Q4: A two dimensional electron gas, or 2DEG, is a system in which non-interacting, non-relativistic, spin-1/2 electrons are tightly trapped at the interface between two different materials such GaAs and GaAlAs. Their motion can be treated as being in strictly in two dimensions.

- a) Write down or derive the energy density-of-states $D(\varepsilon)$ (i.e. the number of quantum states per unit energy, per unit volume) for the electron gas system. Express your answer in terms of the electron's energy ε , the electron mass $m_{\rm e}$, \hbar , and any other relevant quantity. Do not forget to include the degeneracy factor arising from the spin.
- b) If the density of the of electrons is n per unit area, what is the zero-temperature $Fermi\ energy$ of the gas?
- c) What is the numerical value of the Fermi energy (expressed in eV) and the corresponding Fermi temperature (in degrees Kelvin) of a 2DEG gas consisting of $N = 10^{13}$ electrons confined to an area of 1 cm²?

Hint: Recall that there is a table of physical constants at the beginning of the exam.