Q4: Model a binary alloy as a square lattice with $2n$ sites. There are n type-A sites and n type-B sites. These are occupied by $2n$ classical particles: n of type a, and n of type b. The a particles are indistinguishable from each other, as are the b particles. Let m be the number of type-B sites that are filled by a particles (Thus m must also be the number of type-A sites filled by b particles). Set $c = m/n$.

a) For fixed c, and assuming initially that all configurations at fixed c are equally likely, calculate the entropy $S(c)$ of the system for large values of n. (Note that the *sites* are distinguishable.) Use Stirling's approximation $\ln(n!) \approx n \ln n - n$ to reduce your expression to one with no factorials.

b) Now suppose that there is a positive energy cost ε for an a particle to be on a type-B site or a b particle to be on a type-A site. Find the total energy cost $E(c)$ of such a configuration.

c) In thermal equilibrium at a temperature $T \equiv 1/\beta$, write the probability $p(c)$ of such a configuration. Your formula for the probability does not need to be in closed form. You may find it helpful to write down the partition function.

d) Write a formula $F(c)$ for the free energy. Your formula will also depend on temperature and should not have any factorials in it.

e) Find a formula for the equilibrium value of c at temperature T.