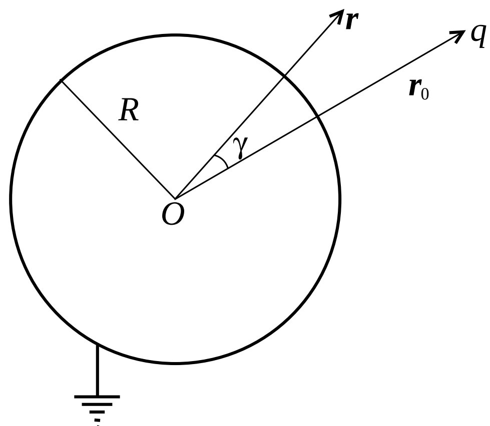
Q2:



A conducting sphere of radius R and center at $\mathbf{r} = 0$ is held at ground potential ($\phi = 0$). A point charge q is located at a point $\mathbf{r}_0$ outside the sphere. A student wishes to find the potential at all points outside the sphere. They remember that this problem can be solved by placing an image charge at the point $\mathbf{r}_1 = (R^2/|\mathbf{r}_0|)\hat{\mathbf{r}}_0$, where $\hat{\mathbf{r}}_0$ is a unit vector in the direction of $\mathbf{r}_0$, but do *not* recall what the charge q' of the image charge should be.

a) Write down the potential due to both the charge q and the image charge q' as a function $\phi(r, \gamma)$ of $r \equiv |\mathbf{r}|$ and the angle γ shown in the figure.

b) Use your result for part (a) to find what q' must be to make the surface of the sphere have potential $\phi(r = R, \gamma) = 0$.

c) Use your result for parts (a) and (b) to determine the surface charge density $\sigma(\gamma)$ on the sphere.

(If you did not do parts (a) and (b) you can get partial credit for part (c) by expressing your answer in terms of a general $\phi(r, \gamma)$.)

d) Integrate your $\sigma(\gamma)$ from part (c) to compute the total surface charge induced on the sphere. Is it equal to q'? If not explain why.

$$\textbf{May be useful}: \int \frac{1}{(a+x)^{3/2}} dx = -\frac{2}{\sqrt{a+x}} + \text{constant}.$$