Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 221 – Calculus I

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: October 7, 4:00-5:30 pm Lucy and Patrick

Session 2: October 8, 6:00-7:20 pm Jiya and Patrick

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/1056
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Answer the following questions:

(a) State the Mean Value Theorem, emphasizing its key conditions.

(b) State Rolle's theorem, emphasizing its key conditions.

2. Suppose that A represents the number of grams of a radioactive substance at time t seconds. Given that $\frac{dA}{dt} = -0.125A$, how long does it take 12 grams of the substance to be reduced to 4 grams?

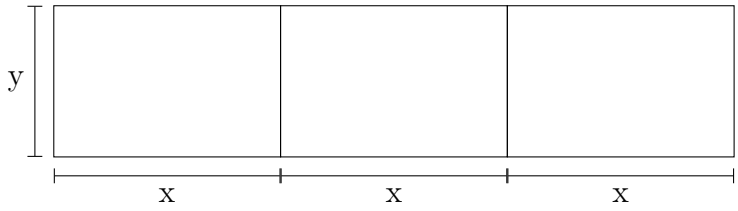
3. A streetlight is mounted at the top of a tall pole with $H = 16.5$ ft. Jennifer's height is $h = 5.5$ ft tall. She walks away from the pole with a speed of 8 ft/s along a straight path. how quickly is the length of her shadow on the ground increasing when she is 15 ft from the pole?

4. The top of a ladder slides down a vertical wall at a rate of 8 m/s. At the moment when the bottom of the ladder is 4 meters from the wall, it slides away from the wall at a rate of 15 m/s. How long is the ladder?

5. Find the absolute minimum y-value of the given function:

$$y = \frac{2x}{\sqrt{x - 81}}$$

6. A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown, but only has 600 feet of fencing available. Determine the values for x and y which will maximize the total area enclosed by these three pens.



7. Without using any kind of computational aid, use a linear approximation to estimate the value of $e^{0.1}$.

8. Given the function $f(x) = 5x^2 - 3x + 15$

(a) Decide whether the function is increasing or decreasing at $x = 0$ and $x = 1$.

(b) Find the critical points of the function and state whether they are maximums or minimums using the first derivative test.

(c) Use the second derivative test to check whether the critical points are maximums or minimums. Does your answer agree with part (b)?

9. A function $f(x)$ has the first derivative $f'(x) = e^{0.5x}(10x - 60)$

(a) Upon which interval is $f(x)$ increasing?

(b) Upon which interval is the graph of $f(x)$ concave down?

10. Evaluate each of the following limits:

(a)
$$\lim_{x\to\infty} \frac{2\ln(x)}{\sqrt[3]{x}}$$

(b)
$$\lim_{x\to 0} \frac{e^{10x} - 1}{5x}$$

(c)
$$\lim_{x\to\infty} \frac{e^{10x} - 1}{5x}$$

11. Evaluate the derivative of the following:

(a)

$$f(x) = sin(x)^{ln(x)}$$

(b)

$$g(x) = \frac{1}{cos^{-1}(x^3 + x)}$$