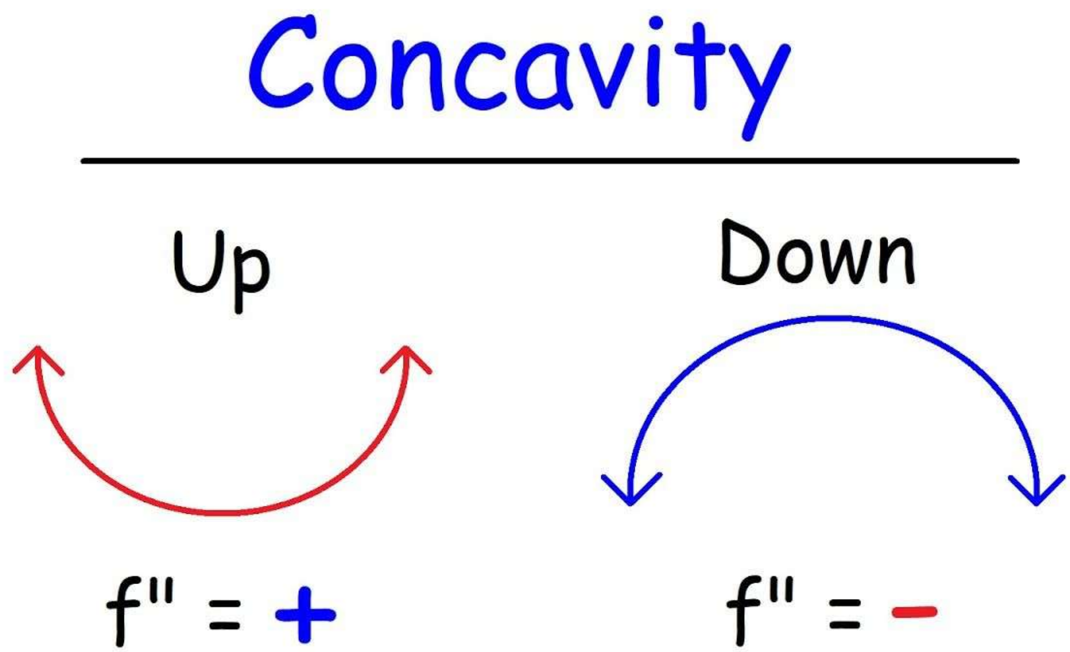
Second derivative:

y'' is positive $\Rightarrow$ Curve is concave up. 

y'' is negative $\Rightarrow$ Curve is concave down. 

y'' is zero $\Rightarrow$ Possible inflection point (where concavity changes). 

Concavity



L'Hospital's Rule

If a limit is being divided by 0, take the derivative of both the numerator and denominator until there is no 0 in the denominator.

Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval containing a, except possibly at a.

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if this limit exists.

A similar rule also works for one sided limits and limits as x goes to infinity.

Common Indeterminate Forms:

$$\boxed{\frac{\infty}{\infty}, \quad \frac{0}{0}, \quad \infty - \infty, \quad 0^0, \quad 0 \cdot \infty, \quad \infty^0, \quad 1^\infty}$$

Logarithms and Exponential Rules

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$
Equality Rule	If $\log_a x = \log_a y$ then $x = y$

Logarithmic Form	Exponential Form
$\log_a^N = x$	$a^x = N$

Exponent Rules For $a \neq 0, b \neq 0$	
Product Rule	$a^x \times a^y = a^{x+y}$
Quotient Rule	$a^x \div a^y = a^{x-y}$
Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Common Derivatives

$$\frac{d}{dx}(x)=1$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x)=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x)=a^x\ln(a)$$

$$\frac{d}{dx}(\mathbf{e}^x)=\mathbf{e}^x$$

$$\frac{d}{dx}(\ln(x))=\frac{1}{x},\quad x>0$$

$$\frac{d}{dx}(\ln|x|)=\frac{1}{x},\quad x\neq 0$$

$$\frac{d}{dx}(\log_a(x))=\frac{1}{x\ln a},\quad x>0$$

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$f(t) = t(6-t)^{\frac{2}{3}}$

A) Find the Inflection points.

B) Classify critical points.

C) Determine intervals of Increase/decrease.

D) Determine Intervals of concavity.

E) Sketch the graph of the function.