# CARE
# MATH 221 MIDTERM 1 REVIEW

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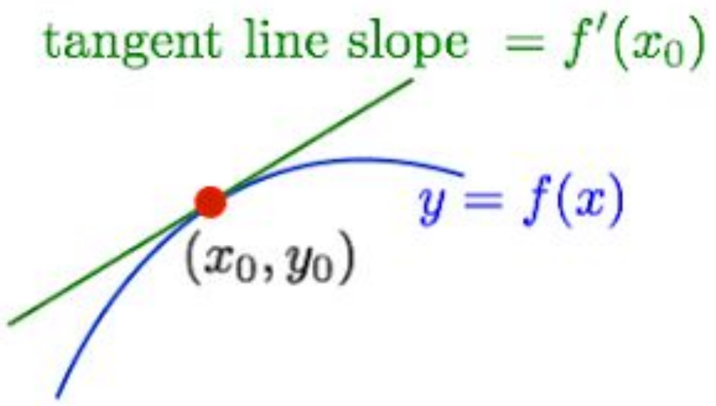
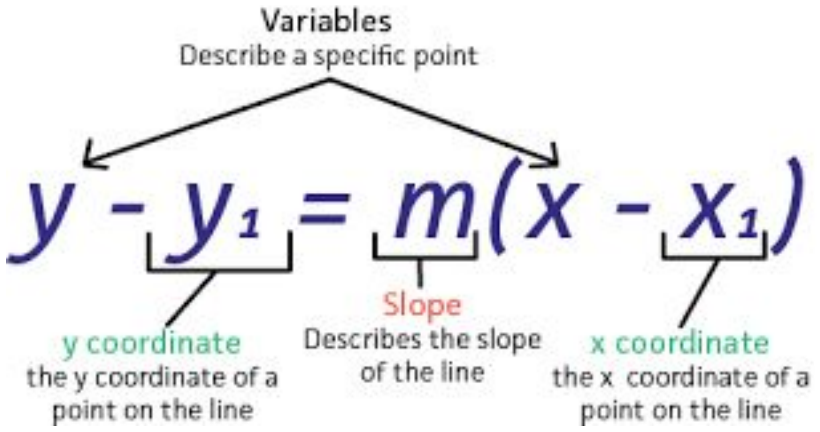
# Slopes of Tangent Lines

**AKA:** The derivative at a certain point along a curve (function)




Point Slope Form:



# Limits

$$\lim_{x \to a} f(x) = L$$

- function: $f(x)$
- $x \to a$: "As you approach $a$ along the x-axis"
- $L$: "What is the y-value getting closer to?"

## Limit and Continuity

### Existence of a Limit

The limit of a function *f(x)* exists if and only if the one sided limits of the function are equal.

$$\lim_{x \to c} f(x) = L$$ **if and only if**

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

## Calculating $\lim_{x \to a} f(x)$

**A. Direct substitution**
Try to evaluate the function directly.

$f(a)$

- $f(a) = \frac{b}{0}$, where ***b*** is not zero
  **B. Asymptote (probably)**
  example:
  $$\lim_{x \to 1} \frac{1}{x-1}$$
  Inspect with a graph or table to learn more about the function at ***x=a***.
- $f(a) = b$, where ***b*** is a real number
  **C. Limit found (probably)**
  example:
  $$\lim_{x \to 3} x^2 = (3)^2 = 9$$
- $f(a) = \frac{0}{0}$
  **D. Indeterminate form**
  example:
  $$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$

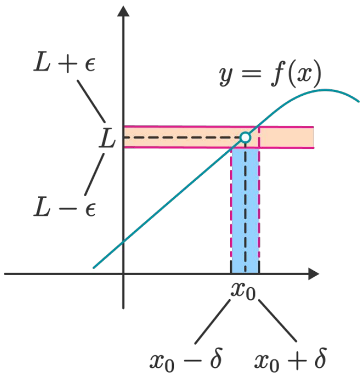
Try rewriting the limit in an equivalent form.

- **E. Factoring**
  example:
  $$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$
  can be reduced to
  $$\lim_{x \to -1} \frac{x-2}{x-3}$$
  by factoring and cancelling.
- **F. Conjugates**
  example:
  $$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$
  can be rewritten as
  $$\lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$
  using conjugates and cancelling.
- **G. Trig identities**
  example:
  $$\lim_{x \to 0} \frac{\sin(x)}{\sin(2x)}$$
  can be rewritten as
  $$\lim_{x \to 0} \frac{1}{2\cos(x)}$$
  using a trig identity.

Try evaluating the limit in its new form.

**H. Approximation**
When all else fails, graphs and tables can help approximate limits.

# Delta-Epsilon Definition




| Definition | Translation |
| --- | --- |
| 1. For every $\varepsilon > 0$, | 1. For every positive distance $\varepsilon$ from $L$, |
| 2. there exists a $\delta > 0$, | 2. There is a positive distance $\delta$ from $a$, |
| 3. such that | 3. such that |
| 4. If $0 < \lvert x-a \rvert < \delta$, then $\lvert f(x)-L \rvert < \varepsilon$. | 4. If $x$ is closer than $\delta$ to $a$ and $x \neq a$, then $f(x)$ is closer than $\varepsilon$ to $L$. |

Let $f(x)$ be a function defined on the interval that contains $x = a$.

Then $\lim_{x \to a} f(x) = L$ if for every number $\varepsilon > 0$ there exists some real number $\delta > 0$ so that if

$$0 < \lvert x - a \rvert < \delta \text{ then } \lvert f(x) - L \rvert < \varepsilon$$

# Limit Laws

**properties of limits**

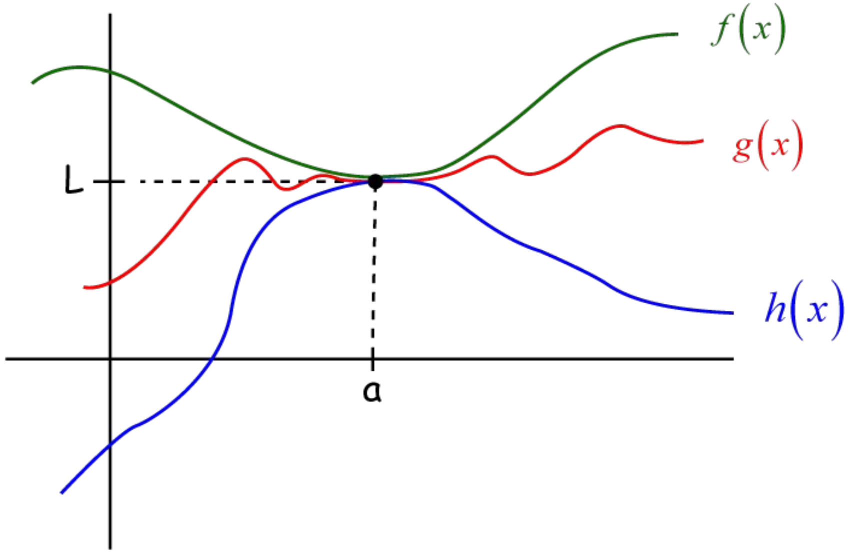
Let $a, k, A,$ and $B$ represent real numbers, and $f$ and $g$ be functions, such that $\lim\limits_{x\to a} f(x) = A$ and $\lim\limits_{x\to a} g(x) = B$. For limits that exist and are finite, the properties of limits are summarized in Table

| | |
|---|---|
| Constant, *k* | $\lim\limits_{x\to a} k = k$ |
| Constant times a function | $\lim\limits_{x\to a}[k \cdot f(x)] = k \lim\limits_{x\to a} f(x) = kA$ |
| Sum of functions | $\lim\limits_{x\to a}[f(x) + g(x)] = \lim\limits_{x\to a} f(x) + \lim\limits_{xtoa} g(x) = A + B$ |
| Difference of functions | $\lim\limits_{x\to a}[f(x) - g(x)] = \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} g(x) = A - B$ |
| Product of functions | $\lim\limits_{x\to a}[f(x) \cdot g(x)] = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x) = A \cdot B$ |
| Quotient of functions | $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{A}{B}, B \neq 0$ |
| Function raised to an exponent | $\lim\limits_{x\to a} [f(x)]^n = \left[\lim\limits_{x\to a} f(x)\right]^n, \quad \text{where } n \text{ is any real number}$ |
| *n*th root of a function, where n is a positive integer | $\lim\limits_{x\to a} f(x) \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a}[f(x)]} = \sqrt[n]{A}$ |
| Polynomial function | $\lim\limits_{x\to a} p(x) = p(a)$ |

# Squeeze Theory

If $h(x) \le g(x) \le f(x)$ when $x$ is near $a$, except possibly at $a$,

and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$.



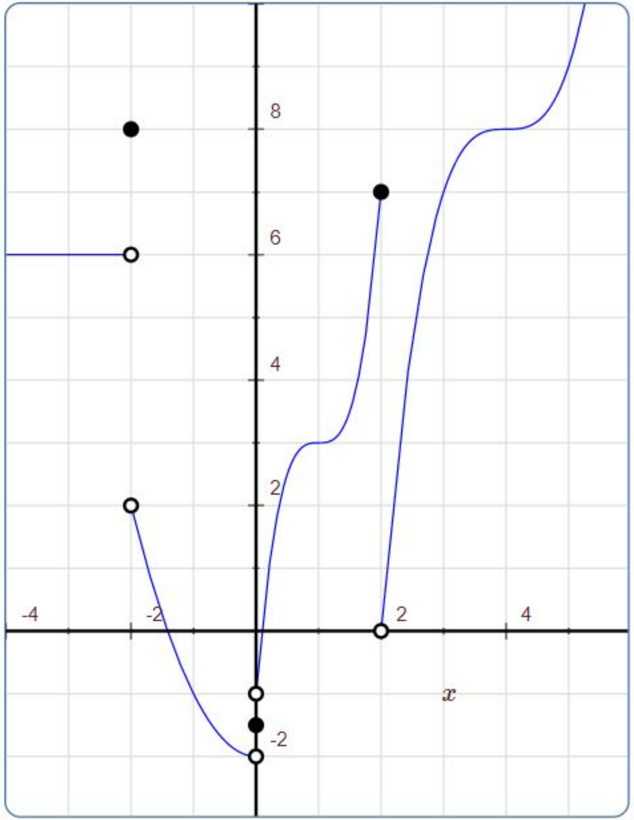
# Criteria for Continuity

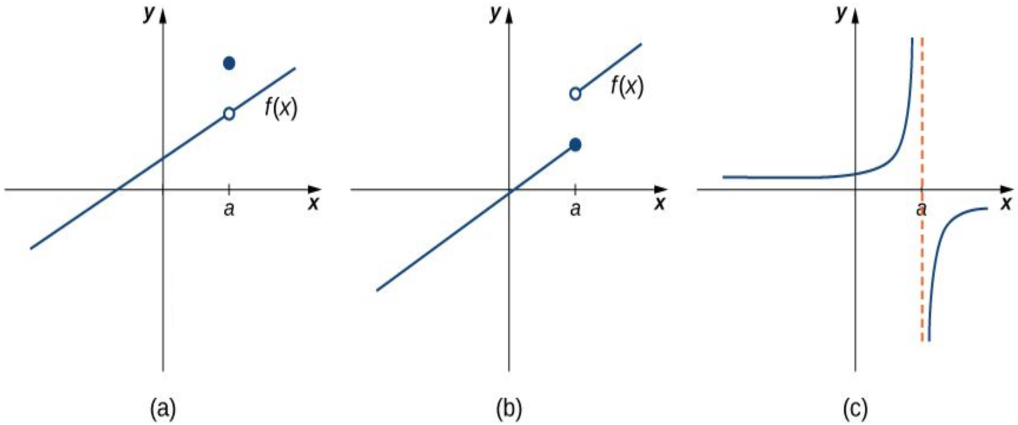
## Continuity

A function is continuous at c when the following three conditions are met:

1. *f(c)* is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

# Types of Discontinuities



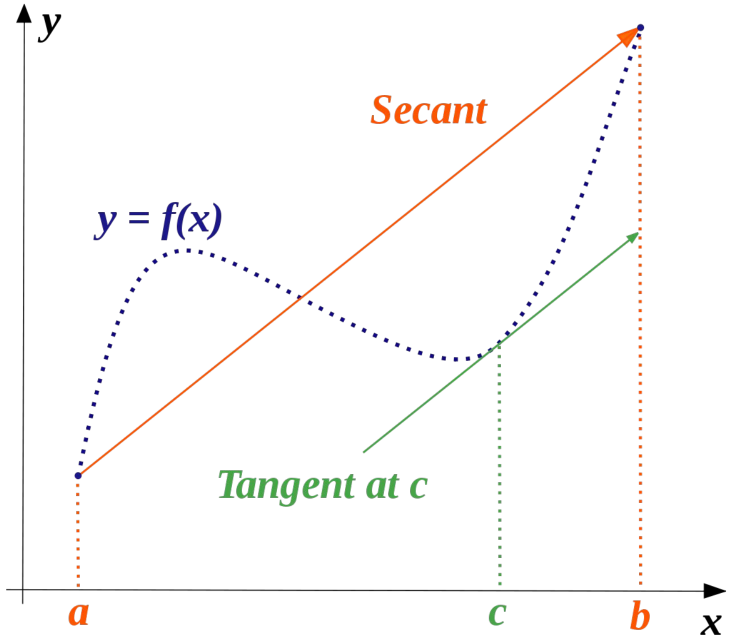
# Limit Definition of a Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

# IVT

## 10 The Intermediate Value Theorem

Suppose that $f$ is continuous on the closed interval $[a, b]$ and let $N$ be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number $c$ in $(a, b)$ such that $f(c) = N$.



# DIFFERENTIABILITY

## Definition of Differentiability

**25.1 DEFINITION** Let $f$ be a real-valued function defined on an interval $I$ containing the point $c$. (We allow the possibility that $c$ is an endpoint of $I$.) We say that $f$ is **differentiable** at $c$ (or has a **derivative** at $c$) if the limit

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

exists and is finite. We denote the derivative of $f$ at $c$ by $f'(c)$ so that

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

whenever the limit exists and is finite. If the function $f$ is differentiable at each point of the set $S \subseteq I$, then $f$ is said to be differentiable on $S$, and the function $f' : S \to \mathbb{R}$ is called the derivative of $f$ on $S$.

# Differentiation

| | |
|---|---|
| Sum rule | $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ |
| Difference rule | $\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$ |
| Constant multiple rule | $\frac{d}{dx}[k \cdot f(x)] = k \cdot \frac{d}{dx}f(x)$ |
| Constant rule | $\frac{d}{dx}k = 0$ |

# Differentiation Rules

$$\frac{d}{dx} c = 0$$ Constant Rule

$$\frac{d}{dx} x^n = nx^{n-1}$$ Power Rule

$$\frac{d}{dx} \sin(x) = \cos(x)$$

Trigonometric Rules

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} b^x = b^x \ln(b)$$ Exponential Rule

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$ Logarithmic Rule

# Product and Quotient Rule

**Product Rule**

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

**Quotient Rule**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

# Trig Derivatives

## Trig Derivatives

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \tan x \sec x$$

$$\frac{d}{dx}\csc x = -\cot x \csc x$$

**Commit these to memory!!**

They will show up a lot throughout the calculus sequence

## Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \frac{d}{dx}\operatorname{arccot} u = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

# Chain Rule

$$\frac{d}{dx}\left[\left(f(x)\right)^{n}\right] = n\left(f(x)\right)^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right)g'(x)$$

# Implicit Differentiation

$$y^5 + 2y = x^2$$

Take the derivative of each term

# Logarithmic Differentiation

## General Workflow

1. Take the natural log of both sides
2. Take the derivative of both sides in terms of X
3. Get dy/dx onto one side
4. Plug in the original equation for any y's in the equation

| | |
|---|---|
| $y = \sin x^{2x}$ | |
| $\ln y = \ln(\sin x)^{2x}$ | *Take the natural log of both sides of the equation* |
| $\ln y = 2x \cdot \ln(\sin x)$ | *Apply logarithmic property* $\log_a M^N = N \log_a M$ |
| $\frac{d}{dx}[\ln y] = \frac{d}{dx}[2x \cdot \ln \sin x]$<br>$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \cdot \left(\frac{\cos x}{\sin x}\right) + \ln(\sin x) \cdot (2)$<br>$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \cot x + 2\ln(\sin x)$ | *Differentiate implicitly* |
| $\frac{dy}{dx} = y(2x \cot x + 2\ln(\sin x))$ | *Solve for* $\frac{dy}{dx}$ |
| $\frac{dy}{dx} = \sin x^{2x}(2x \cot x + 2\ln(\sin x))$ | *Substitute* $y = \sin x^{2x}$ *into final answer* |

# Pre-worksheet

## Practice

***Example 3*** Find the derivative of the following function using the definition of the derivative.

$$R(z) = \sqrt{5z - 8}$$

***Show Solution*** ▸