Exam 1 Review Session
Math 231E

Please join the queue for attendance!

Outline

1. Please join the queue
2. Mini review of some topics covered
3. Practice! → CARE Worksheet, Practice Exams
    a. Please raise hands for questions rather than put them in the queue

Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	12pm-2pm 6pm-8pm	2pm-4pm 6pm-8pm	8pm-10pm	3pm-7pm			2pm-6pm

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Content Review

Complex Numbers

- Numbers of the form: $z = x + yi$
- Modulus: $|z| = \sqrt{x^2 + y^2}$
- Operations:
  - Addition: $z_1 + z_2 = (x_1 + x_2) + (y_1 + y_2)i$
  - Multiplication: $z_1 z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i$ (think of FOIL)
  - Division: $\frac{z_1}{z_2} = \frac{z_1 \bar{z_2}}{|z_2|} = \frac{(x_1 + y_1 i)(x_2 - y_2 i)}{x_2^2 + y_2^2}$
- Euler's Theorem: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

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Taylor Series

- Taylor series help approximate complicated functions into polynomials that are easier to evaluate

$$f(a)+f'(a)(x-a)+f''(a)\frac{(x-a)^2}{2!}+f'''(a)\frac{(x-a)^3}{3!}+...+f^{(n)}(a)\frac{(x-a)^n}{n!}$$

- a is the value our Taylor series is evaluated at
    - Maclaurin series → a = 0

- Big O Notation: indicates which term of the Taylor series is being “cut-off”
    - If we evaluate around a certain value, then any term put into the Big O Notation is insignificant to the polynomial overall

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Common Functions to Use Taylor Series

Function	**Taylor Series**
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$
$\cos(x)$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots + \frac{x^{2n}}{(2n)!}$
$\sin(x)$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots + \frac{x^{2n+1}}{(2n+1)!}$
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n$
$\ln(1+x)$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{(n+1)} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^n \frac{x^{n+1}}{(n+1)}$

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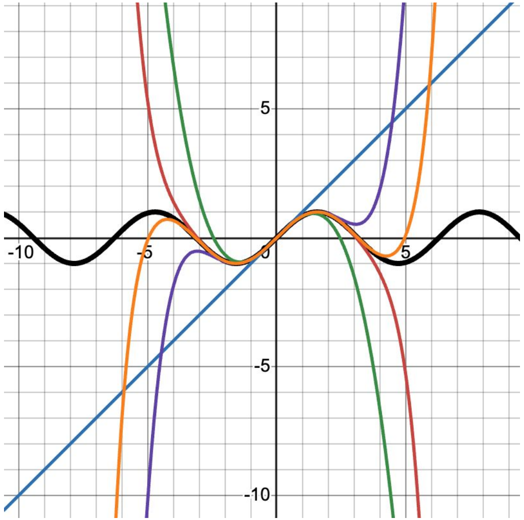
Uses and Applications of Taylor Series



1) Deriving other Taylor Series from the most common
    a) Substitution
    b) Derivatives
    c) Integrals

2) Evaluating limits with Taylor Polynomials versus the initial functions

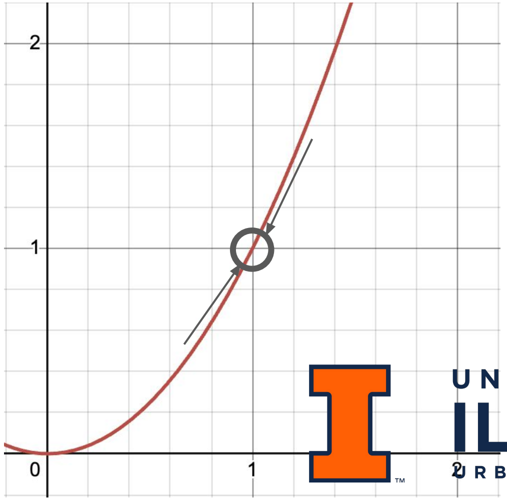
3) Determining convergence or divergence of integrals and series → later in the course

Limits

- As we approach closer and closer to some value "a" from both the left and right hand side, the function gets closer and closer to "L":

$$\lim_{x \to a} f(x) = L$$

- Epsilon-delta definition: For any $\epsilon > 0$, there is a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.
  - a → "target input"
  - Delta → "allowable deviation from the target input"
  - Epsilon → "output tolerance"
  - L → "target output value"



- Infinite Limits: $\lim_{x \to \infty} f(x) = \lim_{x \to 0^+} f(\frac{1}{x})$

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Limit Laws

Operations

- **Addition:** $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- Subtraction: $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
- **Multiplication:** $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
- Division: $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (given the limit of the denominator is not 0)
- **Scaling by a constant:** $\lim_{x\to a}(cf(x)) = c\lim_{x\to a} f(x)$
- Exponentiating: $\lim_{x\to a}(f(x)^n) = (\lim_{x\to a} f(x))^n$

Functions

- **Constant:** $\lim_{x\to a} c = c$
- **Linear:** $\lim_{x\to a} x = a$
- **Power:** $\lim_{x\to a} x^n = a^n$
- Root: $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

These laws can be combined to make finding limits easier!

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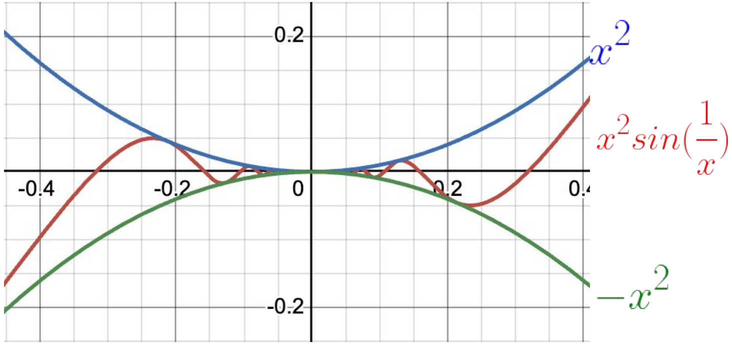
Squeeze Theorem

- We have three functions such that near x: $f(x) \leq g(x) \leq h(x)$

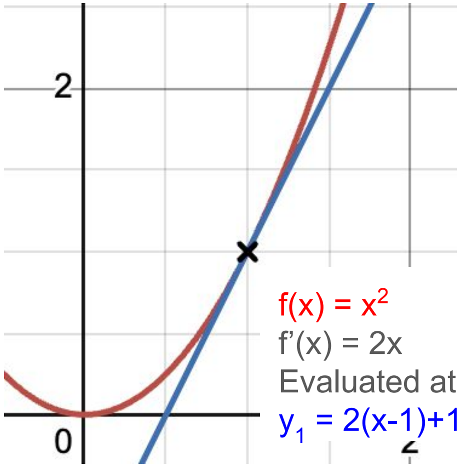
- If $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$

- Great to use for functions that are hard to evaluate with limit laws



Derivatives

- Interpretation: the derivative of a function f(x) at *a* represents the instantaneous rate of change at *a*
  - Slope of tangent line at *a*: $y_a = f'(a)(x-a) + f(a)$
- Power Rule: $\frac{\mathrm{d}}{\mathrm{d}x} x^n = n x^{n-1}$
- Product Rule: $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **Chain Rule:** $\frac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = f'(g(x)) \cdot g'(x)$ (unwrap the layers of the function)



$f(x) = x^2$

$f'(x) = 2x$

Evaluated at x=1

$y_1 = 2(x-1)+1$

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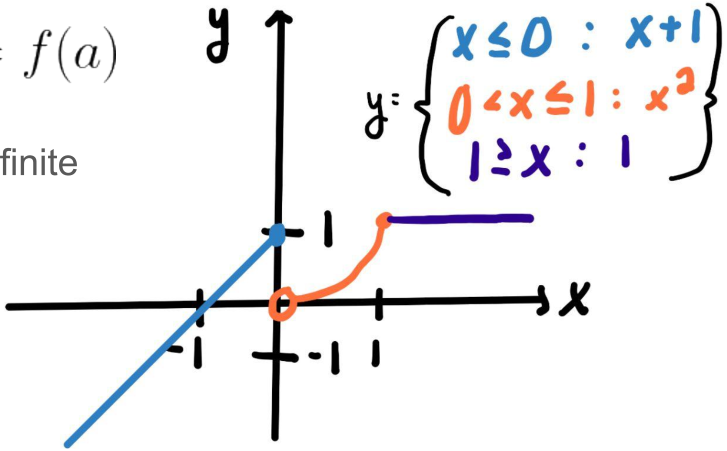
Continuity, Discontinuities, and Intermediate Value Theorem

- A function is continuous at *a* if $\lim_{x \to a} f(x) = f(a)$
    - Types of discontinuities: removable/point, jump, infinite



- Intermediate Value Theorem: If a function f(x) is continuous on a closed interval [a,b] where f(a) and f(b) are different, there is a value z between f(a) and f(b) with some value c such that a < c < b and f(c) = z

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Mean Value Theorem

If:

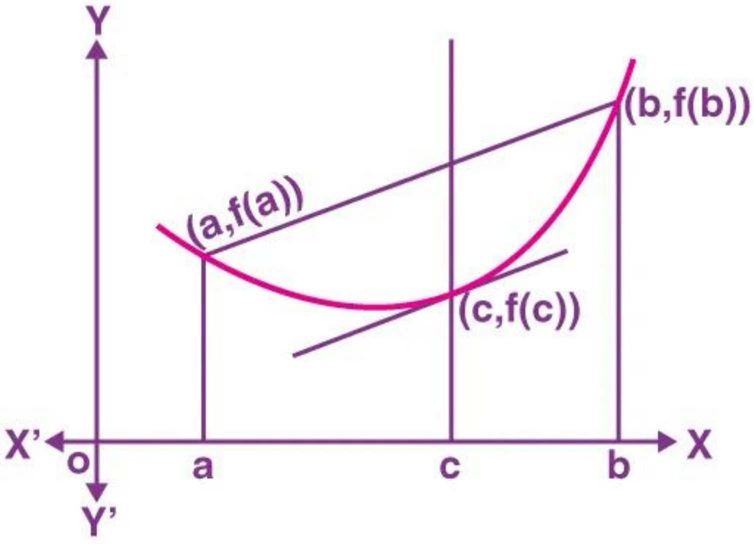
1. *f(x)* is continuous [*a*,*b*]
2. *f(x)* is differentiable (*a*,*b*)

Then:

There is a number *c* such that *a*<*c*<*b* and

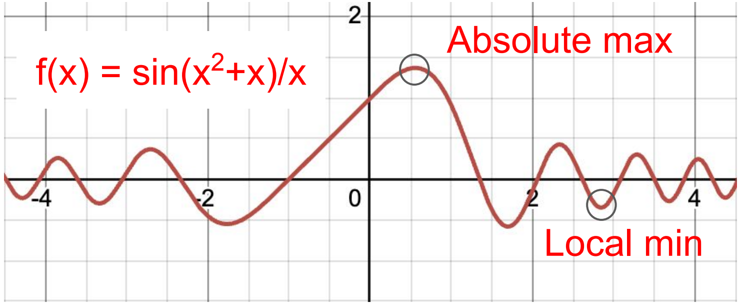
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(b) - f(a) = f'(c)(b - a)$$



Minima and Maxima

- Absolute minima/maxima: smallest/largest outputs a function can produce on an interval
- Local minima/maxima: smallest/largest outputs of a function around a certain point



- Extreme Value Theorem: If **f(x) is continuous on [a,b]**, then f achieves both an **absolute maximum and absolute minimum on [a,b]**
- Fermat's Theorem: If f(x) has a **local minimum/maximum at *c*** and **f' exists** at *c*, then **f'(c) = 0**
  - *We call c a critical point*

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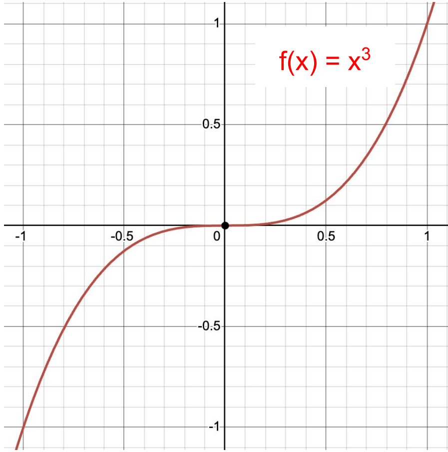
Finding Extreme Values



1. Take the first derivative and set it equal to zero
    a. Solve for all the critical points

2. Plug each critical point back into f(x) and see which is the largest/smallest

-or-

2. Take the second derivative
    a. If f''(c) is > 0 → function is concave up → local minimum
    b. If f''(c) is < 0 → function is concave down → local maximum
    c. If f''(c) = 0 → inconclusive

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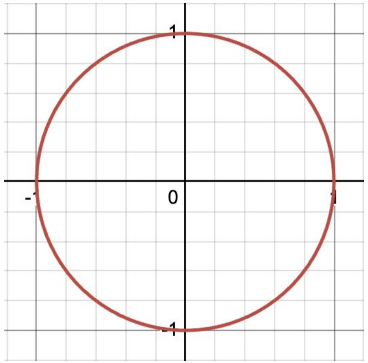
Implicit Differentiation and Differentials

Implicit Differentiation

- Used when functions are defined in terms of both x and y
- Take the derivative of everything with respect to x
  - What is $\frac{\mathrm{d}x}{\mathrm{d}x}$? What is $\frac{\mathrm{d}y}{\mathrm{d}x}$?



Differentials

- For a small change in input Δx, the function will change proportionally to the rate of change:

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Good luck on your exam!

You can use the rest of the time to practice

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