Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 241 – Calculus III

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Sep. 20, 4-5:50 pm Meredith, Gabe, Cami, Liz

Session 2: Sep. 21, 4-4:50pm Lydia, Johail, Pallab, Rick

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/845
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Find $\vec{u} \times \vec{v}$ if $\vec{u} = \langle 3, -4, 1 \rangle$ and $\vec{v} = \langle 5, 2, -6 \rangle$

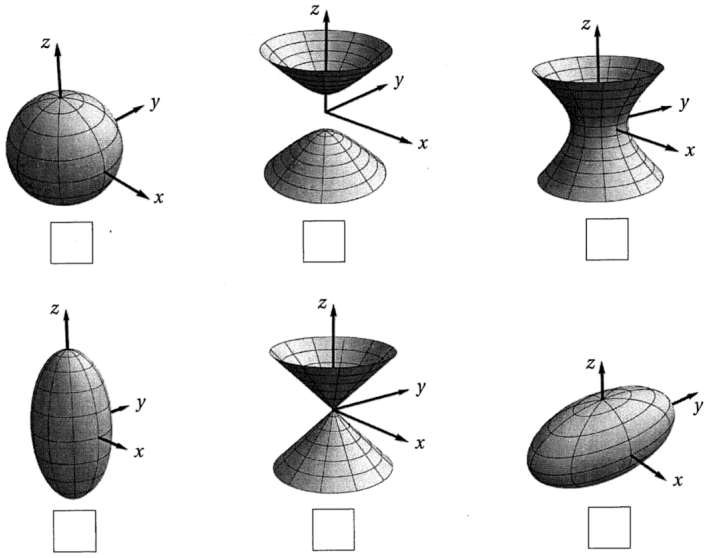
2. Find an equation for the plane that passes through the point $P = (1, 2, 3)$ and contains the line L given by the parametric equation:

$$x(t) = 1 - 3t \qquad y(t) = 3 \qquad z(t) = 6 + 2t$$

for $-\infty < t < \infty$

3. For each equation below, write the corresponding letter in the box next to the picture of the surface it describes

(A) $x^2 + y^2 - z^2 + 1 = 0$ (B) $4x^2 + y^2 + 4z^2 - 1 = 0$



4. Construct an equation for a plane that contains $(2, 1, 3)$, $(0, -1, 0)$, and $(3, 2, 1)$.

5. Consider two points A and B.

$$A = (0,\ 7,\ 2)$$
$$B = (1,\ 2,\ 0)$$

(a) Find the vector that represents the displacement between the points (vector drawn from A to B)

(b) What is the projection of this vector onto $\vec{r} = \langle 5, -2, 7 \rangle$?

(c) What is the projection of the vector from part (a) onto the plane represented by the equation $5x - 2y + 7z = 10$

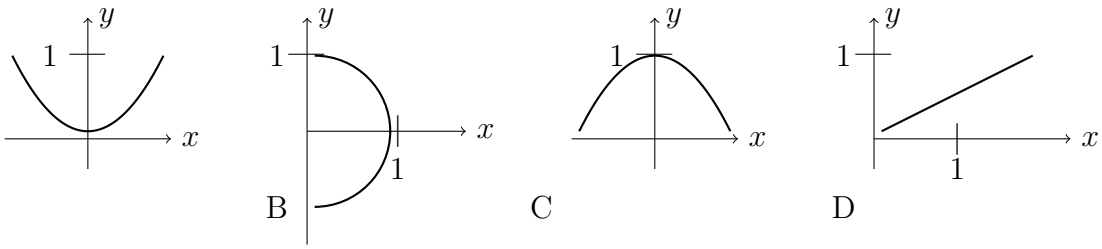
6. Determine if the two lines intersect. If so, find their point of intersection.

$$\begin{cases} x = 3t - 3 \\ y = -2t + 1 \\ z = 4t - 2 \end{cases} \qquad \begin{cases} x = 2s + 3 \\ y = 2s - 1 \\ z = s + 2 \end{cases}$$

7. Find the equation of the line of intersection of the following two planes: $2x + y - 2z = 2$ and $-2x + y + z = 6$.

8. Explain why $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$.

9. Let $r(t) = \langle \sin(t), \cos^2 t \rangle, 0 \leq t \leq 2\pi$. Which graph below represents this curve?

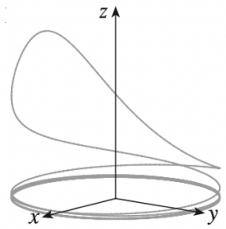


10. Assume you are walking around the surface of a spherical planet with a radius of 2. If you are walking clockwise on the xy-plane, what is the parametrization of the path after circling it twice?

11. Consider the vector function $\vec{r}(t) = \langle 2\cos t, 2\sin t, t \rangle$. Find the length from $(2, 0, 0)$ to $(2, 0, 4\pi)$ of the vector function.

Match the vector functions below to the corresponding graph.

12. (i) $x = t, y = 1/(1+t^2), z = t^2$
    (ii) $x = \cos t, y = \sin t, z = \cos 2t$
    (iii) $x = \cos t, y = \sin t, z = 1/(1+t^2)$
    (iv) $x = t\cos t, y = t, z = t\sin t, t \geq 0$



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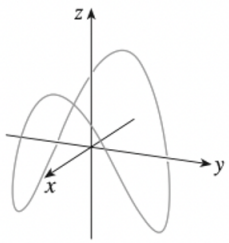

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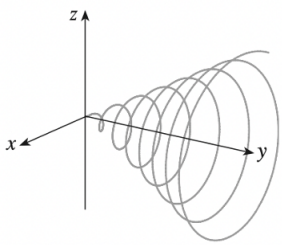

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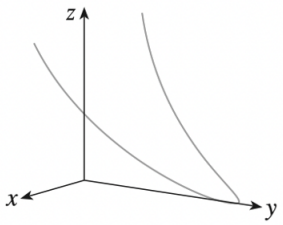

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13. Find the derivatives of the function $f(x, y) = x^2 + y^2 + xy + y^3$.

    (i) f_x, f_y
    (ii) Total differential
    (iii) f_{xx}, f_{yy}
    (iv) f_{xy}, f_{yx}

14. Find the limits for the following expressions:

(i) $$\lim_{(x,y)\to(-1,0)} \frac{x^2+xy+3}{x^2y-5xy+y^2+1}$$

(ii) $$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$$

(iii) $$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$$

(iv) $$\lim_{(x,y)\to(0,0)} \frac{x^6+x^2y^4}{x^2+y^2}$$

15. Match the multi-variable equations to the corresponding graph and contour line.

(i) $z = \sin x - \sin y$

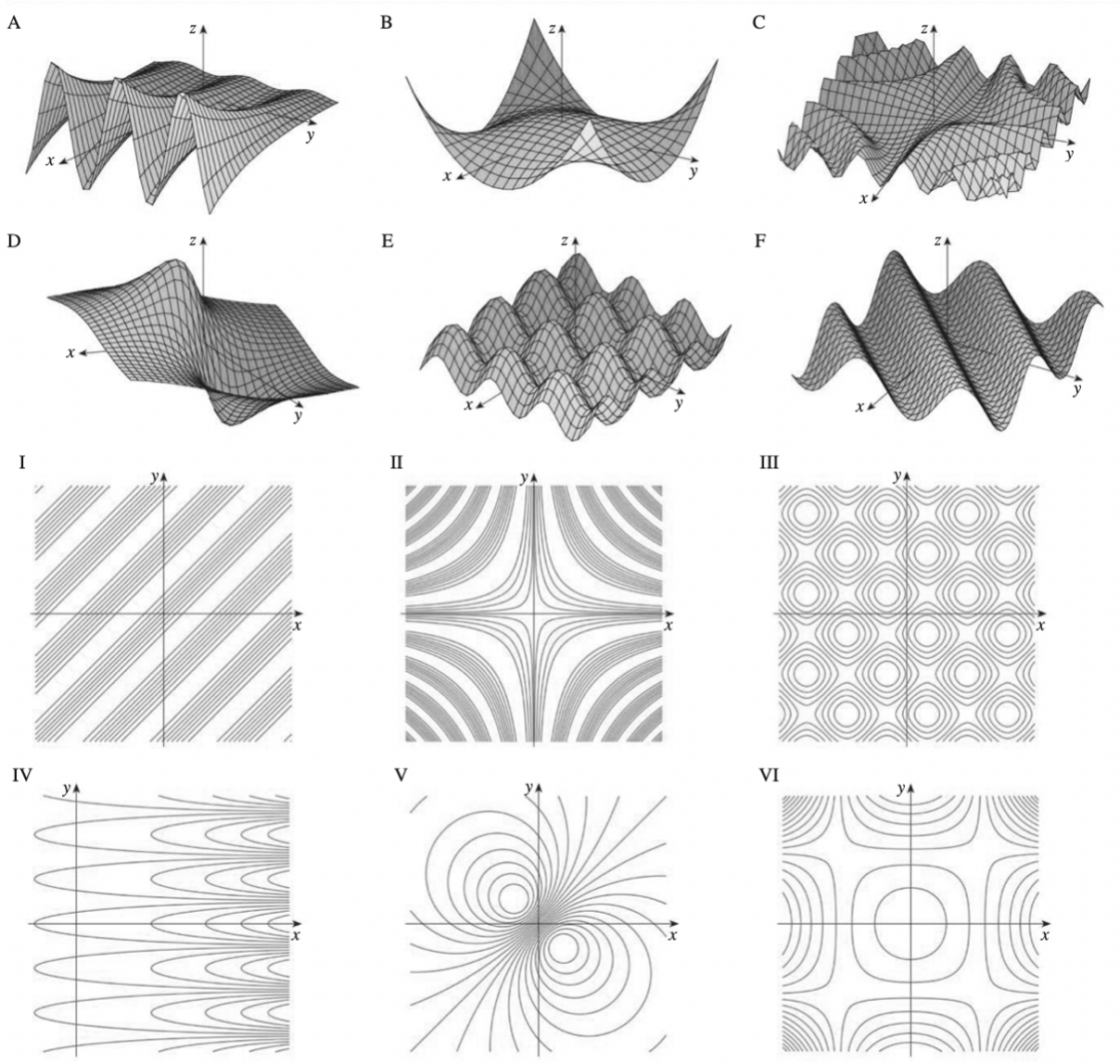
(ii) $z = e^x \cos y$

(iii) $z = (1-x^2)(1-y^2)$

(iv) $z = \sin(xy)$

(v) $z = (x-y)/(1+x^2+y^2)$

(vi) $z = \sin(x-y)$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B
C
D
E
F
I
II
III
IV
V
VI