

MATH 231 REVIEW



B Section Hoffmeister

Topics Covered

- Integration by Parts
- Trigonometric Integrals
- Trigonometric Substitution
- Integration of Rational Functions by Partial Fractions
- Approximate Integration

Integration by Parts

$$\int u dv = uv - \int v du$$

Where

$u = f(x)$	$du = f'(x)$
$dv = g(x)dx$	$v = \int g(x)dx$

Note: May have to repeat process more than once to completely solve

How to Choose “u”?

Use
LIATE!

L-ogarithmic

$\ln(x)$

I-nverse Trig

$\sin^{-1}(x)$

A-lgebraic

$x^2 + 3x$

T-rigonometric functions

$\sin(x)$

E-xponential functions

e^x

Trigonometric Integrals

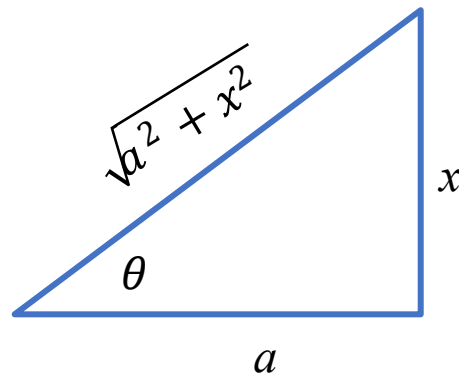
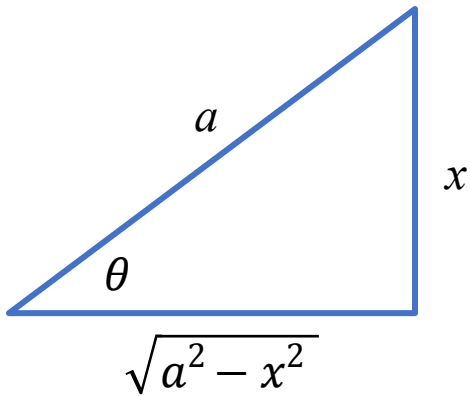
- $\cos^2(x) + \sin^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $\tan^2(x) = \sec^2(x) - 1$
- $\sin(2x) = 2 \sin(x) \cos(x)$
- $\sin^2 x = \frac{1}{2} (1 - \cos(2x))$
- $\cos^2 x = \frac{1}{2} (1 + \cos(2x))$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
 $= 1 - 2\sin^2(x)$
 $= 2\cos^2(x) - 1$

- Used the following trig identities and others to rewrite and simplify trig equations under an integral

Ex: $\int \sin(x)^3 \cos(x)^5 dx$

Trigonometric Substitution

Format	Substitution	Derivative Substitution	Trig Identity
$\sqrt{a^2 - x^2}$	$x = a * \sin(\theta)$	$dx = a * \cos(\theta) d\theta$	$\cos^2(\theta) + \sin^2(\theta) = 1$
$\sqrt{a^2 + x^2}$	$x = a * \tan(\theta)$	$dx = a * \sec^2(\theta) d\theta$	$\tan^2(\theta) + 1 = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a * \sec(\theta)$	$dx = a * \sec(\theta) \tan(\theta) d\theta$	$\tan^2(\theta) = \sec^2(\theta) - 1$



Steps to Solve:

1. Identify format
2. Replace x and dx
3. Simply and/or use trig identity
4. Convert back to numerical using triangle

Integration of Rational Functions by Partial Fractions

Rational Function	Partial Fraction
$\frac{px + q}{(x - a)(x - b)}, a \neq b$	$\frac{A}{(x - a)} + \frac{B}{(x - b)}$
$\frac{px + q}{(x - a)^2}$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2}$
$\frac{px^2 + qx + r}{(x - a)(x - b)(x - c)}$	$\frac{A}{(x - a)} + \frac{B}{(x - b)} + \frac{C}{(x - c)}$
$\frac{px^2 + qx + r}{(x - a)^2(x - b)}$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2} + \frac{C}{(x - b)}$
$\frac{px^2 + qx + r}{(x - a)(x^2 - bx - c)}$	$\frac{A}{(x - a)} + \frac{Bx + C}{(x^2 - bx - c)}$



What integration technique
would you use?

How would you solve these integrals?

1. $\int \frac{1}{\sqrt{16 + x^2}} dx$

2. $\int \sin(x)e^x dx$

3. $\int \cos^3(x)\sin^2(x) dx$

4. $\int \frac{3x + 11}{(x - 3)(x + 2)} dx$

5. $\int \ln(x) dx$

6. $\int (2x + 2)e^{x^2 + 2x + 3} dx$

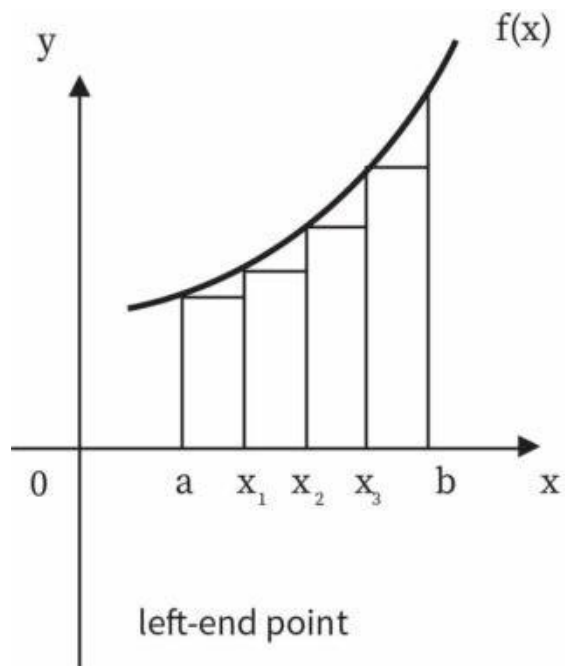
7. $\int \sec(x) dx$

8. $\int \cos(2x) dx$

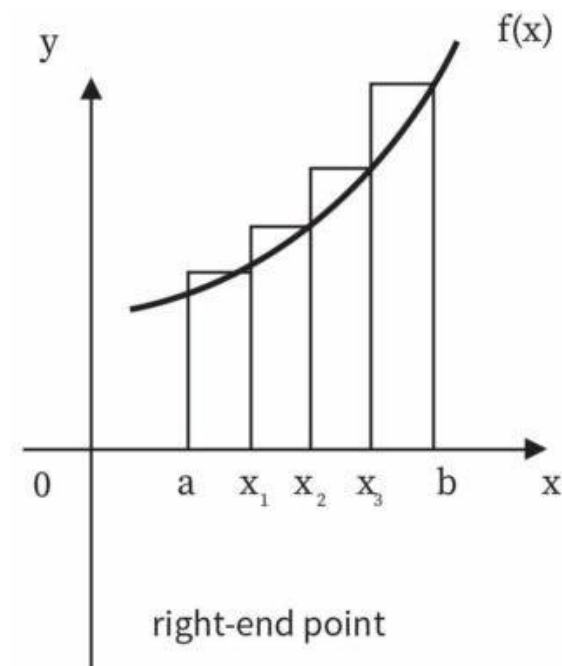
Answers

1. Trig substitution
2. Integration by parts
3. Trigonometric integrals
4. Partial fractions
5. Integration by parts
6. U-substitution
7. Integration by parts with u-sub (or known antiderivative)
8. U-substitution (or known antiderivative)

Approximate Integration



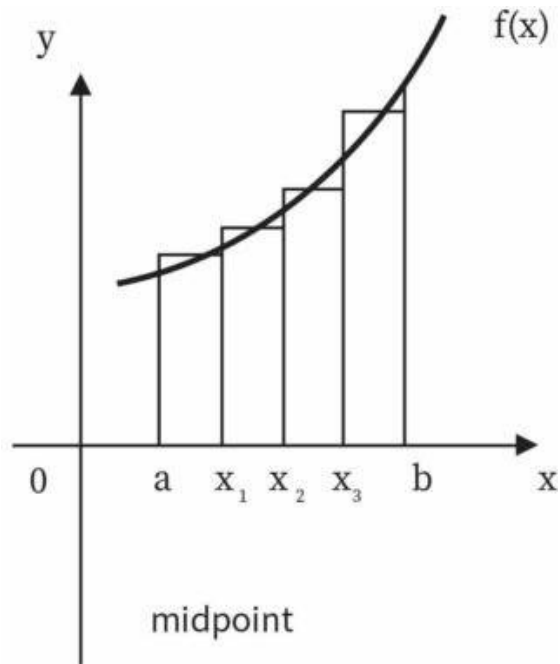
Where $\Delta x = \frac{b-a}{n}$



$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \cdots f(x_{n-1})\Delta x = \sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \cdots f(x_n)\Delta x = \sum_{i=1}^n f(x_i)\Delta x$$

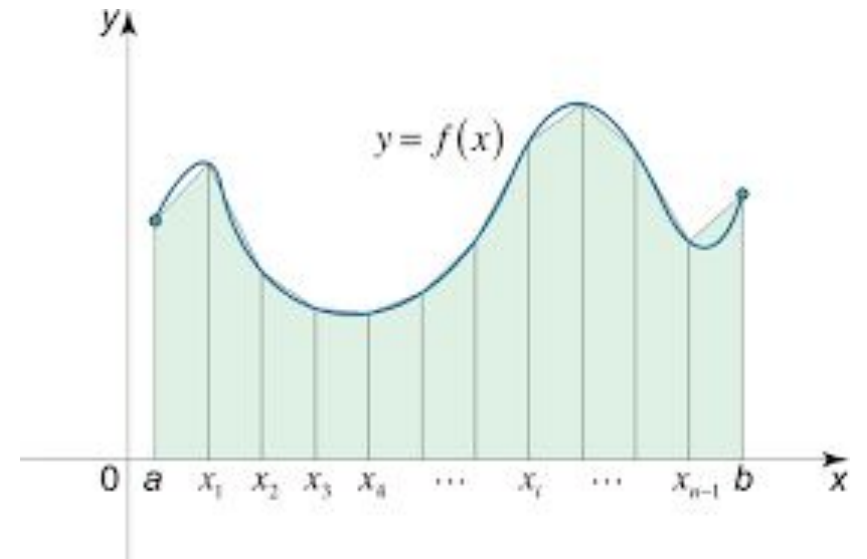
Approximate Integration



$$M_n = f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + \cdots f(\bar{x}_n)\Delta x = \sum_{i=1}^n f(\bar{x}_i)\Delta x$$

$$\text{Where } \bar{x} = \frac{x_{i-1} + x_i}{2}$$

Trapezoidal Rule



$$T_n = \frac{\Delta x}{2} (f(x_0)\Delta x + 2f(x_1)\Delta x + \cdots 2f(x_{n-1}) + f(x_n)) = \sum_{i=1}^n (f(x_{i-1}) + f(x_i)) \frac{\Delta x}{2}$$

Approximate Integration: Simpson's Rule

$$S_n = \frac{\Delta x}{3} (f(x_0)\Delta x + 4f(x_1)\Delta x + 2f(x_2)\Delta x + \cdots f(x_n)) \approx \int_0^n f(x)$$

- Has the lowest error, therefore the most accurate
- Closest of the methods to finding the actual integration or area under the curve

Questions?

