Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Phys 214 – University Physics: Quantum Physics

Quiz 1 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Wednesday, Sept. 10, 7-9 PM: Alex, Camille, Luke

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

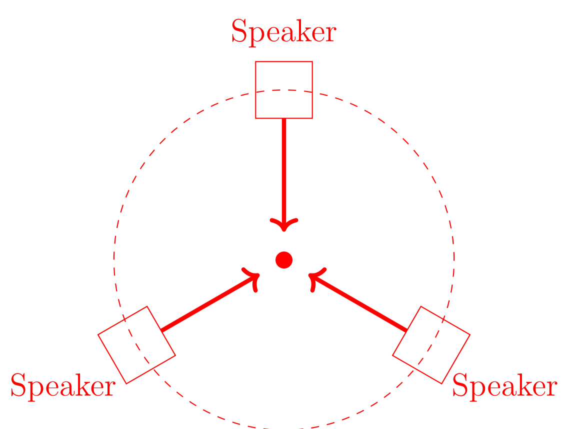
Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

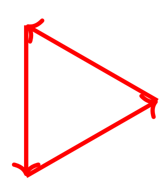
1. In a photoelectric effect demonstration, the intensity of the incident light is gradually increased, but no photocurrent is detected. Provide an explanation for this result.
   It's the energy of a photon, not intensity, that increases the kinetic energy of the photoelectron (more specifically, the *frequency* of a photon). Since no current is detected, the energy of the incoming photons is less than the work function of the material, so no electrons escape. The frequency must be increased to change this.

2. Three speakers lie on the perimeter of a circle. The sound intensity at each source is I_0 while the total intensity at the center of the circle is observed to be zero. Use phasors to determine the relative phase shift of each speaker such that this is possible.
   All three speakers are equidistant from the circle's center. This means that any phase angle between waves is a result of the sources being out of phase, not the path difference. The phasor diagram of this system must be an equilateral triangle so that the sum of the angles between the phasors adds up to 360° (the phase between each speaker can be deduced to be $360/3 = 120°$, 180 - 120 = 60 °).

   Set-up:



   Phasor Diagram:



3. A laser with time-varying frequency is directed at a barrier with a narrow slit followed by a screen. Assuming the laser intensity is constant, as the frequency increases, how does the number of photons per second arriving at the screen change?
   The number of photons arriving at the screen will decrease. Using dimensional analysis:

$$\frac{\text{photons}}{\text{second}} * \frac{\text{joules}}{\text{photon}} = \frac{\text{joules}}{\text{second}}$$

The right hand side is proportional to the intensity (W/m^2), which is held constant. So if the frequency increases, the energy per photon increases, and the left hand side increases (J/photon). To keep intensity constant, the number of photons per second must decrease

4. A laser is directed at a barrier with a narrow slit followed by a screen. Applying a small angle approximation, if the slit width is halved while the wavelength is doubled, by what factor does the location of the first diffraction minimum change? Use small angle approximation.
   Let Δy be the distance from the center to the first minimum of the bright spot. Using small angle approximation:
$$\frac{\Delta y}{L} = \tan(\theta) \approx \theta$$
   Then, using the fact that $\lambda = a\sin(\theta)$, as well as small angle approximation again:
$$\lambda = a\sin(\theta) \approx a\theta \approx a\Delta y/L$$
$$\text{Thus } \delta y = \lambda L/a$$
   So halving the slit width while doubling the wavelength results in the location of the first minimum increasing by a factor of four.

5. A single slit diffraction experiment is set up such that the central bright spot is 10 cm in width, and the screen is 3 m away from the slit. Using light with wavelength of 900 nm, calculate the slit spacing a.
   The full width of the central bright spot is 10 cm in width, meaning the distance from y = 0 to the first minima is 10/2 = 5 cm. Plugging this into our equation for θ:
$$\frac{y}{L} = \tan\theta \implies \theta \approx 0.05/3 = 1.67 \times 10^{-2} \text{ radians}$$
   Solving for the slit width, we convert 900 nanometers to m and get
$$\frac{\lambda}{\sin\theta} = \frac{900 \times 10^{-9}\text{m}}{0.0167} = a = 53.89\mu\text{m}$$

6. Following up on the previous question, calculate the position of the **maximum** of the single-slit diffraction problem.
   It's just $y = 0$! (Central maximum)

7. A wave is traveling along the positive x-axis with speed 5 meters per second. Which equation could describe this wave?

a) $\sin(12x + 2.6t)$

b) $3\cos(x - 0.3\pi t)$

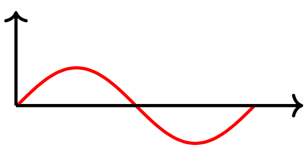
c) $12\cos(x - 5t)$

d) $\sin^2(x + t - \pi)$

The answer is **(c)**. A harmonic wave traveling in the $+\hat{x}$ direction has the general form

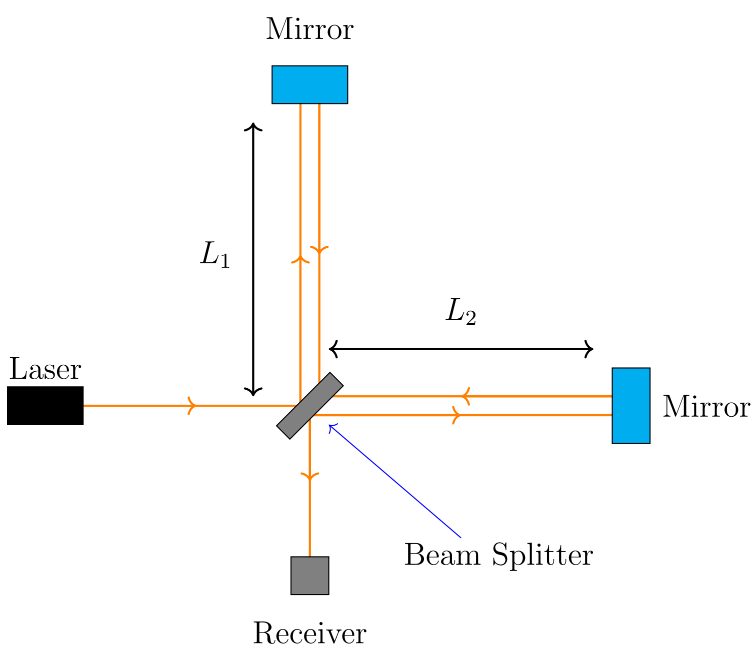
$$A\cos(kx - \omega t + \phi)$$

The velocity of such a wave is given by ω/k. Among the five candidate wave equations, the only choice that has this ratio equaling 5 is C.
To think about the direction of the wave, let's take a look at the general form (same as above).



Let's consider a test point, the max of the wave. If we press play and let our sine wave move in time, in order to stay at the maximum, we need to keep the argument in the sine term constant. To achieve this effect, we need to increase position. Thus, we can see that any wave with differently signed k and ω should be moving in the positive direction, while similarly signed ks and ωs move in the negative direction.

8. An interferometer has two arms of equal length ($L_1 = L_2$). A 200 W laser with wavelength 1064 nm is directed at the central beam splitter (shown below).



What is the minimum increase in L_2 required so that only 100 W goes to the detector? (Note that when the arms are of equal length, the detector receives 200 W).

a) 266 nm

b) 133 nm

c) 532 nm

The answer is **(b)**.
Relevant equation:

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where I_0 is the intensity from a single source. Since the beam goes into the splitter the amplitude is halved, so the intensity is divided by 4 (remember that amplitudes add/subtract, NOT intensity/power!), which is 50 W. Using this, combined with the knowledge that we need the final intensity to be 100 W, we can solve for ϕ, the phase angle.

The equation yields $\phi = \frac{\pi}{2}$. Plugging this into the phase angle equation

$$\phi = k(r_2 - r_1) = k\delta = \frac{2\pi\delta}{\lambda}$$

we get that the minimum distance, δ, is 266 nm. Taking into account the fact that the beam bounces off the mirror, the distance it travels along that path is doubled. So we get that the minimum distance we need to move the mirror is 133 nm.

9. A wave propagating through the ocean is measured by a sensor and can be described by the equation $f(x,t) = \cos(0.4x - 2t)$.

   (i) What is the wavelength, frequency and amplitude of the wave?

   (ii) In which direction is the wave traveling?

   a) $-x$

   b) $+x$

   c) The direction is time dependent

(i) $\lambda = 15.71m$, $f = 0.318Hz$, $A = 1$
The general harmonic wave equation is

$$A\cos(kx - \omega t + \phi)$$

This can be used with $f = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ to get the frequency and wavelength while the amplitude is the coefficient A.

(ii) The answer is **(b)**. As we increase time, the value of x must also increase for the argument of the wave equation to stay constant.

10. A spacecraft is being pushed by a laser of wavelength 400 nm emitting photons at a rate of 10^{22} photons per second. Calculate the acceleration of the spacecraft given its mass is 4000 kg. Values are given in meters per second.

a) 3.25×10^{-6}
b) 1.53×10^{-3}
c) 4.14×10^{-9}
d) 1.7×10^{-5}
e) 5.5×10^{-4}

The answer is **(c)**. The photon's momentum can be found by

$$p = \frac{h}{\lambda}$$

which can be multiplied by the rate at which photons are emitted, R_γ to give us the recoiling force by Newton's second law:

$$F = \frac{dp}{dt} = pR_\gamma \equiv \frac{\text{momentum}}{\text{photon}} * \frac{\text{photons}}{\text{second}} = \frac{\text{momentum}}{\text{sceond}} = \text{Newtons}$$

This force, divided by the ship's mass giving us the acceleration

11. Light with wavelength 100 nm is incident on a metal. The speed of the ejected photoelectrons is measured to be 10^6 meters per second. Find the work function of this metal.

a) 1.99×10^{-18}
b) 1.53×10^{-18}
c) 4.55×10^{-18}

The answer is **(b)**. Through conservation of energy, we know that

$$hf = \frac{1}{2}mv^2 + \Phi$$

The frequency of this light can be found by dividing the speed of light by its wavelength

$$f = \frac{3 * 10^8}{100 * 10^{-9}} = 3 \times 10^{15} \text{ Hz}$$

Multiplying this with Planck's constant h gives us the total energy of the incoming photons:

$$1.988 \times 10^{-18} \text{ Joules}$$

The kinetic energy of the ejected electrons is

$$\frac{1}{2}mv^2 = 4.55 \times 10^{-19} \text{ Joules}$$

The difference in these values given the work function of this metal, using the above conservation equation.

$$\boxed{1.988 \times 10^{-18} - 4.55 \times 10^{-19} = 1.52 \times 10^{-18} \text{ J}}$$

12. An interferometer with equal arm lengths is sourced by a laser of wavelength 700 μm. If the length of one arm is increased by 0.12 mm, by what amount are the waves out of phase?
To convert from a difference in distance to a difference in phase, we can use

$$\Delta\phi = k(r_2 - r_1) = \Delta r$$

It may be tempting to use 0.12 mm as Δr, but we should use 0.24 mm instead. This is because when we increase the arm length by a distance Δx, the light beam path length increases by *twice* that distance (i.e. it must travel to the mirror and back), so we use $\Delta r = 2\Delta x$.

$$\Delta\phi = k\Delta r = \frac{2\pi\Delta r}{\lambda} \rightarrow \Delta r = 0.24 \times 10^{-3} \text{ m}$$

$$\boxed{\Delta\phi = 2.15 \text{ radians}}$$

13. Continuing from the previous question, assuming that the intensity received at the detector was 4 W/m^2 when the arm lengths were equal, what is the new intensity?
Values are given in W/m^2

a) 2.95
b) 0
c) 0.898
d) 1.21
e) 4

The answer is **(c)**. Since the detector received 4 W/m^2 when the arm lengths were equal, we know that the source intensity is 4. We established the phase difference between these two waves in the previous question to be 2.15 radians. Using the equation

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

we can determine the new intensity in this interferometer. Remember that I_0 in this case is 1 because the beam is first split from 4 to 2, then 2 to 1 after the light passes the beam splitter a second time.

14. The distance to the first minimum of a circular diffraction pattern is found to be 0.012 cm from the center. Assuming the distance to the screen is 10 mm and the diameter of the opening is 200 μm, what is the wavelength of the light used? Values are given in μm

a) 1.97
b) 2.28

c) 0.94

The answer is **(a)**. For a circular diffraction pattern, the equation

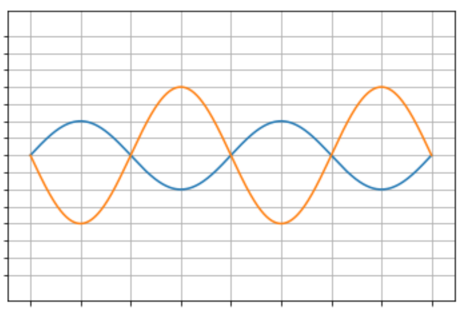
$$1.22\lambda = D\sin\left(\arctan\left(\frac{y}{L}\right)\right)$$

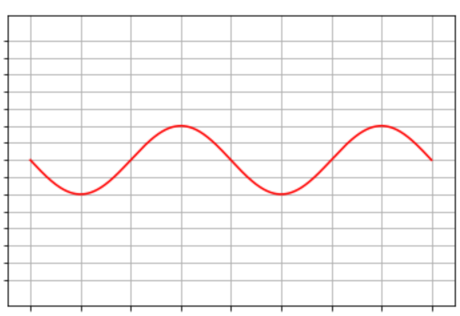
is used. However, a small angle approximation would be appropriate here since the angle is really small.

$$\lambda \approx \frac{Dy}{1.22L} = \frac{200\times10^{-6}(0.012\times10^{-2})}{1.22(10\times10^{-3})} = 1.97\times10^{-6}\text{ m}$$

Once converted to micrometers we get 1.97 μm

15. Below are two waves undergoing interference. Sketch the resulting waveform and determine the amplitude. What would the amplitude of this wave be if interference was constructive?





The resulting waveform will be the sum of the interfering waves. The amplitude is $4-2=2$. If we changed this so that the two were interfering constructively, the amplitude would be $4+2=6$.

16. A material with work function $\Phi = 3.4$ eV has a laser beam with $\lambda = 200$ nm and power $P = 2.3\times10^{-4}$ W.

a) Calculate N_γ, the number of photons hitting the material per second.

b) Calculate the energy E_{e^-}, the maximum energy of each ejected electron.

c) Say we have a device that detects the power of the ejected electrons. Calculate the maximum power P this device could measure (assuming every photon ejects an electron, and each electron has maximum energy).

a) $\boldsymbol{N_\gamma = 2.319 \times 10^{14}}$ photons/s. We know the power, and the energy of each photon is equal to $hc/\lambda = (1240 \text{ eV} \cdot \text{nm})/(200 \text{ nm}) = 6.2 \text{ eV} = 9.92 \times 10^{-19}$ J. Performing units analysis:

$$N_\gamma = \frac{2.3 \times 10^{-4} \text{ J}}{\text{s}} \cdot \frac{\text{photon}}{9.92 \times 10^{-19} \text{ J}} = 2.319 \times 10^{14} \text{ photons/s}.$$

b) $\boldsymbol{E_{e^-} = 2.8}$ eV. The energy of each photon is equal to 6.2 eV. Subtracting the work function gives $E_{e^-} = 2.8$ eV.

c) $\boldsymbol{1.039 \times 10^{-4}}$ W. If each photon is ejecting an electron of maximum energy, then 2.319×10^{14} electrons/s are being ejected, each with energy 2.8 eV. Multiplying these values yields $P = 6.492 \times 10^{14}$ eV/s $= 1.039 \times 10^{-4}$ W.

17. A bird watcher wants to take a picture of two red birds across a lake. The lake is 1.5 km long, and the birds are 10 cm apart. The wavelength of red light the bird emit is $\lambda = 700$ nm. Approximating the image of the birds as two bright spots, calculate the minimum aperture diameter D required to be able to distinguish the two birds as separate spots.
$\boldsymbol{D = 1.3}$ **cm**. If we want to be able to distinguish the bright spots, then the center of one bright spot must have a greater value of θ than the first intensity minimum $\theta_\circ$ of the other (i.e. the center of one bright spot cannot overlap with another bright spot).
We use the equation

$$D \sin(\theta_\circ) = 1.22\lambda$$

which represents the first 0 intensity angle of one bright spot. We want $\theta_\circ \leq \theta$, θ being the angle between the two birds.
We know $\tan(\theta) = 10 \text{ cm}/1.5 \text{ km} = 6.67 \times 10^{-5} \approx \theta$ using small angle approximation. Plugging in this angle into the circular aperture equation:

$$D \sin(\theta) \approx D\theta = 1.22\lambda$$
$$D = \frac{1.22\lambda}{\theta}$$
$$D = \frac{1.22(700 \text{ nm})}{6.67 \times 10^{-5}} = 0.0128 \text{ m} \approx 1.3 \text{ cm}$$

18. Express the Law of Cosines formula (listed below) in terms of Intensities I_1, I_2, and I_{tot}:

$$A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A_{tot}^2$$

$2I_1 + 2I_2 + 4\sqrt{I_1I_2}\cos\phi = 2I_{\text{tot}}$.

To convert from amplitude to intensity we must use the following relation: $I = \frac{A^2}{2}$.

This implies that $A^2 = 2I$ and $A = \sqrt{2I}$. Now use those two equations to substitute for A^2 and A.