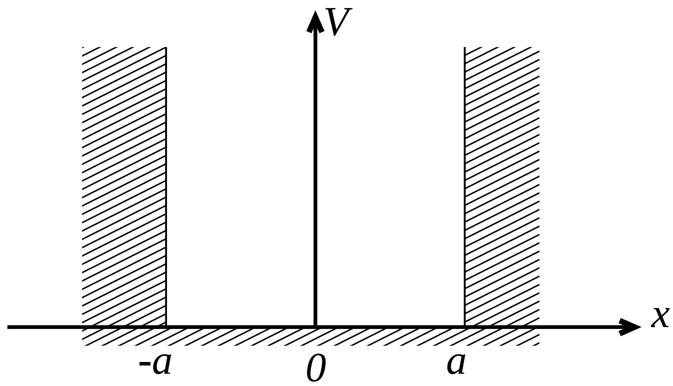
Q1: Consider the quantum mechanics of a particle of mass m confined in a one-dimensional potential well of width $2a$ and with infinitely-high walls — as shown in the figure:



a) Copy the figure into your answer book and sketch the wavefunctions $\psi(x)$ corresponding to the three lowest energy states.

b) Write down the Schrödinger equation for the system, state the appropriate boundary conditions, and find all possible energy eigenvalues and eigenfunctions.

Now we add a delta-function perturbation $V(x) = \lambda\delta(x)$ to the center of the potential well.

c) To lowest order in λ find the shift in each allowed energy E_n, and state the condition for this change to be nonzero.

d) Perturbation theory will be useful when λ is small — but small compared to *what combination* of other parameters in the problem?

e) Assume now that λ is no longer small, so we cannot use perturbation theory. Write down the conditions that $\psi(x)$ must obey at $x = 0$, and hence derive a transcendental equation whose solutions will give the energy levels.

f) Show that when λ is small your transcendental equation recovers your results from part (c).